

Graduate Macro Theory II: The Basics of Financial Constraints

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1 Introduction

The recent Great Recession has highlighted the potential importance of financial market imperfections for macroeconomic fluctuations. This note briefly discusses how to incorporate financial market imperfections in a tractable way to an otherwise standard RBC model.

Modeling financial frictions in a fully micro-founded way is technically challenging. If fully micro-founded, to get financial frictions into a model one needs some degree of market incompleteness and some level of heterogeneity. The financial system aggregates the savings of households and funnels it to finance investment in physical capital by firms. This funneling of savings can either be equity or debt, and as we have shown absent some other frictions it does not matter which. Indeed, in the standard RBC model it doesn't matter if we think about the household directly making capital accumulation decisions rather than firms, and making this counterfactual assumption often makes life easier by making the problem of a firm effectively static.

Financial market imperfections, or financial market frictions, generically refer to situations in which the funneling of saving from households to firms gets messed up somehow. In the real world, a very large fraction of firm financing comes in the form of conventional debt. Without formally modeling why, in models with financial frictions we will assume that firms must finance some of their operations via debt. In reality, part of this is because debt finance is more efficient than equity finance because financial intermediaries play important informational roles (i.e. banks making loans know a lot more about a company than individuals thinking about purchasing new equity).

There are two popular ways to throw a wrench into the use of debt to finance operations. The first dates back to Townsend (1979) and is known as the “cost state verification” (CSV) problem. This has been brought to the fore in macroeconomic models in Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999). Here the problem is one of information asymmetry. Borrowers have private information about their projects and have limited liability. This gives rise to an optimal contracting problem between a lender and a firm. This gives rise to an external finance premium, wherein firms pay a higher interest rate to raise money from external sources (e.g. a bank). The more net worth a firm has, the more financing it

can do internally. Hence, fluctuations in asset prices (which affect net worth) affect firm investment decisions.

The alternative setup is sometimes known as “costly enforcement” and dates back to Kiyotaki and Moore (1997). Here there is no asymmetric information per se, but there is costly enforcement. In the event of default, the lender can only recover a fraction of the debt. Like the CSV framework, this also leads to an optimal contracting problem. What ends up happening is there ends up being a collateral constraint wherein a borrower can only borrow up to a fraction of the value of his assets, which in this case is the stock of capital. We can think about a financial market shock as a shock to the fraction of the value of assets a borrower can borrow. Furthermore, asset price fluctuations will affect the borrowing limit. In this way, just like in the CSV framework, fluctuations in asset prices can directly affect firm investment decisions.

The collateral constraint version of the model is much easier to deal with, particularly as a reduced form way to think about financial frictions. It is the route I am going to take here; the CSV approach is much more intensive. In the model we’re going to work with, I will simply specify (without formally deriving) what a firm has to finance out of debt, and will then include a constraint in which total borrowing cannot exceed a fraction of the value of the firm’s capital stock (i.e. its collateral). This will allow us to illustrate most of the ideas of financial frictions without digging into the weeds of the CSV approach. We will think about a financial shock as an exogenous change in the fraction of the value of a firm’s capital that it can borrow. There are some downsides to the collateral constraint approach. In this framework, there is no default in equilibrium (whereas there is the CSV framework as well as in reality). Furthermore, there are no credit spreads in the collateral constraint framework. Finally, we can’t think about the role of idiosyncratic risk in the way one might with the CSV framework.

In the next section I will focus on a setup in which the firm has to finance its payments to labor with an intraperiod loan (what is called working capital), and the amount it can finance is tied to the value of its capital. Because this financial friction directly affects the firm’s choice of labor, it generates a labor wedge and financial shocks can induce co-movement among aggregate variables. In subsequent sections I will consider the case where investment in new physical capital must be financed via either intra or interperiod debt. This will also lead to interesting dynamics but it will be difficult to generate co-movement among consumption and labor in response to a financial shock, at least within the context of an otherwise standard RBC type model. In addition to a financial shock, I will also analyze how the presence of a financial constraint impacts the responses to a productivity shock.

2 A Simple Collateral Constraint Model

The household problem is fairly standard. Differently than what we’ve been doing, we assume that the firm owns the capital stock and makes capital accumulation decisions. We are going to require that the firm borrow to finance its payments to labor (what is known as “working capital”) via an intra-period loan. The amount that the firm can borrow depends on the value of its capital stock.

