Name: **Version #1**

Instructor: Bullwinkle

**Exam I**
**September 24, 2019**

- The Honor Code *is* in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 11 pages *of* the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

| | | | | | |
|---|---|---|---|---|---|
| 1. | (•) | (b) | (c) | (d) | (e) |
| 2. | (•) | (b) | (c) | (d) | (e) |
| 3. | (•) | (b) | (c) | (d) | (e) |
| 4. | (•) | (b) | (c) | (d) | (e) |
| 5. | (•) | (b) | (c) | (d) | (e) |
| 6. | (•) | (b) | (c) | (d) | (e) |
| 7. | (•) | (b) | (c) | (d) | (e) |
| 8. | (•) | (b) | (c) | (d) | (e) |
| 9. | (•) | (b) | (c) | (d) | (e) |
| 10. | (•) | (b) | (c) | (d) | (e) |

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|---|---|
| **Multiple Choice** | |
| 11. | |
| 12. | |
| 13. | |
| Total | |

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|---|---|---|---|---|---|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4 | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

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| **Multiple Choice** | ________ |
| 11. | ________ |
| 12. | ________ |
| 13. | ________ |
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## Multiple Choice

**1.**(5pts) Suppose $A$ is a $3 \times 5$ matrix and we know that the reduced row echelon form of $A$ has exactly one row of zeroes. What is the dimension of the null space of $A$?

(a) 3 (b) 1 (c) 2 (d) 4 (e) 5

**Solution.** By assumption, the RREF of $A$ has $2 = 3 - 1$ pivots, so there are two basic variables and 3 free variable rows, which means that the dimension of the null space is 3. Alternatively, the two pivots give the rank of $A$ equal to 2. By the rank-nullity theorem, the dimension of the null space of $A$ equals $5 - 2 = 3$.

**2.**(5pts) Let $A$ be a $3 \times 3$ square matrix. Suppose that the linear system $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions. Which of the following statements **must** be true, given what we know about $A$?

(a) The homogeneous linear system $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution $\mathbf{x} \neq \mathbf{0}$.

(b) The linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(\mathbf{x}) = A\mathbf{x}$ is onto.

(c) There is an $3 \times 3$-matrix $B$ with $AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(d) The linear system $A\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ has a unique solution.

(e) The rank of $A$ equals 3.

**Solution.** Since $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions, $A$ is not invertible and so (a) is true while the others are not.

**3.**(5pts) Let $\mathcal{B}$ denote the basis of $\mathbb{R}^3$ given by $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$ and let $\mathbf{v}$ denote the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$. The coordinate vector $[\mathbf{v}]_{\mathcal{B}}$ of $\mathbf{v}$ with respect to $\mathcal{B}$ is $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Which of the following is the value of $b$?

(a) $-2$ (b) $2$ (c) $1$ (d) $\frac{1}{2}$ (e) $-1$

**Solution.** Let us consider a generic vector $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and plug in the given values at the end of the computation. We have to solve the linear system with augmented matrix the first matrix in: $\begin{bmatrix} 1 & 0 & 1 & x \\ 1 & 1 & -1 & y \\ 1 & 2 & 0 & z \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & x \\ 0 & 1 & -2 & y-x \\ 0 & 2 & -1 & z-x \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & x \\ 0 & 1 & -2 & y-x \\ 0 & 0 & 3 & x-2y+z \end{bmatrix} \sim$ $\begin{bmatrix} 1 & 0 & 1 & x \\ 0 & 1 & -2 & y-x \\ 0 & 0 & 1 & (x-2y+z)/3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & (2/3)x+(2/3)y-(1/3)z \\ 0 & 1 & 0 & (-1/3)x-(1/3)y+(2/3)z \\ 0 & 0 & 1 & (x-2y+z)/3 \end{bmatrix}$. If we let $x = 1, y = 1, z = -2$, the coordinate vector is $\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$ so $b = -2$.

**4.**(5pts) Find the inverse of the matrix $\begin{bmatrix} -6 & -11 \\ 2 & 4 \end{bmatrix}$.

(a) $\begin{bmatrix} -2 & -11/2 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -1 \\ 11/2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -6 & 2 \\ -11 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 11 \\ -2 & -6 \end{bmatrix}$ (e) $\begin{bmatrix} 3 & 11/2 \\ -1 & -2 \end{bmatrix}$

**Solution.** The inverse of a $2 \times 2$ matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by $\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. This yields (a) as the answer.

4.

**5.**(5pts) Find the reduced echelon form of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 & -3 \\ 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 2 & 2 & 3 & -8 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 2 & 0 & 2 & 0 & -2 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 1 & -3 \\ 0 & 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 3 & 1 & 4 & -9 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
(e) $\begin{bmatrix} 0 & 1 & -1 & 0 & 1 \\ 1 & 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

**Solution.** $\begin{bmatrix} 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 2 & 2 & 3 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & -3 \\ 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 2 & 2 & 3 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & -3 \\ 0 & 1 & -1 & 2 & -3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & -1 & 2 & -3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ where $\sim$ denotes row equivalence.

5.

