New Keynesian Model with Price and Wage Stickiness

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June 19, 2020

This note describes a basic New Keynesian model (no capital and no other real frictions) where both prices and wages are sticky. Rather than doing the setup of Erceg, Henderson, and Levin (2000, *JME*), I do a union setup where I can assume a representative household.

1 Household

Household flow utility is given by:

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi}$$

Flow utility is discounted by β. The budget constraint facing the household, written in nominal terms, is:

$$P_t C_t + B_t \leq MRS_t L_t + R_{t-1} B_{t-1} + DIV_t$$

The household can save via a one period bond with gross nominal interest rate R_t. MRS_t is the nominal remuneration for supply labor to unions. A Lagrangian is:

$$\mathbb{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} + \mu_t \left[MRS_t L_t + R_{t-1} B_{t-1} + DIV_t - P_t C_t - B_t \right] \right\}$$

The first order conditions are:

$$C_t^{-\sigma} = \mu_t P_t$$

$$\psi L_t^{\chi} = \mu_t MRS_t$$

$$\mu_t = \beta R_t \, \mathbb{E}_t \, \mu_{t+1}$$

Re-written in real terms, where $\Pi_t = P_t / P_{t-1}$, we have:

$$\psi L_t^{\chi} = C_t^{-\sigma} mrs_t \tag{1}$$

$$1 = R_t \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1} \tag{2}$$

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \tag{3}$$

$mrs_t = MRS_t/P_t$ is the real remuneration for supply labor. $\Lambda_{t,t+1}$ is the real stochastic discount factor.

2 Labor Markets

There are a continuum of labor unions indexed by $l \in [0, 1]$. They hire labor from the household at MRS_t and sell to a labor packer at $W_t(h)$. The labor packer combines union labor into a final labor input available to firms via a CES technology. In particular:

$$L_{d,t} = \left[\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

Profit maximization yields a demand curve for each union's labor and an aggregate wage index:

$$L_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} L_{d,t}$$

$$W_t^{1-\epsilon_w} = \int_0^1 W_t(l)^{1-\epsilon_w} dl$$

Unions simply repackage labor from the household one-for-one for resale to the packer. Nominal dividends are:

$$DIV_{u,t}(l) = W_t(l) L_t(l) - MRS_t L_t(l)$$

Plugging in the demand function:

$$DIV_{u,t}(l) = W_t(l)^{1-\epsilon_w} W_t^{\epsilon_w} L_{d,t} - MRS_t W_t(l)^{-\epsilon_w} W_t^{\epsilon_w} L_{d,t}$$

Dividing by P_t to put this into real terms:

$$div_{u,t}(l) = W_t(l)^{1-\epsilon_w} W_t^{\epsilon_w} P_t^{-1} L_{d,t} - mrs_t W_t(l)^{-\epsilon_w} W_t^{\epsilon_w} L_{d,t}$$

With probability $1 - \phi_w$, a union can update its wage. The problem for a union given the opportunity to update is to pick $W_t(l)$ to maximize the present discounted value of real dividends, where discounting is by the household's SDF as well as the probability that a price chosen today will be in effect in the future. The problem is:

$$\max_{W_t(l)} \; \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} \left\{ W_t(l)^{1-\epsilon_w} W_{t+j}^{\epsilon_w} P_{t+j}^{-1} L_{d,t+j} - mrs_{t+j} W_t(l)^{-\epsilon_w} W_{t+j}^{\epsilon_w} L_{d,t+j} \right\}$$

The FOC is:

$$(1-\epsilon_w)W_t(l)^{-\epsilon_w}\,\mathbb{E}_t\sum_{j=0}^{\infty}\phi_w^j\Lambda_{t,t+j}W_{t+j}^{\epsilon_w}P_{t+j}^{-1}L_{d,t+j}+\epsilon_w W_t(l)^{-\epsilon_w-1}\,\mathbb{E}_t\sum_{j=0}^{\infty}\phi_w^j\Lambda_{t,t+j}mrs_{t+j}W_{t+j}^{\epsilon_w}L_{d,t+j}=0$$