For this to make sense, we need the firm to own the capital stock.

The household problem is:

$$\max_{C_{t+j}, N_{t+j}, B_{t+j+1}} E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln C_{t+j} - \theta \frac{N_{t+j}^{1+\chi}}{1+\chi} \right\}$$

s.t.

$$C_{t+j} + B_{t+j+1} - B_{t+j} \leq w_{t+j}N_{t+j} + \Pi_{t+j} + r_{t+j-1}B_{t+j}$$

The terms here are all standard. B_t is the stock of savings/debt with which a household enters a period, with r_{t-1} the real interest rate on that savings. Π_t is a dividend distribution from the firm. A Lagrangian is:

$$\mathbb{L} = E_t \sum_{j=0}^{\infty} \left\{ \ln C_{t+j} - \theta \frac{N_{t+j}^{1+\chi}}{1+\chi} + \lambda_{t+j} [w_{t+j}N_{t+j} + \Pi_{t+j} + (1+r_{t+j-1})B_{t+j} - C_{t+j} - B_{t+j+1}] \right\}$$

The first order conditions are standard:

$$\frac{\partial \mathbb{L}}{\partial C_t} = 0 \Leftrightarrow \frac{1}{C_t} = \lambda_t \quad (1)$$

$$\frac{\partial \mathbb{L}}{\partial N_t} = 0 \Leftrightarrow \theta N_t^\chi = \lambda_t w_t \quad (2)$$

$$\frac{\partial \mathbb{L}}{\partial B_{t+1}} = 0 \Leftrightarrow \lambda_t = \beta E_t \lambda_{t+1} (1+r_t) \quad (3)$$

These are standard. The multiplier can be eliminated, leaving:

$$\theta N_t^\chi = \frac{1}{C_t} w_t \quad (4)$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (1+r_t) \quad (5)$$

The firm produces output using a standard Cobb-Douglas production function:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \quad (6)$$

There is a capital adjustment cost. Capital accumulates according to:

$$K_{t+1} = I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t + (1-\delta)K_t \quad (7)$$

Investment is financed out of dividends, and for simplicity assume that the firm issues no *intertemporal* debt. The dividend payout is then:

$$\Pi_t = Y_t - w_t N_t - I_t$$

So far, this is all fairly standard. What differentiates the setup relative to a frictionless RBC model is the inclusion of a working capital constraint. In particular, we assume that the firm must pay labor in advance of producing output, and finances this labor payment with an *intraproduct* loan. Since the loan is intratemporal (as opposed to the standard intertemporal debt), there is no interest on this loan.

We assume that the amount the firm can borrow equals a fraction of the value of its capital. In particular, the firm faces the following constraint:

$$w_t N_t \leq \xi_t q_t K_t \quad (8)$$

Here, $w_t N_t$ denotes total wage payments, q_t is the value of capital, K_t is how much capital the firm has, and ξ_t is an exogenous and stochastic borrowing limit. The basic idea is that, if the borrower defaults on the working capital loan, the lender can only seize a fraction, $\xi_t < 1$, of the firm's assets. This setup is sometimes known as the *costly enforcement* model – since a firm might renege on its debt, a lender will only allow a firm to borrow up to a fraction of its debt.

Let's setup a Lagrangian for the firm. The firm discounts future profit flows by the stochastic discount factor of the household, $E_t \beta^j \frac{\lambda_{t+j}}{\lambda_t}$. There are two constraints. Let q_t be the multiplier on the accumulation equation (since the units of the firm's problem are goods, q_t has the interpretation of reflecting how many goods the firm would give up for an additional unit of installed capital). The firm's Lagrangian is:

$$\begin{aligned} \mathbb{L} = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ A_{t+j} K_{t+j}^\alpha N_{t+j}^{1-\alpha} - w_{t+j} N_{t+j} - I_{t+j} + \dots \right. \\ \left. q_{t+j} \left[I_{t+j} - \frac{\phi}{2} \left(\frac{I_{t+j}}{K_{t+j}} - \delta \right)^2 K_{t+j} + (1-\delta) K_{t+j} - K_{t+j+1} \right] + \mu_{t+j} [\xi_{t+1} q_{t+j} K_{t+j} - w_{t+j} N_{t+j}] \right\} \end{aligned}$$

