Equilibrium in an Endowment Economy

ECON 30020: Intermediate Macroeconomics

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Readings

- GLS Ch. 10

General Equilibrium

- We previously studied the optimal *decision problem* of a household. The outcome of this was an optimal *decision rule* (the consumption function)
- The decision rule takes prices as given. In two period consumption model, the only price is r_t
- Three modes of economic analysis:
  1. Decision theory: derivation of optimal decision rules, taking prices as given
  2. Partial equilibrium: determine the price in one market, taking the prices in all other markets as given
  3. General equilibrium: simultaneously determine all prices in all markets
- Macroeconomics is focused on general equilibrium
- How do we go from decision rules to equilibrium? What determines prices?

Competitive Equilibrium

- Webster's online dictionary defines the word equilibrium to be "a state in which opposing forces or actions are balanced so that one is not stronger or greater than the other."
- In economics, an equilibrium is a situation in which prices adjust so that (i) all parties are content supplying/demanding a given quantity of goods or services at those prices and (ii) markets clear
- If parties were not content, they would have an incentive to behave differently. Things wouldn't be "balanced" to use Webster's terms
- A competitive equilibrium is a set of prices and allocations where (i) all agents are behaving according to their optimal decision rules, taking prices as given, and (ii) all markets simultaneously clear

Competitive Equilibrium in an Endowment Economy

- An endowment economy is a fancy term for an economy in which there is no endogenous production – the amount of income/output is exogenously given
- With fixed quantities, it becomes particularly clear how price adjustment results in equilibrium
- Basically, what we do is take the two period consumption model:
  - Optimal decision rule: consumption function
  - Market: market for saving, S_t
  - Price: r_t (the real interest rate)
  - Market-clearing: in aggregate, saving is zero (equivalently, $Y_t = C_t$)
  - Allocations: C_t and C_{t+1}
- This is a particularly simple environment, but the basic idea carries over more generally

Setup

- There are L total agents who have identical preferences, but potentially different levels of income. Index households by j
- Each household can borrow/save at the same real interest rate, r_t
- Each household solves the following problem:

$$\max_{C_t(j), C_{t+1}(j)} \quad U(j) = u(C_t(j)) + \beta u(C_{t+1}(j))$$

$$\text{s.t.}$$

$$C_t(j) + \frac{C_{t+1}(j)}{1 + r_t} = Y_t(j) + \frac{Y_{t+1}(j)}{1 + r_t}$$

- Optimal decision rule is the standard consumption function:

$$C_t(j) = C^d(Y_t(j), Y_{t+1}(j), r_t)$$

Market-Clearing

- In this context, what does it mean for markets to clear?
- *Aggregate* saving must be equal to zero:

$$S_t = \sum_{j=1}^{L} S_t(j) = 0$$

- Why? One agent's saving must be another's borrowing and vice-versa
- But this implies:

$$\sum_{j=1}^{L} (Y_t(j) - C_t(j)) = 0 \Rightarrow \sum_{j=1}^{L} Y_t(j) = \sum_{j=1}^{L} C_t(j)$$

- In other words, aggregate income must equal aggregate consumption:

$$Y_t = C_t$$

Everyone the Same

- Suppose that all agents in the economy have identical endowment levels in both period t and $t+1$
- Convenient to just normalize total number of agents to $L = 1$ – representative agent. Can drop j references
- Optimal decision rule:

$$C_t = C^d(Y_t, Y_{t+1}, r_t)$$

- Market-clearing condition:

$$Y_t = C_t$$

- Y_t and Y_{t+1} are exogenous. Optimal decision rule is effectively one equation in two unknowns – C_t (the allocation) and r_t (the price)
- Combining the optimal decision rule with the market-clearing condition allows you to determine both r_t and C_t