The reset wage doesn't depend upon l indexes, so I will call the optimal reset wage $W_t^{\#}$. The FOC can be written:

$$W_t^{\#}=\frac{\epsilon_w}{\epsilon_w-1}\frac{\mathbb{E}_t\sum_{j=0}^{\infty}\phi_w^j\Lambda_{t,t+j}mrs_{t+j}W_{t+j}^{\epsilon_w}L_{d,t+j}}{\mathbb{E}_t\sum_{j=0}^{\infty}\phi_w^j\Lambda_{t,t+j}W_{t+j}^{\epsilon_w}P_{t+j}^{-1}L_{d,t+j}}$$

This can be written recursively:

$$W_t^{\#}=\frac{\epsilon_w}{\epsilon_w-1}\frac{F_{1,t}}{F_{2,t}}$$

$$F_{1,t}=mrs_tW_t^{\epsilon_w}L_{d,t}+\phi_w\,\mathbb{E}_t\,\Lambda_{t,t+1}F_{1,t+1}$$

$$F_{2,t}=W_t^{\epsilon_w}P_t^{-1}L_{d,t}+\phi_w\,\mathbb{E}_t\,\Lambda_{t,t+1}F_{2,t+1}$$

Write $F_{1,t}$ and $F_{2,t}$ in terms of real variables by multiplying and dividing by powers of P_t:

$$F_{1,t}=mrs_tw_t^{\epsilon_w}P_t^{\epsilon_w}L_{d,t}+\phi_w\,\mathbb{E}_t\,\Lambda_{t,t+1}F_{1,t+1}$$

$$F_{2,t}=w_t^{\epsilon_w}P_t^{\epsilon_w-1}L_{d,t}+\phi_w\,\mathbb{E}_t\,\Lambda_{t,t+1}F_{2,t+1}$$

Define $f_{1,t}=F_{1,t}/P_t^{\epsilon_w}$ and $f_{2,t}=F_{2,t}/P_t^{\epsilon_w-1}$. We then have:

$$w_t^{\#}=\frac{\epsilon_w}{\epsilon_w-1}\frac{f_{1,t}}{f_{2,t}}\tag{4}$$

$$f_{1,t}=mrs_tw_t^{\epsilon_w}L_{d,t}+\phi_w\,\mathbb{E}_t\,\Lambda_{t,t+1}\Pi_{t+1}^{\epsilon_w}f_{1,t+1}\tag{5}$$

$$f_{2,t}=w_t^{\epsilon_w}L_{d,t}+\phi_w\,\mathbb{E}_t\,\Lambda_{t,t+1}\Pi_{t+1}^{\epsilon_w-1}f_{2,t+1}\tag{6}$$

3 Production

Production is split into three sectors. A representative wholesale firm hires labor from the labor packer and produces output, selling it to a continuum of retail firms at $P_{w,t}$. The retail firms purchase wholesale output at $P_{w,t}$, costly repackage it, and sell it to a competitive final goods firm at $P_t(f)$, where retailers are indexed by $f\in[0,1]$. The final goods firm combines retail output into a final output good.

Retail output is transformed into final output via:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon_p - 1}{\epsilon_p}} df \right]^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

Profit maximization by the final goods firm yields a demand for each retail output and a price index.