The first order conditions are:

$$\frac{\partial \mathbb{L}}{\partial N_t} = 0 \Leftrightarrow w_t (1 + \mu_t) = (1 - \alpha) A_t K_t^\alpha N_t^{-\alpha} \quad (9)$$

$$\frac{\partial \mathbb{L}}{\partial I_t} = 0 \Leftrightarrow 1 = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] \quad (10)$$

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial K_{t+1}} = 0 \Leftrightarrow q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + \dots \right. \\ \left. q_{t+1} \left((1-\delta) + \mu_{t+1} \xi_{t+1} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right] \quad (11) \end{aligned}$$

If we look at these first order conditions, if the collateral constraint did not bind (i.e. $\mu_t = 0$), then we would have identical conditions to a standard model with a capital adjustment cost. One

can see here that if the constraint does bind, however, then there is a wedge between the wage and the marginal product of labor (and hence a wedge between the MRS and the marginal product of labor, hence a labor wedge). If the constraint becomes “tighter,” we would expect the multiplier on the constraint to become bigger, which will function much like an increase in a tax on labor income.

The market-clearing conditions are standard. Since the firm issues no intertemporal debt, then the household cannot have any stock of savings. The full set of equilibrium conditions can therefore be written:

$$\frac{1}{C_t} = \lambda_t \quad (12)$$

$$\theta N_t^\chi = \lambda_t w_t \quad (13)$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + r_t) \quad (14)$$

$$w_t (1 + \mu_t) = (1 - \alpha) A_t K_t^\alpha N_t^{-\alpha} \quad (15)$$

$$1 = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] \quad (16)$$

$$q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + q_{t+1} \left((1 - \delta) + \mu_{t+1} \xi_{t+1} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right] \quad (17)$$

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \quad (18)$$

$$Y_t = C_t + I_t \quad (19)$$

$$K_{t+1} = I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t + (1 - \delta) K_t \quad (20)$$

$$w_t N_t \leq \xi_t q_t K_t \quad (21)$$

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \epsilon_{A,t} \quad (22)$$

$$\ln \xi_t = (1 - \rho_\xi) \ln \xi + \rho_\xi \ln \xi_{t-1} + s_\xi \epsilon_{\xi,t} \quad (23)$$

I assume that the exogenous productivity variable obeys the usual AR(1) in the log. In addition, I assume that the fraction of the value of a firm’s capital that may be borrowed, ξ_t , follows a stationary AR(1) process, with non-stochastic mean of ξ . There are twelve equations and twelve variables – C_t , λ_t , w_t , N_t , μ_t , r_t , K_t , A_t , Y_t , I_t , ξ_t , and q_t .

2.1 Non-Stochastic Steady State

Let us solve for the non-stochastic steady state. Variables without time subscripts denote steady state values. We will consider two regions – one in which the collateral constraints binds and the other in which it does not. If the constraint does not bind, then $\mu = 0$ and the steady state is the same as in the standard model.

From the first order condition for investment as well as the capital accumulation equation, it is clear that $q = 1$. We can then solve for the steady state capital-labor ratio from the dynamic capital supply Euler equation:

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1 - \delta) - \mu\xi} \right)^{\frac{1}{1-\alpha}} \quad (24)$$

Note that this expression holds regardless of whether the constraint binds or not. If the constraint does not bind, then this is the usual expression for the capital-labor ratio. If the constraint binds, then $\mu > 0$ and the capital-labor ratio will be larger than in the standard case. This may seem odd that capital-labor would be larger in the constrained case, but note that the constraint applies to labor; so a more binding constraint will tend to reduce N , which drives up the capital-labor ratio.

