

Exam B (Part II)**Name**

Important: Show your work in the spaces provided. (Answers alone are not enough.) For maximum credit present your solutions in a well organized, neat, and legible way. Calculators may be used *only* in elementary computational mode and trig mode, but not in calculus mode (*even for explorations*).

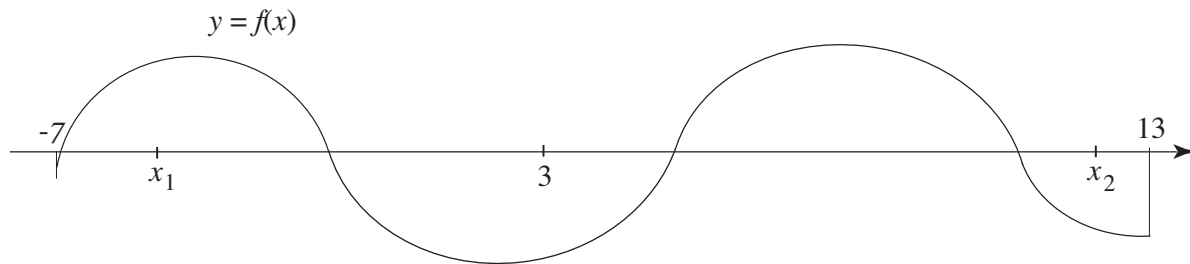
1. Let $y = f(x) = x^4 + 8x^3 + 10x^2$. Find all the critical numbers for this function and determine the intervals over which its graph is increasing or decreasing.

critical numbers:

intervals of increase:

and decrease:

2. The figure below shows the graph of a continuous function $y = f(x)$ defined on the interval $[-7, 13]$. This function has exactly one antiderivative $y = F(x)$ that satisfies $F(3) = 0$. Define this function $y = F(x)$. To do so consider the two typical points x_1 and x_2 and use the figure to explain what $F(x_1)$ and $F(x_2)$ are equal to.



$F(x_1) =$

$F(x_2) =$

3. The sum $(1)^{\frac{1}{3}} \cdot \frac{1}{1000} + \left(1 + \frac{1}{1000}\right)^{\frac{1}{3}} \frac{1}{1000} + \left(1 + \frac{2}{1000}\right)^{\frac{1}{3}} \frac{1}{1000} + \cdots + \left(7 + \frac{999}{1000}\right)^{\frac{1}{3}} \frac{1}{1000}$ follows the pattern that the first three terms and the last term establish. Consider the sum involved in the definition of $\int_a^b f(x) dx$. For what n , $a = x_0, x_1, \dots, x_{n-1}, x_n = b$, and $y = f(x)$ is the sum above of this form? Find a number that closely approximates the value of the sum above.

the number =

4. Evaluate the integral $\int_{-1}^2 (-x^3 + \cos(\pi x)) dx$.

$$\int_{-1}^2 (-x^3 + \cos(\pi x)) dx =$$

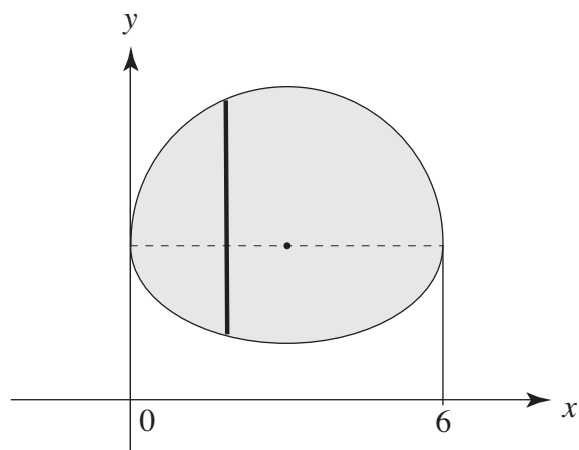
5. Consider the definite integral $\int_1^3 \sqrt{1+x} dx$.

i. For what function $f(x)$ does this definite integral represent an area under the graph of $f(x)$.

ii. Find a function $g(x)$ with the property that this definite integral represents the length graph of $f(x)$.

iii. Find a function $h(x)$ with the property that the integral represents the volume obtained by rotating a region under the graph of $h(x)$ one revolution about the x -axis; and

6. The figure below shows the upper half of a circle of radius 3 and the lower half of an ellipse with semimajor axis 3 and semiminor axis 2. Both the circle and the ellipse have center $(3, 3)$.



a. Find the area between the circle and the ellipse (the area of the shaded region). [Best without integrals.]

area =

b. Express the volume obtained by revolving the shaded region one revolution around the x -axis as a definite integral.

$V_x = \int$

- c. Express the volume obtained by revolving the shaded region one revolution around the y -axis as a definite integral.

$$V_y = \int$$

- d. Why is the vertical strip a better choice than a horizontal strip for the above two problems?

- e. Express the surface area S of the solid obtained by rotating the shaded region one revolution around the x -axis as a sum of two definite integrals.

$$S = \int + \int$$

7. Consider the hyperbolic functions $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$.

- a. Show that $(\cosh x)^2 - (\sinh x)^2 = 1$.

- b. Show that the derivative of $\sinh x$ is $\cosh x$, and that the derivative of $\cosh x$ is $\sinh x$.

- c. Compute the length of the graph of $y = \cosh x$ between the points with x -coordinates 0 and $\ln 7$.

this length is:

8. The Golden Gate Bridge has a center span of 4,200 feet and a total length of 6,450 feet. It has two main cables, one deck, a dead load of 21,300 pounds per foot, and a live load capacity of 4,000 pounds per foot. The towers have a height of 746 feet and the sag in each of the two main cables is 470 feet.

- i. Compute the maximal and minimal tensions in each main cable over the center span.

- ii. Compute the angle that the cable makes with the horizontal at the tower .

- ii. Compute the compression that a cable over the center span produces in the tower.

Formulas and expressions:

$$\int u \, dv = uv - \int v \, du \quad Ay'' + By' + Cy = 0 \quad y = D_1 e^{r_1 x} + D_2 e^{r_2 x} \quad y = D_1 e^{2x} + D_2 x e^{2x}$$

$$y = e^{ax}(D_1 \cos bx + D_2 \sin bx) \quad x = r \cos \theta \quad y = r \sin \theta$$

$$a = \frac{d}{1-\varepsilon^2} \quad b = \frac{d}{\sqrt{1-\varepsilon^2}} \quad a = \frac{d}{\varepsilon^2-1} \quad b = \frac{d}{\sqrt{\varepsilon^2-1}} \quad f'(\theta) = f(\theta) \cdot \tan(\gamma - \frac{\pi}{2})$$

$$f(x) = \frac{s}{d^2} x^2, \quad \tan \alpha = \frac{2s}{d}, \quad T(x) = w \sqrt{\left(\frac{d^2}{2s}\right)^2 + x^2}, \quad T_d = wd \sqrt{\left(\frac{d}{2s}\right)^2 + 1}, \quad T_0 = \frac{wd^2}{2s}.$$

$$ab\pi \quad a^2 = b^2 + c^2 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \kappa = \frac{ab\pi}{T} \quad F = ma \quad f(t) \cdot r = I \cdot \alpha(t) \quad \frac{d}{dx} \sin x = \cos x$$

$$\int_a^b \sqrt{1+f'(x)^2} \, dx \quad \int_a^b \pi f(x)^2 \, dx \quad 2\pi \int_a^b f(x) \sqrt{1+f'(x)^2} \, dx$$