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon_p} Y_t$$

$$P_t^{1-\epsilon_p} = \int_0^1 P_t(f)^{1-\epsilon_p} df$$

Retailers costlessly transform wholesale output into retail output. Their nominal dividend is:

$$DIV_{r,t}(f) = P_t(f)Y_t(f) - P_{w,t}Y_t(f)$$

Using the demand function, this is:

$$D_{r,t}(f) = P_t(f)^{1-\epsilon_p} P_t^{\epsilon_p} Y_t - P_{w,t} P_t(f)^{-\epsilon_p} P_t^{\epsilon_p} Y_t$$

Or, in real terms:

$$d_{r,t}(f) = P_t(f)^{1-\epsilon_p} P_t^{\epsilon_p - 1} Y_t - P_{w,t} P_t(f)^{-\epsilon_p} P_t^{\epsilon_p - 1} Y_t$$

Retailers can only adjust their price with probability $1 - \phi_p$. This makes their price-setting problem dynamic, where future real dividends are discounted by the household's stochastic discount factor as well as the probability that a price chosen in period t remains in effect in the future. The price-setting problem is:

$$\max_{P_t(f)} \; \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} \Big\{ P_t(f)^{1-\epsilon_p} P_{t+j}^{\epsilon_p - 1} Y_{t+j} - P_{w,t+j} P_t(f)^{-\epsilon_p} P_{t+j}^{\epsilon_p - 1} Y_{t+j} \Big\}$$

The first order condition is:

$$(1-\epsilon_p) P_t(f)^{-\epsilon_p} \, \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{t+j}^{\epsilon_p - 1} Y_{t+j} + \epsilon_p P_t(f)^{-\epsilon_p - 1} \, \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{w,t+j} P_{t+j}^{\epsilon_p - 1} Y_{t+j} = 0$$

The optimal reset price does not depend on f. Call it $P_t^{\#}$. We can re-write the FOC as:

$$P_t^{\#} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{w,t+j} P_{t+j}^{\epsilon_p - 1} Y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{t+j}^{\epsilon_p - 1} Y_{t+j}}$$

This can be written recursively:

$$P_t^{\#} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{X_{1,t}}{X_{2,t}}$$

$$X_{1,t} = p_{w,t} P_t^{\epsilon_p} Y_t + \phi_p \Lambda_{t,t+1} X_{1,t+1}$$

$$X_{2,t} = P_t^{\epsilon_p - 1} Y_t + \phi_p \Lambda_{t,t+1} X_{2,t+1}$$

Where $p_{w,t} = P_{w,t}/P_t$ and is interpretable as real marginal cost. Define $x_{1,t} = X_{1,t}/P_t^{\epsilon_p}$ and $x_{2,t} = X_{2,t}/P_t^{\epsilon_p - 1}$. We have:

$$x_{1,t} = p_{w,t} Y_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_p} x_{1,t+1} \tag{7}$$

$$x_{2,t} = Y_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_p - 1} x_{2,t+1} \tag{8}$$

$$\Pi_t^{\#} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{x_{1,t}}{x_{2,t}} \tag{9}$$

Where $\Pi_t = P_t/P_{t-1}$ and $\Pi_t^{\#} = P_t^{\#}/P_t$.

The wholesale firm produces output according to:

$$Y_{W,t} = A_t L_{d,t} \tag{10}$$

Its nominal dividend is:

$$DIV_{W,t} = P_{w,t} Y_{W,t} - W_t L_{d,t}$$

The optimality condition is:

$$W_t = P_{w,t} A_t$$

Or, in real terms:

$$w_t = p_{w,t} A_t \tag{11}$$

4 Monetary Policy

Assuming the gross nominal rate, R_t, is set according to a Taylor type rule:

$$\ln R_t = (1 - \rho_R) \ln R + \rho_R \ln R_{t-1} + (1 - \rho_R)\theta_\pi (\ln \Pi_t - \ln \Pi) + s_R \varepsilon_{R,t} \tag{12}$$

Variables without time subscripts denote non-stochastic steady state values.