How would we know what μ is? It is either 0 (non-binding) or positive. Suppose that the collateral constraint binds. Then from the constraint evaluated in steady state we know:

$$w = \xi \frac{K}{N}$$

From the first order condition for labor demand, we also know that:

$$w = \frac{1}{1 + \mu} (1 - \alpha) \left(\frac{K}{N} \right)^\alpha$$

Combining these, we have:

$$\xi \frac{K}{N} = \frac{1}{1 + \mu} (1 - \alpha) \left(\frac{K}{N} \right)^\alpha$$

Or:

$$\xi(1 + \mu) = (1 - \alpha) \left(\frac{K}{N} \right)^{\alpha-1}$$

Using what we now know about $\frac{K}{N}$, this can be written:

$$\xi(1 + \mu) = (1 - \alpha) \frac{\frac{1}{\beta} - (1 - \delta) - \mu\xi}{\alpha}$$

Then solving for μ , we get:

$$\mu = \frac{1 - \alpha}{\xi} \left(\frac{1}{\beta} - (1 - \delta) \right) - \alpha \quad (25)$$

We can see here that the multiplier is decreasing in ξ , which is the fraction of the value of capital that can be borrowed against. This makes sense – the bigger is the fraction, the “less binding” is the collateral constraint, and hence the lower will be the multiplier. We can also solve for the cutoff value of ξ for which the constraint will in fact be borrowing. This is found by solving for

the values of ξ where $\mu > 0$:

$$\xi < \frac{1 - \alpha}{\alpha} \left(\frac{1}{\beta} - (1 - \delta) \right) \quad (26)$$

If ξ is bigger than this, then the constraint will not bind and we will be back in the normal case.

Now let us proceed. Combine the labor supply and labor demand conditions evaluated in the steady state:

$$\theta N^\chi = \frac{1}{C} \frac{1}{1 + \mu} (1 - \alpha) \left(\frac{K}{N} \right)^\alpha$$

Multiply both sides by N :

$$\theta N^{1+\chi} = \frac{N}{C} \frac{1}{1 + \mu} (1 - \alpha) \left(\frac{K}{N} \right)^\alpha$$

From the resource constraint, we know that:

$$\frac{C}{N} = \left(\frac{K}{N} \right)^\alpha - \delta \frac{K}{N}$$

Hence, we can solve for N as:

$$N = \left[\frac{1}{\theta} (1 + \mu)^{-1} \frac{(1 - \alpha) \left(\frac{K}{N} \right)^\alpha}{\left(\frac{K}{N} \right)^\alpha - \delta \frac{K}{N}} \right]^{\frac{1}{1+\chi}} \quad (27)$$

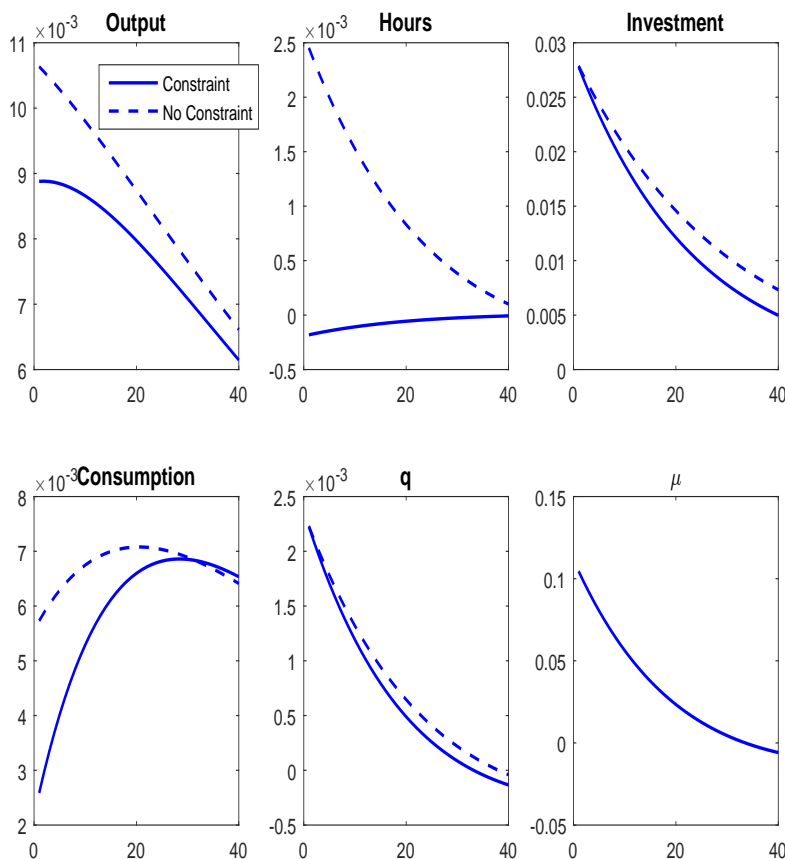
How does the value of ξ affect N ? There is both a direct effect and an indirect effect. A higher ξ makes μ smaller, which makes $(1 + \mu)^{-1}$ bigger, and therefore works to make N bigger. A higher ξ also makes $\frac{K}{N}$ bigger, which works in the same direction. We would therefore expect N to be increasing in ξ . Once we know N , it is straightforward to solve for the rest of the steady state.

