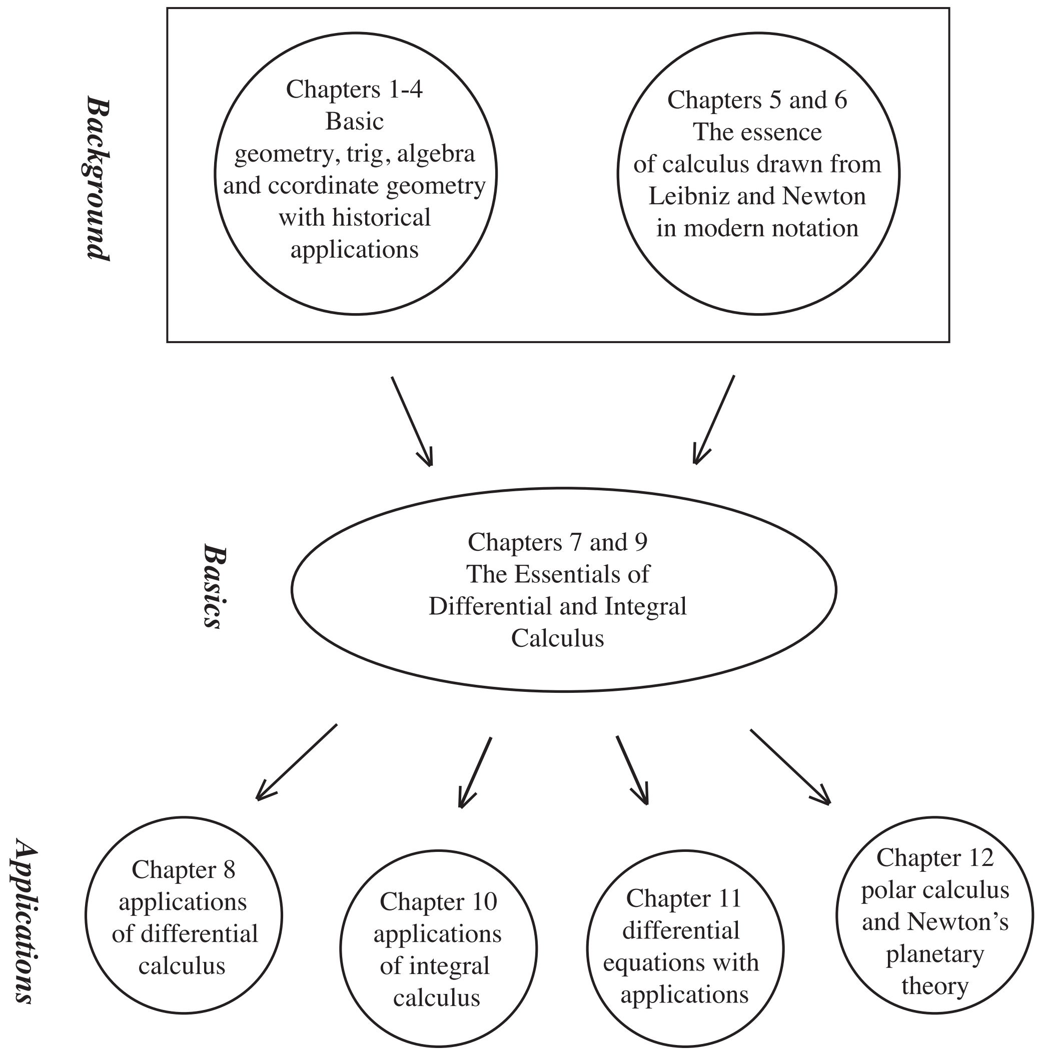
Overview of the Content of the Text

The first four chapters present the basics of standard geometry, trigonometry, algebra, and coordinate geometry. This discussion is framed by the contributions to geometry and astronomy from ancient Greece (Archimedes and Ptolemy, for instance) to the scientific revolution (including those of Copernicus, Galileo, Kepler, and Descartes). Chapter 5 draws a brief, basic calculus course from the pages of Leibniz and Chapter 6 adds fundamental insights and applications from the work of Newton. With its extensive Problems and Projects sections, Part I of the text is a complete essential calculus and the precalculus preliminaries in 270 pages. The subject is framed by its historical context, but the notation and the selection of



topics is current. If calculus and precalculus are the primary concerns of a course, then Chapter 2 with its focus on area computations of Archimedes and Chapter 3 with its goal of developing the Sun-centered model of the motion of the planets can be skipped. If included, these two chapters show Greek geometry and trigonometry in action. The scientific revolution reached its culmination in the latter part of the 17th century with the publication of

Newton's *Principia* and its explanation of the planetary orbits with a combination of fundamental laws of motion and force and the strategies of calculus. A discussion of the central aspects of this challenging work—it too can be omitted if the target is calculus—concludes the first part of the text.

