

Please remember: You may use your calculator only in elementary and trig modes but not in calculus mode.

- Of the numbers listed below, the one that approximates the area of the region bounded by the graph of $y = \ln x$, the x -axis, and the lines $x = 1$ and $x = 5$ best is:
A. 2.5 B. 3 C. 3.5 D. 4 E. 4.5
- The value of the integral $\int_0^{\frac{\pi}{2}} x (\sin \frac{x}{2}) dx$ is:
A. $2\sqrt{2} - \pi$ B. $2\sqrt{2} - \frac{\pi}{2}\sqrt{2}$ C. $\sqrt{2} - \sqrt{2}\pi$ D. $2\sqrt{2} - \sqrt{2}\pi$ E. $\frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{2}\pi$
- Let $y = f(x)$ be a solution of the differential equation $x \frac{dy}{dx} = 2y + x^3 \cos x$ (where $x > 0$) that satisfies $f(\pi) = 1$. Then $f(\frac{\pi}{2})$ is equal to:
A. $\frac{1}{4} - (\frac{2}{\pi})^2$ B. $\frac{2}{\pi} + \pi$ C. $\frac{1}{\pi} + 2$ D. $1 - (\frac{2}{\pi})^2$ E. $(\frac{\pi}{2})^2 + \frac{1}{4}$
- Let $y = f(x)$ be a solution of the differential equation $y \frac{dy}{dx} = xe^{x^2}$ that satisfies $f(x) > 0$ and $f(0) = 0$. Then $f(1)$ is equal to:
A. $\sqrt{e-1}$ B. $\sqrt{e+1}$ C. $\sqrt{e^2-1}$ D. $\sqrt{2e^2+1}$ E. $\sqrt{e^2+1}$
- Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 20y = 0$. Then find the particular solution $y = f(x)$ that satisfies $f(0) = 0$ and $f'(0) = 4$. The value of $f(\frac{\pi}{8})$ is
A. $2e^{\frac{\pi}{4}}$ B. $2e^{\frac{\pi}{4}}(1 + \frac{\sqrt{2}}{2})$ C. $e^{\frac{\pi}{4}}$ D. $3e^{\frac{\pi}{4}}(1 + \frac{\sqrt{2}}{2})$ E. $e^{\frac{\pi}{2}}$
- You are given a complex plane with a polar coordinate system. Take the numbers $c_1 = (2, \frac{1}{4}\pi)$, $c_2 = (5, \frac{9}{4}\pi)$, and $c_3 = (-5, \pi)$. The number $c_1c_2 + c_3$ is equal to
A. $-5 + 10i$ B. 10 C. $10 - 5i$ D. $10 + 5i$ E. $5 + 10i$
- You are given a plane with a polar coordinate system and the equation $r = \frac{5}{1+3\cos\theta}$. Because $3 > 1$ its graph is a hyperbola. The box in the polar plane that determines the graph of this hyperbola has center (in polar coordinates) and dimensions given respectively by:
A. $(0, \frac{15}{8}); \frac{5}{8} \times \frac{5}{\sqrt{8}}$ B. $(\frac{5}{8}, 0); \frac{5}{8} \times \frac{5}{\sqrt{8}}$ C. $(-\frac{15}{8}, 0); \frac{5}{8} \times \frac{5}{\sqrt{8}}$ D. $(\frac{15}{8}, 0); \frac{5}{8} \times \frac{5}{8}$ E. $(\frac{15}{8}, 0); \frac{5}{8} \times \frac{5}{\sqrt{8}}$

Problems 8 and 9 deal with a polar plane and the spiral $r = f(\theta) = \theta^{\frac{3}{2}}$ where $\theta \geq 0$. Any angle θ determines a ray in the polar plane starting at the origin O . For $\theta = 0$, let P be the origin O . For any $\theta > 0$ let $P \neq O$ be the first point of intersection of the ray and the spiral.

- Rotate the ray from $\theta = 0$ to $\theta = 2\pi$. The part of the area that the segment OP traces out that falls outside the circle of radius 1 is equal to:
A. $2\pi^4$ B. $2\pi^4 - \pi$ C. $2\pi^4 - \pi + \frac{1}{2}$ D. $2\pi^4 - \pi + \frac{3}{8}$ E. $2\pi^4 - \pi^2$
- As the segment OP rotates from $\theta = \frac{\pi}{2}$ to $\theta = 2\pi$, the positive angle α that the tangent to the spiral at P makes with OP :

- A. increases by about 30° B. decreases by about 15° C. decreases by about 45°
D. increases by about 45° E. decreases by about 30°

Problems 10 and 11 deal with the graph of the polar equation $r = f(\theta) = \frac{6}{3 \sin \theta + 2 \cos \theta}$.

10. The integral $\int_0^{\frac{3\pi}{4}} \frac{1}{2} f(\theta)^2 d\theta$ is equal to

A. 6 B. 7 C. 8 D. 9 E. 10

11. The integral $\int_0^{\frac{3\pi}{4}} \sqrt{f(\theta)^2 + f'(\theta)^2} d\theta$

A. $\sqrt{113}$ B. $\sqrt{115}$ C. $\sqrt{117}$ D. $\sqrt{119}$ E. $\sqrt{121} = 11$

$$\begin{array}{l}
\text{Formulas and expressions: } \int u \, dv = uv - \int v \, du \quad Ay'' + By' + Cy = 0 \quad y = D_1 e^{r_1 x} + D_2 e^{r_2 x} \\
y = D_1 e^{2x} + D_2 x e^{2x} \quad y = e^{ax} (D_1 \cos bx + D_2 \sin bx) \quad e^{i\theta} = \cos \theta + i \sin \theta \quad x = r \cos \theta \quad y = r \sin \theta \\
a = \frac{d}{1-\varepsilon^2} \quad b = \frac{d}{\sqrt{1-\varepsilon^2}} \quad a = \frac{d}{\varepsilon^2-1} \quad b = \frac{d}{\sqrt{\varepsilon^2-1}} \quad f'(\theta) = f(\theta) \cdot \tan(\alpha - \frac{\pi}{2}) \quad \int_a^b \sqrt{f(\theta)^2 + f'(\theta)^2} \, d\theta \\
\frac{1}{2} r^2 \theta \quad \int_a^b \frac{1}{2} f(\theta)^2 \, d\theta \quad \frac{d\theta}{dt} = \frac{2\kappa}{r^2} \quad 2 \frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2} = 0
\end{array}$$