# Equilibrium with Production and Endogenous Labor Supply

## ECON 30020: Intermediate Macroeconomics

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# Readings

- GLS Chapter 11

# Production and Labor Supply

- We continue working with a two period, optimizing, equilibrium model of the economy
- No uncertainty over future, although it would be straightforward to entertain this
- We augment the model with which we have been working along the following two dimensions:
  1. We model production and an investment decision
  2. Model endogenous labor supply
- The production side is very similar to the Solow model

# Firm

- There exists a representative firm. The firm produces output using capital, $K_t$, and labor, $N_t$, according to the following production function:

$$Y_t = A_t F(K_t, N_t)$$

- $A_t$ is exogenous productivity variable. Abstract from trend growth
- $F(\cdot)$ has the same properties as assumed in the Solow model – increasing in both arguments, concave in both arguments, both inputs necessary. For example:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1$$

# Capital Accumulation

- Slightly differently than the Solow model, we assume that the firm makes the capital accumulation decisions
- We assume that the firm must borrow from a financial intermediary in order to finance its investment
- "Equity" versus "debt" finance would be equivalent absent financial frictions, which we will model
- Furthermore, ownership of capital wouldn't make a difference absent financial frictions (i.e. firm makes capital accumulation decision vs. household owning capital and leasing it to firms)
- Current capital, $K_t$, is predetermined and hence exogenous. Capital accumulates according to:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

- Exactly same accumulation equation as in Solow model

# Prices Relevant for the Firm

- Firm hires labor in a competitive market at (real) wage $w_t$ (and $w_{t+1}$ in the future)
- Firm borrows to finance investment at:

$$r_t^I = r_t + f_t$$

- $r_t^I$ is the interest rate relevant for the firm, while $r_t$ is the interest rate relevant for the household
- $f_t$ is (an exogenous) variable representing a financial friction. We will refer to this as a credit spread
- During financial crises observed credit spreads rise significantly

# Dividends

- The representative household owns the firm. The firm returns any difference between revenue and cost to the household each period in the form of a dividend
- Dividend is simply output (price normalized to one since model is real) less cost of labor in period $t$ (since borrowing cost of investment is borne in future):

$$D_t = Y_t - w_t N_t$$

- Terminal condition for the firm: firm wants $K_{t+2} = 0$ (die with no capital). This implies $I_{t+1} = -(1-\delta)K_{t+1}$, which we can think of as the firm "liquidating" its remaining capital after production in $t+1$
- This is an additional source of revenue for the firm in $t+1$. In addition, firm has to pay interest plus principal on its borrowing for investment in $t$:

$$D_{t+1} = Y_{t+1} + (1-\delta)K_{t+1} - w_{t+1}N_{t+1} - (1+r_t^I)I_t$$

# Firm Valuation and Problem

- Value of the firm: PDV of flow of dividends:

$$V_t = D_t + \frac{1}{1+r_t} D_{t+1}$$

- Firm problem is to pick $N_t$ and $I_t$ to maximize $V_t$ subject to accumulation equation:

$$\max_{N_t, I_t} \; V_t = D_t + \frac{1}{1+r_t} D_{t+1}$$

s.t.

$$\begin{aligned} K_{t+1} &= I_t + (1-\delta)K_t \\ D_t &= A_t F(K_t, N_t) - w_t N_t \\ D_{t+1} &= A_{t+1} F(K_{t+1}, N_{t+1}) + (1-\delta)K_{t+1} - w_{t+1} N_{t+1} - (1 + r_t^I) I_t \end{aligned}$$

# First Order Conditions

- Two first order conditions come out of firm problem:

$$w_t = A_t F_N(K_t, N_t)$$

$$1 + r_t^I = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta)$$

- Intuition: MB = MC
- Wage condition exactly same as Solow model expression for wage
- Investment condition can be re-written in terms of earlier notation by noting $R_{t+1} = A_{t+1} F_K(K_{t+1}, N_{t+1})$ and:

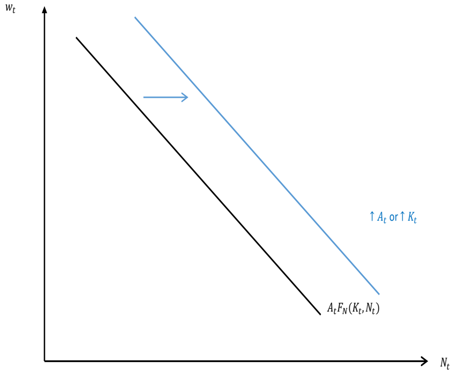
$$R_{t+1} = r_t^I + \delta = r_t + f_t + \delta$$

- Return on capital, $R_{t+1}$, closely related to real interest rate, $r_t$
- These FOC implicitly define labor and investment demand functions

# Labor Demand

- Labor FOC implicitly characterizes a downward-sloping labor demand curve:

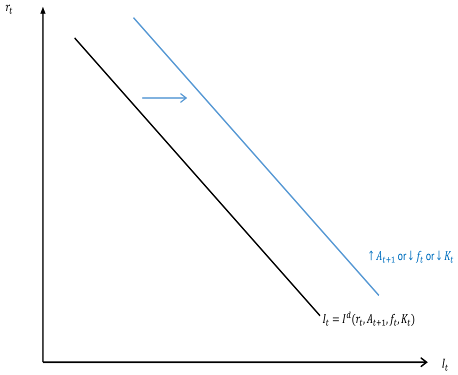
$$N_t = N^d(\underset{-}{w_t}, \underset{+}{A_t}, \underset{+}{K_t})$$



# Investment Demand

- Second first order condition implicitly defines a demand for $K_{t+1}$, which can be used in conjunction with the accumulation equation to get an investment demand curve:

$$I_t = I^d(\underset{-}{r_t}, \underset{+}{A_{t+1}}, \underset{-}{f_t}, \underset{-}{K_t})$$



# Household

- There exists a representative household. Households gets utility from consumption and leisure, where leisure is $L_t = 1 - N_t$, with $N_t$ labor and available time normalized to 1
- Lifetime utility:

$$U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$

- Example flow utility functions:

$$u(C_t, 1 - N_t) = \ln C_t + \theta_t \ln(1 - N_t)$$
$$u(C_t, 1 - N_t) = \ln \left[ C_t + \theta_t \ln(1 - N_t) \right]$$

- Here, $\theta_t$ is an exogenous "labor supply shock" governing utility from leisure (equivalently, disutility from labor)
- Notation: $u_C$ denotes marginal utility of consumption, $u_L$ marginal utility of leisure (marginal utility of labor is $-u_L$)

# Budget Constraints

- Household faces two flow budget constraints, conceptually the same as before, but now income is partly endogenous:

$$C_t + S_t \leq w_t N_t + D_t$$
$$C_{t+1} + S_{t+1} - S_t \leq w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1} + r_t S_t$$

- Household takes $D_t$, $D_{t+1}$, and $D^I_{t+1}$ (dividend from financial intermediary) as given (ownership different than management)
- Terminal condition: $S_{t+1} = 0$. Gives rise to IBC:

$$C_t + \frac{C_{t+1}}{1+r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}{1+r_t}$$

# First Order Conditions

- Do the optimization in the usual way. The following first order conditions emerge:

$$u_C(C_t, 1 - N_t) = \beta(1 + r_t)u_C(C_{t+1}, 1 - N_{t+1})$$

- This is the usual Euler equation, only looks different to accommodate utility from leisure

$$u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$$
$$u_L(C_{t+1}, 1 - N_{t+1}) = w_{t+1} u_C(C_{t+1}, 1 - N_{t+1})$$

- Discussion and intuition

# Optimal Decision Rules

- Can go from first order conditions to optimal decision rules
- Cutting a few corners, we get the same consumption function as before:

$$C_t = C^d(\underset{+}{Y_t}, \underset{+}{Y_{t+1}}, \underset{-}{r_t})$$

- Or, if there were government spending, with Ricardian Equivalence we'd have:

$$C_t = C^d(Y_t \underset{+}{-} G_t, Y_{t+1} \underset{+}{-} G_{t+1}, \underset{-}{r_t})$$