Part II of the text starts by engaging calculus again, this time much more fully and deeply. Chapter 7 consists of a comprehensive treatment of the essentials of differential calculus, and Chapter 9 does the same for integral calculus. All basic aspects of calculus are dealt with in 107 pages plus 32 pages of problems. A course may well start with Part II and use the discussions of Part I—especially Chapters 1 and 4—for reference and review. After the presentation of the basics of calculus, Part II turns its attention to applications. At this point, an instructor's strategy vis-à-vis the topics presented in this text should shift more into a "pick and choose" mode. If the power of calculus to inform is to be illustrated convincingly and beyond such trivial (or contrived) situations as "the farmer and the fence," then explanations of some of the very basic aspects of science and engineering that calculus informs need to be included. The fact is that the explanation of such central concepts as acceleration, force, momentum, torque, moments of inertia, and the properties of lenses require core ideas of calculus. Chapters 8 and 10 apply calculus (differential in the first case and integral in the second) to mechanics (in particular to statics and dynamics), optics, architecture, the geometry of the pseudosphere, planetary motion, and ballistics. Chapter 11 provides the fundamentals of differential equations and then turns to applications. The focus is on first- and second-order differential equations, complex numbers, power series, and applications to motion with air resistance and the action of shock absorbers. Chapter 12 starts with a study of polar functions and then completes the agenda of the text by returning to Newton's orbital mechanics and applying the calculus of polar functions to reach its important conclusions. The various applied segments of the text do not build on each other but are independent. Whether they are selected or not is entirely up to the intentions and preferences of the instructor. The number of pages involved in Part II gives some sense of the way the material is emphasized. The applications of calculus in Chapters 8, 10, 11, and 12 take up 200 pages and their Problems and Projects sections—many of which are study segments—about 70 pages more.

Expressed with the metaphor of language, this book combines the vocabulary and basic grammar of calculus with much of the poetry and the stories that have been written with it. The notation used in the text is modern throughout, the mathematics is current and relevant, and the wide-ranging Problem and Projects segments provide the student with skill-sharpening opportunities. An instructor should exercise careful judgment when assigning the problems, as the level of difficulty varies.

How this Text Can be Used

In the hands of an engaged instructor, these materials and their organization are suitable for two-semester calculus courses of different degrees of difficulty and intensity and a wide range of student involvement. A course can be more elementary—for example, by starting with and incorporating discussions from Part I—or more advanced—for instance, by beginning with the more theoretical approach to calculus of Part II and then turning to the applications in the later chapters.

I have taught the topics of this text at the University of Notre Dame for many years (to a large extent from my earlier book *Basic Calculus: From Archimedes to Newton to Its Role in Science* [Springer-Verlag, 1998]). My approach was to build the material from the ground up in standard lecture format and to provide students with lots of opportunities to engage questions and to solve (mostly routine) problems on the board. The students responded well to this approach and learned a lot. The fact that students are increasingly exposed to calculus in high school moved me recently to use a much different strategy. In the spirit of the *flipped classroom* and *inquiry-based learning*, I split the class into teams (of three, sometimes two) and handed each team a "packet" with an applied topic from Chapters 8 and 10. The assignment for each team was to "reverse engineer" the material of the packet (mostly outside of class). The students were to begin by identifying, reviewing, studying, and learning the fundamental aspects of calculus that their topic involved from earlier versions of Chapters 7 and 9. They would then expand their investigations to relevant background materials and to the mathematical particulars. I assisted their efforts during office hours and with clarifying lectures in class. After a final "go through" with me, the student teams presented their topics to the class (typically taking two or three class periods). The students were first-year Notre Dame honors program students and hence among the best at Notre Dame, but their interests and intended majors were primarily in arts and letters disciplines. The results were positive for this small class of about 30. Students gained an understanding and a sense of ownership of strategies of calculus and an appreciation of its applications within meaningful contexts. They stepped to the board confidently and, often with PowerPoint assistance, presented cogent explanations. Briefly put, this was a flipped classroom based on the text (rather than on instructor-selected mini-videos).

Letting students dive in, select a topic, wrestle with it, and prepare it for presentation—with the instructor providing both guidance during office hours and clarifying comments in class—can be a very profitable approach. Does it not have to be one of the primary aims of a college education to enable students to engage sophisticated written materials, to extract the essence of the matter, and, finally, to present their findings articulately in front of an audience?

The Role of Technology

This text's emphasis is on the mathematics and not on computation and related technology. In this regard, the text makes use of a number of the websites with powerful and wide ranging computational capacity. Internet searches will display many more examples of such websites.

`http://web2.0calc.com` (a good basic calculator with memory)

`http://www.desmos.com/calculator` (a good graphing calculator) `http://www.integral-calculator.com` (computes definite and indefinite integrals)

`http://www.emathhelp.net/calculators/calculus-2/trapezoidal-rule-calculator/` (computes trapezoidal approximations)

`http://www.emathhelp.net/calculators/calculus-2/simpsons-rule-calculator/` (computes Simpson approximations)

`https://www.desmos.com/calculator/hmmutc9ija` (computes Taylor polynomials)

`https://www.symbolab.com/solver/ordinary-differential-equation-calculator` (solves differential equations)

`http://www.math-cs.gordon.edu/~senning/desolver/` (applies Euler's method)

`https://www.desmos.com/calculator/ms3eghkkgz` (polar graphing calculator)

With a few exceptions, the units used throughout the book are the standard metric units—meter, kilogram, and second (MKS)—and units, such as the newton, that are derived from them. The most important exception involves the discussion about the American suspension bridges, including the Brooklyn Bridge, the George Washington Bridge, and the Golden Gate Bridge. They are discussed as they were built—in feet and pounds.