

Math 20580
Practice Midterm 2
March 5, 2015

Name: _____
Instructor: _____
Section: _____

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an \times through one letter for each problem on this answer sheet.

Sign the pledge. “On my honor, I have neither given nor received unauthorized aid on this Exam”:

1. a b c d e

2. a b c d e

3. a b c d e

4. a b c d e

5. a b c d e

6. a b c d e

7. a b c d e

8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Let V be the vector space of all functions $f(x)$ where $f : \mathbb{R} \rightarrow \mathbb{R}$.

Which of the following are subspaces of V ?

A. the constant functions;

B. functions with $\lim_{x \rightarrow \infty} f(x) = 3$;

C. functions with $f(1) = 1$;

D. functions with $f(0) = 0$.

- (a) A, B, C and D (b) A, B and C only (c) B, C and D only
(d) B and D only (e) A and D only.

2. Let \mathbb{P}_2 be the vector space of polynomials of degree at most 2, and let \mathcal{B} be the basis

$$\mathcal{B} = \{1, 1 + x, 1 + x^2\}.$$

Find the \mathcal{B} -coordinates $[p]_{\mathcal{B}}$ of the polynomial $p(x) = (1 - x)^2$.

- (a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

3. Let H be a subspace of a vector space V , and suppose that V has dimension d . Which of the following statements are true?

- A. $\dim(H) \leq \dim(V)$;
- B. a linearly independent set of vectors in H is also linearly independent in V ;
- C. d vectors which span V will be linearly independent;
- D. d vectors which span H will also span V .

- (a) A, B, C and D (b) A, B and C only (c) B, C and D only
(d) B and D only (e) A and D only.

4. A linear transformation

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

has outputs $T \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Find $T \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

- (a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4/5 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

5. The vector $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is an eigenvector for the matrix

$$\begin{bmatrix} 2 & 5 & 1 \\ 1 & 7 & -1 \\ 1 & 0 & 2 \end{bmatrix}$$

What is the corresponding eigenvalue?

- (a) 3 (b) 1 (c) 0 (d) -1 (e) 2

6. What are the eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ -2 & -1 \end{bmatrix}$?

- (a) 0, 1 (b) 5, -1 (c) 1, 3 (d) 0, 2 (e) -1, -3

7. Suppose A is a 3×3 matrix, that has $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ as an eigenvector with eigenvalue 2, and

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ as an eigenvector with eigenvalue -1 . Compute $A^3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

- (a) $\begin{bmatrix} 8 \\ 16 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 8 \\ 16 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$
(e) cannot be computed from the given information

8. Let P_3 be the vector space of polynomials of degree at most 3. Find the dimension of the subspace of P_3 spanned by $1 + x^2$, $x + 2x^2 + x^3$ and $1 + x + x^3$.

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

Part II: Partial credit questions (11 points each). Show your work.

9. Find a basis for the Row space, $\text{Row}(A)$, of the matrix

$$A = \begin{bmatrix} 0 & 1 & 3 & 2 \\ 1 & 0 & 2 & 3 \\ 1 & 1 & -5 & 6 \\ 1 & -1 & -1 & 2 \end{bmatrix}.$$

10. Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\}$ be two bases of \mathbb{R}^2 .

Find the change of coordinate matrix $\mathcal{P}_{\mathcal{C} \leftarrow \mathcal{B}}$ sending a \mathcal{B} coordinate vector $[\vec{x}]_{\mathcal{B}}$ to the \mathcal{C} coordinate vector $[\vec{x}]_{\mathcal{C}}$.

11. Is the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. If A is not diagonalizable, explain why not.

(Hint: The eigenvalues of A are 1 and 3.)

12. Let P_3 be the vector space of polynomials of degree at most 3 and P_2 be the space of polynomials of degree at most 2.

Consider the linear transformation

$$T : P_3 \rightarrow P_2$$

given by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_3x^2$.

(a) Write down bases \mathcal{B}_3 and \mathcal{B}_2 of P_3 and P_2 respectively.

(b) Find the matrix of T relative to the bases \mathcal{B}_3 and \mathcal{B}_2 .