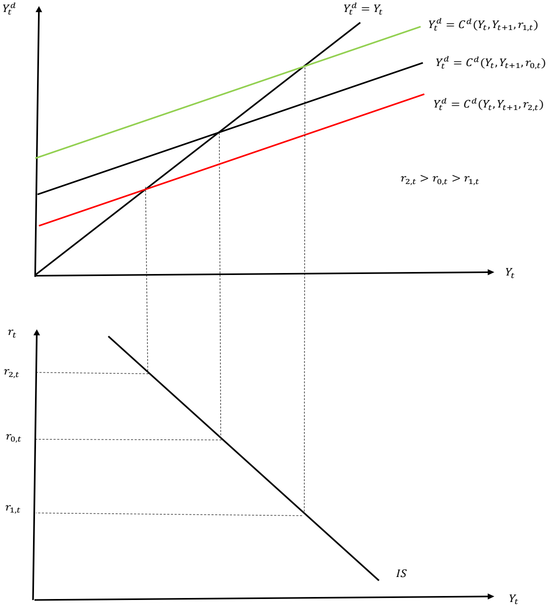
Graphical Analysis

- Define total desired expenditure as equal to consumption:

$$Y_t^d = C^d(Y_t, Y_{t+1}, r_t)$$

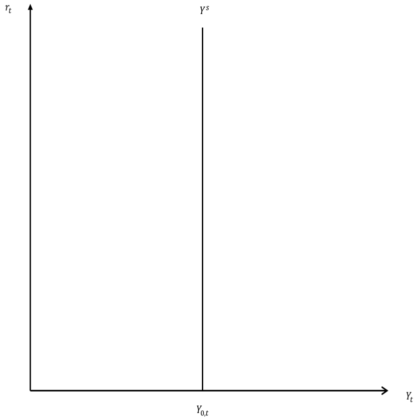
- Total desired expenditure is a function of income, Y_t
- But income must equal expenditure in any equilibrium
- Graph desired expenditure against income. Assume total desired expenditure with zero current income is positive – i.e. $C^d(0, Y_{t+1}, r_t) > 0$. This is sometimes called "autonomous expenditure"
- Since MPC < 1, there will exist one point where income equals expenditure
- *IS* curve: the set of (r_t, Y_t) pairs where income equals expenditure assuming optimal behavior by household. Summarizes "demand" side of the economy. Negative relationship between r_t and Y_t

Derivation of the IS Curve



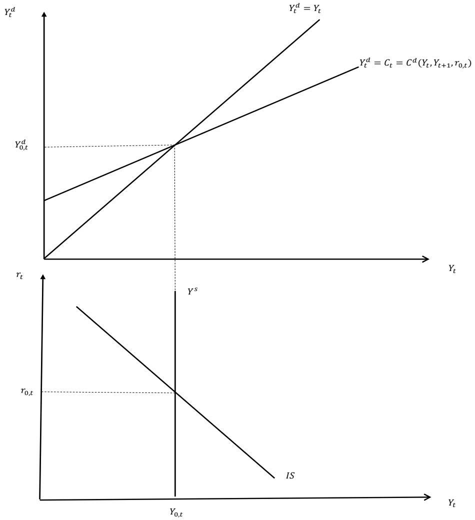
The Y^s Curve

- The Y^s curve summarizes the production side of the economy
- In an endowment economy, there is no production! So the Y^s curve is just a vertical line at the exogenously given level of Y_t