2.2 Impulse Responses to Shocks

I assume the following parameter values: $\beta = 0.99$, $\alpha = 1/3$, $\delta = 0.02$, $\phi = 4$, $\theta = 7.71$, $\chi = 1$, $\rho_A = 0.98$, $s_A = 0.01$, $\rho_\xi = 0.90$, and $s_\xi = 0.01$. If the collateral constraint were not binding, with these parameter values labor hours would be $1/3$ in steady state. With these parameter values, the cutoff value of ξ where the constraint binds is $\xi = 0.0602$. Because I want to solve the model using perturbation techniques, I went to approximate about a point where the constraint binds, and then implicitly ignore any possibility of the constraint perhaps not binding at some point in the future. I therefore assume $\xi = 0.05$, which means that in the steady state the constraint binds.

In what follows, I show impulse responses for two cases: one where the constraint binds (the parameterization described above), and another in which it does not. This allows us to compare how the economy reacts to the two kinds of shocks with and without the constraint binding. Below are impulse responses to the productivity shock:

Figure 1: IRFs to Productivity Shock



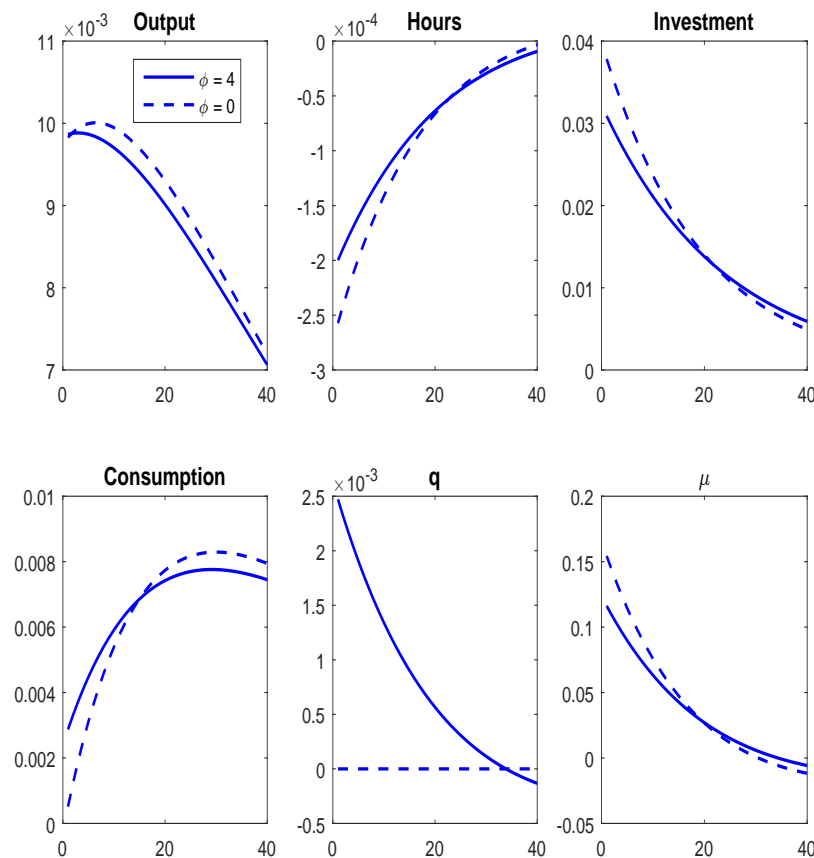
First, note that μ_t increases significantly after the productivity shock. This means that the collateral constraint is “tighter.” The increase in q_t is pretty similar whether the constraint binds or not. Hence, what the increase in μ_t means is that the firm would like to increase total payments to labor ($w_t N_t$) by more than the value of the firm’s capital increases. The increase in μ_t is isomorphic to an increase in the tax on labor. This means that N_t increases less than it would in the unconstrained case. Here we actually see that N_t decreases on impact in the constrained case. As a result, output goes up by significantly less than in the unconstrained case. Even though output goes up by quite a bit less, the impact response of investment is quite similar in the constrained and unconstrained cases. What accounts for this? The firm has an incentive to increase its investment by a lot, because this results in more capital in the future, which allows the firm to borrow more in the future to finance labor payments. In essence, investment today helps relax the credit constraint in the future. Hence, investment increases similarly after the productivity shock regardless of whether the collateral constraint binds or not. Since output goes up by significantly less, but the investment response is roughly the same as before, it follows that consumption increases by