# Labor Supply

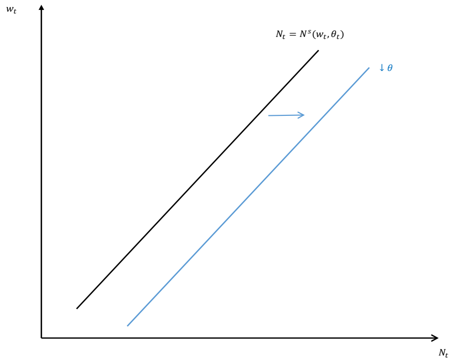
- First order condition for $N_t$ can be characterized by an indifference curve / budget line diagram similar to the two period consumption case:
- Things are complicated for a few reasons:
  - Competing income and substitution effects of $w_t$
  - Non-wage income and expectations about future income (including through an interest rate channel) can affect current labor supply
- We will sweep most of this stuff under rug: substitution effect dominates and other things (other than exogenous variable $\theta_t$) are ignored
- Can be motivated explicitly with preference specification due to Greenwood, Hercowitz, and Huffman (1988):

$$u(C_t, 1 - N_t) = \ln\left[C_t + \theta_t \ln(1 - N_t)\right]$$

# Labor Supply Curve

- Labor supply function under these assumptions:

$$N_t = N^s(\underset{+}{w_t}, \underset{-}{\theta_t})$$



# Financial Intermediary

- Will not go into great detail
- In period $t$, takes in deposits, $S_t$, from household; issues loans in amount $I_t$ to firm
- Pays $r_t$ for deposits, and earns $r_t^I = r_t + f_t$ on loans
- $f_t$ is exogenous, and $f_t > 0$ means intermediary earns profit in $t+1$, which is returned to household as dividend:

$$D_{t+1}^I = (r_t + f_t)I_t - r_t S_t$$

# Market-Clearing

- Market-clearing requires $S_t = I_t$ (i.e. funds taken in by financial intermediary equal funds distributed to firm for investment)
- This implies:

$$Y_t = C_t + I_t$$

- If there were a government levying (lump sum) taxes on household period $t$ resource constraint would just be:

$$Y_t = C_t + I_t + G_t$$

- Can show that period $t+1$ constraint is the same:

$$Y_{t+1} = C_{t+1} + I_{t+1}$$

# Equilibrium

- The following conditions must all hold in period $t$ in equilibrium:

$$
\begin{aligned}
C_t &= C^d(Y_t, Y_{t+1}, r_t) \\
N_t &= N^s(w_t, \theta_t) \\
N_t &= N^d(w_t, A_t, K_t) \\
I_t &= I^d(r_t, A_{t+1}, f_t, K_t) \\
Y_t &= A_t F(K_t, N_t) \\
Y_t &= C_t + I_t
\end{aligned}
$$

- Endogenous: $C_t$, $N_t$, $Y_t$, $I_t$, $w_t$, and $r_t$
- Exogenous: $A_t$, $A_{t+1}$, $K_t$, $f_t$, $\theta_t$. Will talk about $Y_{t+1}$ and $K_{t+1}$ later
- Four optimal decision rules, two resource constraints: income = production and income = expenditure

# Competitive Equilibrium

- There are now *two* prices – $r_t$ (intertemporal price of goods) and $w_t$ (price of labor)
- Different ways to think about what the markets are. One is clear – market for labor, which $w_t$ adjusts to clear (i.e. labor supply = demand)
- Can think about either market for goods (i.e. $Y_t = C_t + I_t$) or a loanable funds market $S_t = I_t$ as being the other market, which $r_t$ adjusts to clear. We will focus on market for goods
- Endowment economy special case of this if $N_t$ and $I_t$ are held fixed
- Will be possible to do some consumption smoothing in equilibrium here, however. Suppose household wants to increase $S_t$. It can do this if $r_t$ falls to incentivize more $I_t$ (whereas in endowment economy $I_t = 0$, so $S_t$ must remain fixed at 0).