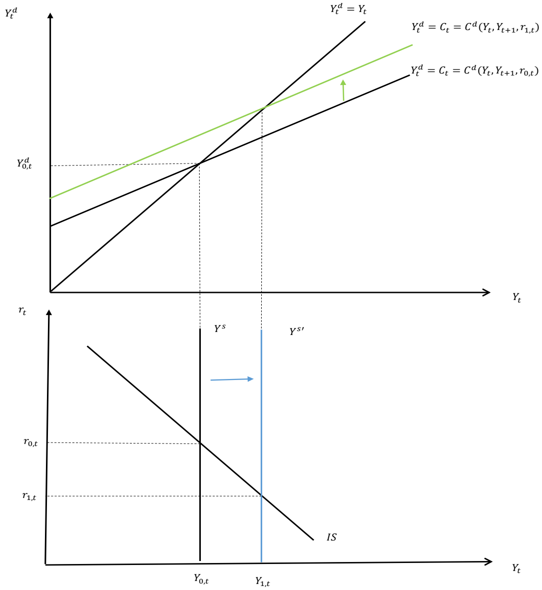


Equilibrium

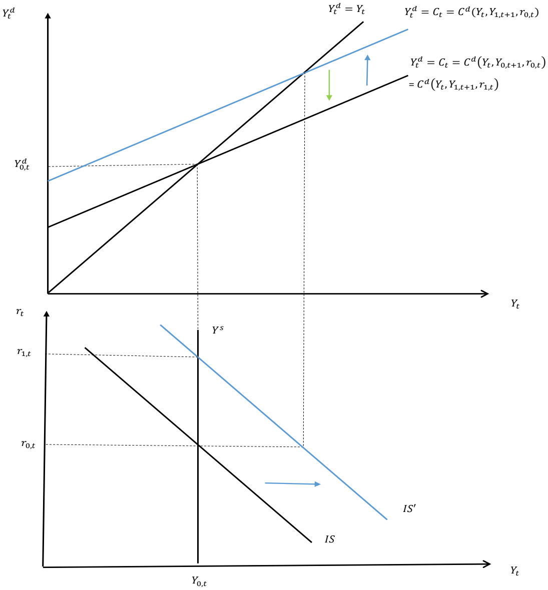
- Must have income = expenditure (demand side) = production (supply-side). Find the r_t where IS and Y^s cross



Supply Shock: $\uparrow Y_t$



Demand Shock: $\uparrow Y_{t+1}$



Discussion

- Market-clearing requires $C_t = Y_t$
- For a given r_t, household does not want $C_t = Y_t$. Wants to smooth consumption relative to income
- But in equilibrium cannot
- r_t adjusts so that household is content to have $C_t = Y_t$
- r_t ends up being a measure of how plentiful the future is expected to be relative to the present

Example with Log Utility

- With log utility, equilibrium real interest rate comes out to be (just take Euler equation and set $C_t = Y_t$ and $C_{t+1} = Y_{t+1}$):

$$1 + r_t = \frac{1}{\beta} \frac{Y_{t+1}}{Y_t}$$

- r_t proportional to expected income growth
- Potential reason why interest rates are so low throughout world today: people are pessimistic about the future. They would like to save for that pessimistic future, which ends up driving down the return on saving

Agents with Different Endowments

- Suppose there are two types of agents, 1 and 2. L_1 and L_2 of each type
- Identical preferences
- Type 1 agents receive $Y_t(1) = 1$ and $Y_{t+1}(1) = 0$, whereas type 2 agents receive $Y_t(2) = 0$ and $Y_{t+1}(2) = 1$
- Assume log utility, so consumption functions for each type are:

$$C_t(1) = \frac{1}{1+\beta}$$
$$C_t(2) = \frac{1}{1+\beta}\frac{1}{1+r_t}$$

- Aggregate income in each period is $Y_t = L_1$ and $Y_{t+1} = L_2$

Equilibrium

- With this setup, the equilibrium real interest rate is:

$$1 + r_t = \frac{1}{\beta} \frac{L_2}{L_1}$$

- Noting that $L_2 = Y_{t+1}$ and $L_1 = Y_t$, this is the same as in the case where everyone is the same!
- In particular, given aggregate endowments, equilibrium r_t does not depend on distribution across agents, only depends on aggregate endowment
- Amount of income heterogeneity at micro level doesn't matter for macro outcomes. Example of "market completeness" and motivates studying representative agent problems more generally