significantly less when the collateral constraint binds.

Hence, the collateral constraint here dampens the responses to a productivity shock. Is there a so-called “financial accelerator” mechanism present in the model? The basic idea of the financial accelerator is that exogenous shocks (such as productivity shocks) impact the value of capital, q_t , which affects the ability to borrow (i.e. credit markets). This is somewhat mechanical here – changes in q_t affect the borrowing limit. Hence, the idea of the financial accelerator is that exogenous shocks might have *bigger* effects on output through an effect on asset prices and hence credit conditions than if financial frictions were absent. Given that output responds significantly less to the productivity shock when the constraint binds than when it does not, that does not seem to be the case here.

To better see if there is a financial accelerator mechanism, it is useful to consider two cases where the financial constraint binds in both. But in one, assume that $\phi = 0$, so that $q_t = 1$. In the other, let $\phi > 0$. The responses are shown below in Figure 2.

Figure 2: IRFs to Productivity Shock, Adj. Cost vs. No Adj. Cost

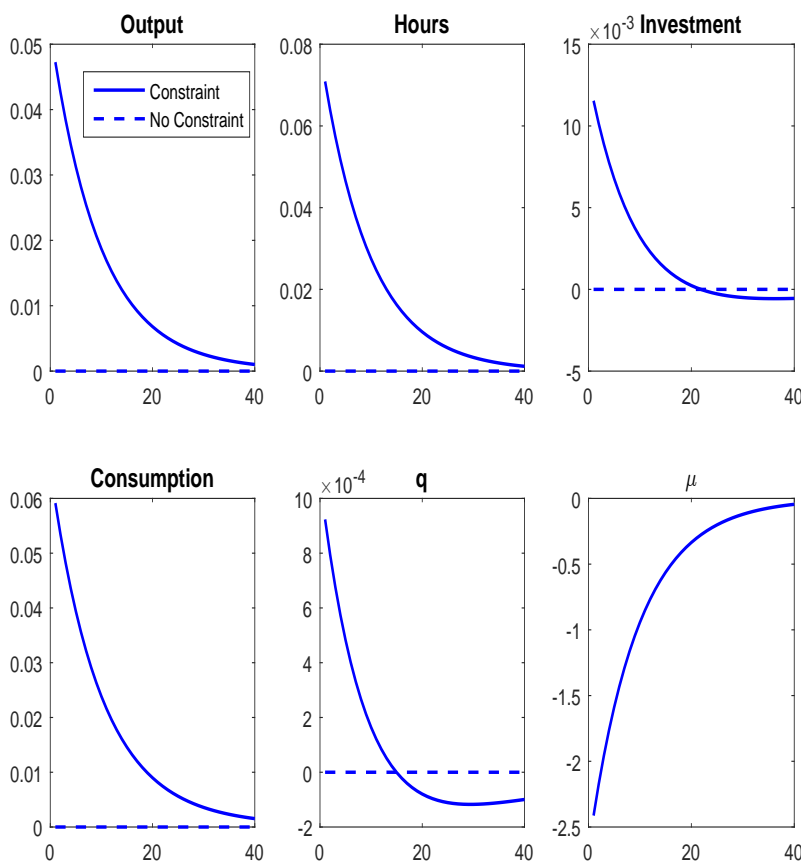


The difference between these two cases, of course, is that q_t can react when $\phi > 0$, and it is constant when there is no adjustment cost. If you look closely enough at the figure, you will see that output reacts more on impact (but only by a little) when $\phi = 4$ than when $\phi = 0$. What is happening here is that N_t falls by less. The mechanism is straightforward. When $\phi = 0$, total labor payments cannot change on impact because $q_t K_t$ is then predetermined. But when $\phi > 0$, $w_t N_t$ can go up on impact, even though K_t cannot. This means that if the shock can affect q_t (the price of capital), it can have a bigger effect than if it cannot. This is loosely a financial accelerator mechanism. The financial accelerator mechanism here turns out to be pretty weak, however, which is often true in models of this sort. What drives this is that the model does not predict a very big increase in q_t after the productivity shock. It is often true that macro models have difficulty generating lots of asset price volatility, so that the financial accelerator mechanism is weak.

Another thing is worth pointing out when looking at Figures 1-2. In particular, you will note that the model with a binding collateral constraint produces a bit of a hump-shaped response of output to the productivity shock. Indeed, in Figure 2 we see that the apparent hump-shape is even stronger without an adjustment cost to capital than with one. What is driving this? For this, it is easiest to focus on the responses shown in Figure 1. When the constraint binds, the output response is much smaller than when the constraint does not bind. The same is not true for investment. But differently, the binding collateral constraint results in increase in the volatility of investment relative to output. This is fairly intuitive. When productivity improves, absent the constraint the firm would like to hire more labor. One thing the firm can do to ease the constraint is to accumulate more capital. While this won't let the firm hire more labor immediately, it will once the capital comes on line. This means that the firm has an incentive to do more investment, which strengthens the model's internal propagation mechanism. The extra capital that is accumulated (relative to the initial increase in output) generates more interesting output dynamics than in the model with no collateral constraint. Put another way, binding financial constraints often have the effect of increasing propagation and persistence, in a way somewhat similar to the inclusion of capital/investment adjustment costs. This is an older point that is made in Carlstrom and Fuerst (1997) in the context of a costly state verification (CSV) setup.