5 Aggregation

The aggregate inflation rate and real wage evolve according to the following expressions, which can be derived using properties of Calvo pricing:

$$1 = (1-\phi_p)\left(\Pi_t^{\#}\right)^{1-\epsilon_p} + \phi_p \Pi^{\epsilon_p - 1} \tag{13}$$

$$w_t^{1-\epsilon_w} = (1-\phi_w)\left(w_t^{\#}\right)^{1-\epsilon_w} + \phi_w \Pi_t^{\epsilon_w - 1} w_{t-1}^{1-\epsilon_w} \tag{14}$$

Goods market-clearing requires that wholesale output by sold to unions in the aggregate, or:

$$Y_{W,t} = \int_0^1 Y_t(f) df$$

Given the demand function for each retailers output, this works out to:

$$Y_{W,t} = Y_t v_t^p \tag{15}$$

Where v_t^p is a measure of price dispersion:

$$v_t^p = (1-\phi_p)\left(\Pi_t^{\#}\right)^{-\epsilon_p} + \phi_p \Pi_t^{\epsilon_p} v_{t-1}^p \tag{16}$$

Labor market-clearing requires that labor supplied by the household equal the sum total of labor employed by unions:

Labor market clearing: labor supplied by the household must equal labor used by the union:

$$L_t = \int_0^1 L_{u,t}(l) dl$$

Given the demand for union labor, this works out to:

$$L_t = L_{d,t} v_t^w \tag{17}$$

Where v_t^w is a measure of wage dispersion:

$$v_t^w = (1-\phi_w)\left(\frac{w_t^{\#}}{w_t}\right)^{-\epsilon_w} + \phi_w \Pi_t^{\epsilon_w}\left(\frac{w_t}{w_{t-1}}\right)^{\epsilon_w} v_{t-1}^w \tag{18}$$

To get the aggregate resource constraint, first aggregate dividends from retail firms

$$DIV_{r,t} = \int_0^1 DIV_{r,t}(f) = P_t^{\epsilon_p} Y_t \int_0^1 P_t(f)^{1-\epsilon_p} df - P_{w,t} Y_t \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon_p} df$$

Which, given the price index and definition of price dispersion is:

$$DIV_{R,t} = P_t Y_t - P_{w,t} Y_t v_t^p$$

Now aggregate dividends from unions:

$$DIV_{u,t} = \int_0^1 DIV_{u,t}(l)dl = W_t^{\epsilon_w} L_{d,t} \int_0^1 W_t(l)^{1-\epsilon_w} dl - MRS_t L_{d,t} \int_0^1 \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} dl$$

Which given the wage index works out to:

$$DIV_{u,t} = W_t L_{d,t} - MRS_t L_{P,t} v_t^w$$

The dividend from wholesale firm is:

$$DIV_{W,t} = P_{w,t} Y_{w,t} - W_t L_{d,t}$$

Total dividends received by the household are then:

$$DIV_t = DIV_{r,t} + DIV_{w,t} + DIV_{W,t}$$

Summing these up:

$$DIV_t = P_t Y_t - P_{w,t} Y_t v_t^p + W_t L_{d,t} - MRS_t L_{d,t} v_t^w + P_{w,t} Y_{w,t} - W_t L_{W,t}$$

But then using facts about labor and goods market-clearing, we have:

$$DIV_t = P_t Y_t - P_{w,t} Y_{W,t} + W_t L_{d,t} - MRS_t L_t + P_{w,t} Y_{W,t} - W_t L_{d,t}$$

But then stuff cancels, leaving:

$$DIV_t = P_t Y_t - MRS_t L_t$$

Then if we impose bonds in zero supply (which is innocuous, we could have different kinds of firms buying/selling debt and it wouldn't affect anything), we get the standard resource constraint:

$$Y_t = C_t \tag{19}$$

6 Exogenous Process

A_t is the only exogenous variable. Assume it follows an AR(1) in the log with non-stochastic mean normalized to unity:

