Problem Set 2

ECON 40364: Monetary Theory and Policy
Prof. Sims
Spring 2022

Instructions: Please answer all questions to the best of your ability. You may consult with other members of the class, but each student is expected to turn in his or her own assignment. This problem set is due in class on February 10.

1. **Seigniorage Laffer Curve:** Suppose that the demand for real money balances in an economy is given by (omitting time subscripts):

$$m = \exp(-b \times i) \times Y$$

Here Y is real expenditure/income, $b > 0$ is a parameter, i is the nominal interest rate, and m is real money balances. $\exp(x)$ is the exponential operator (also often written e^x). As discussed in class, seigniorage revenue can be approximated by:

$$S = g^M m = g^M \exp(-b \times i) \times Y$$

From the Fisher relationship, we have $i = r + \pi$. Assume that $g^M = \pi$. This means we can write seigniorage as:

$$S = g^M \exp(-b \times (r + g^M)) \times Y$$

(a) Assume that $Y = 1$, $b = 1$, and $r = 2$. In Microsoft Excel, create a grid of possible values of g^M, ranging from 0 to 6 with a space of 0.1 between (i.e. 0, 0.1, 0.2, ...). Calculate S as a function of g^M. Create a plot graphing S (vertical axis) against g^M. Print out and turn in your plot.

(b) Now, consider alternative values of b: 0.4, 0.6, and 1.2. Repeat the exercise from part (a) for each of these values of b. Create *one* graph plotting total seigniorage revenue against g^M for each of these values of b. Turn in this graph. Comment on how the "peak" of the Laffer curve seems to change with b.

(c) Now, analytically (i.e. in terms of parameters, not numbers) for the general case use calculus to derive an expression for the seigniorage-maximizing growth rate of money. Verify that your analytical answer accords with the graphs you produced in parts (a) and (b).

2. **Precautionary Money Demand:** Suppose that a household begins life in period t with 1 unit of income (which is completely exogenous). This household has no need to consume anything in period t, but it will need to consume in period $t+1$. The household can save through two different assets, the proceeds from which may be used to finance its consumption in $t+1$ (the household receives no income in $t+1$ other than from its assets). One of these assets is money, which we will denote M_t. The other asset is risky and we will denote the quantity the household chooses to hold as A_t. There are two possible states of the world which could materialize in period $t+1$. Call these state 1 (probability of occurring $0 \leq p \leq 1$) and

state 2 (probability of occurring of $1-p$). If the household saves one unit of money in period t, it has one unit of money available to consume in period $t+1$ *regardless* of whether state 1 or state 2 materializes (this is the sense in which money is riskless in this model). This is not true for the other asset. In state 1, one unit of this asset generates income of $\frac{1}{2}$ for the household. But in state 2, one unit of this asset generates income of 2 for the household.

The household must choose how much money and how much of the alternative asset to hold in period t. It does so to maximize expected utility:

$$\max_{M_t, A_t} \quad U = p \ln C_{t+1}(1) + (1-p) \ln C_{t+1}(2)$$

The (1) and (2) index the realization of the state. We abstract from any form of utility discounting. The budget constraint facing the household in period t is:

$$M_t + A_t = 1$$

In period t, the household simply decides how to split its one unit of income between holdings of money and the other asset. Consumption in each state of the world in $t+1$ must satisfy:

$$C_{t+1}(1) = M_t + \frac{1}{2} A_t$$
$$C_{t+1}(2) = M_t + 2A_t$$

In other words, consumption in $t+1$ must equal income from asset holdings *in both possible states of the world*. What differs across states is the payout on the risky asset, A_t. Assume that the household cannot hold negative values of either asset, i.e. $M_t \geq 0$ and $A_t \geq 0$ are constraints.

(a) What is the expected (net) return on holding the risky asset expressed as a function of p? What about the expected net return on holding money? At what value of p does money offer a higher expected return than the risky asset?

(b) Use calculus to solve for the optimal quantities of M_t and A_t (as a function of p). Note that you may need to worry about "corner solutions."

(c) Use Microsoft Excel (or a similar program) to create a graph of the optimal level of M_t plotted against p, where p values can range from 0 to 1 with a "step" of 0.01 between entries (i.e. create a column of p values ranging from 0, 0.01, 0.02, ..., 1, and for each entry calculate the optimal M_t).

(d) Next, create a graph of the optimal level of M_t against the expected return on A_t for each of the p values you created in the previous part. What is the relationship between the demand for M_t and the expected return on the alternative asset? Does this make sense?

(e) In a couple of sentences, intuitively explain why there exist values of p where (i) the expected return on A_t exceeds the expected return on M_t yet (ii) the household still desires to hold some M_t.

3. **Monetary Policy Shocks in the Short Run and Long Run:** Consider a simplified version of the AD-AS model. There is only consumption and investment (no government spending or taxes and no net exports). The IS, MP, and AS curves are:

$$\begin{aligned} \text{IS} \quad & Y = \frac{1}{1 - mpc}\bar{A} - \frac{d}{1 - mpc} r \\ \text{MP} \quad & r = \bar{r} + \lambda \pi \\ \text{AS} \quad & \pi = \pi^e + \gamma (Y - Y^P) \end{aligned}$$

In writing these, $\bar{A} = \bar{C} + \bar{I}$ since there is no government spending, taxes, or net exports. I have also abstracted from the inflation shock by setting $\rho = 0$.

(a) *Graphically* show how Y, r, and π react to an increase in $\bar{r}$ in the *short run*.

(b) Now graphically show how Y, r, and π adjust to the increase in $\bar{r}$ in the *long run*.

(c) How does the *nominal interest rate*, i, react to the increase in $\bar{r}$ in the long run?

(d) Explain why the sign of the reaction of the nominal interest rate to the increase in $\bar{r}$ is ambiguous in the short run. Provide some intuition concerning the parameter values in which i is likely to rise in the short run and those in which it is likely to decline after the increase in $\bar{r}$.