**6.**(5pts) Give a parametrization for the points in the solution set for $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 0 & 0 & 2 & 4 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 6 \\ -2 \\ 2 \end{bmatrix}?$$

(a) $\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

(b) $\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

(c) $\mathbf{x} = \begin{bmatrix} 6 \\ -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

(d) $\mathbf{x} = \begin{bmatrix} 6 \\ -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

(e) $\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

**Solution.** The reduced echelon form of the augmented matrix for this problem is $\begin{bmatrix} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 & 1 & 1 \end{bmatrix}$.
Changing this back into equations and solving for the pivot variables we get $x_1 = 1 + 2x_4$, $x_2 = 4 - x_4$, and $x_3 = 1 - 2x_4 - x_5$, and that $x_4$ and $x_5$ are free.

6.

**7.**(5pts) Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$. Which of the following sets forms a basis for Col $A$?

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

**Solution.** The answer is a).

**8.**(5pts) Determine which of the following sets of vectors is linearly independent.

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -4 \end{bmatrix}, \right\}$ (b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ -6 \\ -8 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 7 \end{bmatrix} \right\}$

**Solution.** Putting the vectors in (a) into a matrix as columns gives us a matrix already in echelon form, there are 3 pivots and 3 columns so these columns are linearly independent.

(b) Not linearly independent; $\mathbf{v}_2 = -2\mathbf{v}_1$.

(c) Not linearly independent; $\mathbf{v}_3 = 2\mathbf{v}_1 + \mathbf{v}_2$

(d) Not linearly independent; contains the zero vector.

(e) Three vectors in $\mathbb{R}^2$ must be linearly dependent.

7.

**9.**(5pts) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by

$$T(x_1, x_2, x_3) = (2x_1 + 3x_3, -x_2 + 5x_3, x_1 - 2x_2 + 4x_3).$$

Which of the following matrices is the standard matrix of $T$?

(a) $\begin{bmatrix} 2 & 0 & 3 \\ 0 & -1 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & -2 \\ 3 & 5 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 & 0 \\ -1 & 5 & 0 \\ 1 & -2 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 2 & 3 \\ 5 & -1 & 0 \\ 1 & -2 & 4 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 5 \\ 1 & -2 & 4 \end{bmatrix}$

**Solution.** The first column of the standard matrix of $T$ should be $T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, the second column should be $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$, and the third column should be $T(\mathbf{e}_3) = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$.

**10.**(5pts) Which of the following matrices can be the standard matrix for a one-to-one linear transformation?

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

**Solution.** The matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ is the only one where the number of pivots equals the number of columns, so it is the only one representing a one-to-one linear transformation.

**Partial Credit**

**11.**(14pts) Let $A = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 2 & 1 & 0 & 1 & 2 \\ -1 & 0 & 1 & 0 & 1 \end{bmatrix}$ be a $3 \times 5$ matrix.

(1) Find a basis for the null space of $A$.

(2) Compute the dimension of the null space of $A$.

(3) Compute the rank of $A$.

9.

**Solution.** Row reduce: $\begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 2 & 1 & 0 & 1 & 2 \\ -1 & 0 & 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 4 \\ 1 & 0 & -1 & 0 & -1 \end{bmatrix} \sim$
$\begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$

We see that the linear system has free variables $x_3$ and $x_5$ and bound variables $x_1$, $x_2$, and $x_4$. We get the equations

$$\begin{aligned} x_1 &= x_3 + x_5 \\ x_2 &= -2x_3 - 3x_5 \\ x_4 &= -x_5 \end{aligned}$$

expressing the bound variables in terms of the free variables. In parametric form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ -3 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad x_2, x_4 \in \mathbb{R},$$

and a basis for the nullspace of $A$ is given by those two vectors. The dimension is also seen to be 2, the number of free variables.

**12.**(14pts) Determine all real numbers $a$ for which the system of linear equations in 3 variables

$$\begin{cases} x_1 & + x_2 & - x_3 & = & 3 \\ -x_1 & & +2x_3 & = & -1 \\ 2x_1 & & +ax_3 & = & 1 \end{cases}$$

has a unique solution.

**Solution.**

$$\begin{bmatrix} 1 & 1 & -1 & 3 \\ -1 & 0 & 2 & -1 \\ 2 & 0 & a & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & a+2 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & a+4 & -1 \end{bmatrix}.$$

Thus, the given linear system has a unique solution for $a \neq -4$ .

11.

**13.**(14pts) Compute the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 3 & 2 & 2 \end{bmatrix}$.

**Solution.** Row-reduce $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & -1 & -1 & -3 & 0 & 1 \end{bmatrix} \sim$ $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -3 & 0 & 1 \\ 0 & 2 & 3 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & -1 & -1 & -3 & 0 & 1 \\ 0 & 0 & 1 & -6 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & -1 & 0 & -9 & 1 & 3 \\ 0 & 0 & 1 & -6 & 1 & 2 \end{bmatrix}$ The inverse is $\begin{bmatrix} -2 & 0 & 1 \\ 9 & -1 & -3 \\ -6 & 1 & 2 \end{bmatrix}$.