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t} \tag{20}$$

7 Full Set of Equilibrium Conditions

- Household:

$$\psi L_t^{\chi} = C_t^{-\sigma} mrs_t \tag{21}$$

$$1 = R_t \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1} \tag{22}$$

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \tag{23}$$

- Wage-setting:

$$w_t^{\#} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{f_{1,t}}{f_{2,t}} \tag{24}$$

$$f_{1,t} = mrs_t w_t^{\epsilon_w} L_{d,t} + \phi_w \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_w} f_{1,t+1} \tag{25}$$

$$f_{2,t} = w_t^{\epsilon_w} L_{d,t} + \phi_w \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_w - 1} f_{2,t+1} \tag{26}$$

- Price-setting:

$$x_{1,t} = p_{w,t} Y_t + \phi_p \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_p} x_{1,t+1} \tag{27}$$

$$x_{2,t} = Y_t + \phi_p \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon_p - 1} x_{2,t+1} \tag{28}$$

$$\Pi_t^{\#} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{x_{1,t}}{x_{2,t}} \tag{29}$$

- Wholesale firm:

$$Y_{W,t} = A_t L_{d,t} \tag{30}$$

$$w_t = p_{w,t} A_t \tag{31}$$

- Monetary policy:

$$\ln R_t = (1 - \rho_R) \ln R + \rho_R \ln R_{t-1} + (1 - \rho_R) \theta_\pi \left(\ln \Pi_t - \ln \Pi \right) + s_R \varepsilon_{R,t} \tag{32}$$

- Aggregate conditions:

$$1 = (1 - \phi_p) \left(\Pi_t^{\#} \right)^{1 - \epsilon_p} + \phi_p \Pi^{\epsilon_p - 1} \tag{33}$$

$$w_t^{1 - \epsilon_w} = (1 - \phi_w) \left(w_t^{\#} \right)^{1 - \epsilon_w} + \phi_w \Pi_t^{\epsilon_w - 1} w_{t-1}^{1 - \epsilon_w} \tag{34}$$

$$Y_{W,t} = Y_t v_t^p \tag{35}$$

$$v_t^p = (1-\phi_p)\left(\Pi_t^{\#}\right)^{-\epsilon_p} + \phi_p \Pi_t^{\epsilon_p} v_{t-1}^p \tag{36}$$

$$L_t = L_{d,t} v_t^w \tag{37}$$

$$v_t^w = (1-\phi_w)\left(\frac{w_t^{\#}}{w_t}\right)^{-\epsilon_w} + \phi_w \Pi_t^{\epsilon_w}\left(\frac{w_t}{w_{t-1}}\right)^{\epsilon_w} v_{t-1}^w \tag{38}$$

$$Y_t = C_t \tag{39}$$

- Exogenous process:

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t} \tag{40}$$

This is 20 variables $\left\{C_t, Y_t, Y_{W,t}, L_t, L_{d,t}, \Lambda_{t,t+1}, R_t, mrs_t, w_t, p_{w,t}, \Pi_t, \Pi_t^{\#}, w_t^{\#}, x_{1,t}, x_{2,t}, f_{1,t}, f_{2,t}, A_t, v_t^p, v_t^w\right\}$ in 20 equations.

8 Steady State

It is easiest to assume zero net inflation in steady state. This means $\Pi = 1$, which then implies $\Pi^{\#} = v^p = v^w = 1$, $R = 1/\beta$, and $w^{\#} = w$. Note also we have already assumed $A = 1$.

From the FOC for price-setting, we get:

$$p_w = w = \frac{\epsilon_p - 1}{\epsilon_p} \tag{41}$$

From the FOC for wage-setting, we see:

$$mrs = \frac{\epsilon_w - 1}{\epsilon_w} w \tag{42}$$

Combining these, we get:

$$mrs = \frac{\epsilon_w - 1}{\epsilon_w}\frac{\epsilon_p - 1}{\epsilon_p} \tag{43}$$

In an efficient allocation, we would have $mrs = 1$. Market-power in both labor and goods distort this, with $mrs < 1$.

I will calibrate the model to be consistent with $L = 1$ in steady state. This means that ψ must satisfy:

$$\psi = mrss \tag{44}$$