Next, let us consider the responses to a financial shock (a shock to ξ_t). This is obviously only relevant in the situation in which the collateral constraint binds. The responses are shown below in Figure 3.

Figure 3: IRFs to Financial Shock



The increase in ξ_t makes the collateral constraint “less binding” and consequently results in a fall in μ_t . Mechanically in terms of how the other equations of the model look, this looks like a decrease in a distortionary tax rate on labor income. As a result, firms hire more labor and output rises. This results in higher consumption. Investment (and also q_t) also rise, albeit not by very much. In other words, a shock to the fraction of collateral a firm can borrow against for working capital (what we can loosely interpret as a financial shock) can lead to a broad-based movement in economic activity. Hence, a reduction in ξ_t (a negative financial shock) could generate a broad-based decline in output and other aggregates. It can do so precisely because it generates a labor wedge and hence looks like a time-varying tax on labor income.

3 Alternative Setups

Next I’m going to consider two alternative setups. These involve different factors which must be financed by debt, and hence different things which are directly affected by the collateral constraint.

First, I will assume that both payments to labor and investment must be financed by working capital. In the second setup, I will assume there is no working capital requirement, but that investment must be financed with intertemporal debt, and that this intertemporal debt is subject to the collateral requirement.

3.1 Investment Financed by Working Capital as Well

Let's consider the same setup as above, but this time assume that the firm must use working capital to finance its investment as well as its labor input. This leaves the firm problem unaffected relative to our baseline setup, but the collateral constraint is now:

$$w_t N_t + I_t \leq \xi_t q_t K_t \quad (28)$$

The only first order condition affected by this difference is the one over investment, which now becomes:

$$1 + \mu_t = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] \quad (29)$$

This conveys something interesting. In particular, even if $\phi = 0$, the price of capital, q_t , could be different than one in steady state, and fluctuate outside of steady state, if the collateral constraint binds.

3.1.1 Steady State

Let's solve for the steady state of the revised problem. From the accumulation equation, it must still be the case that $I = \delta K$. This means that steady state q is:

$$q = 1 + \mu \quad (30)$$

The expression for the steady state capital stock is the same as earlier:

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1 - \delta) - \mu \xi} \right)^{\frac{1}{1-\alpha}} \quad (31)$$

As before, begin by assuming that the collateral constraint binds. With $I = \delta K$, after plugging in for q we can then get an expression for the steady state real wage:

$$w = (\xi(1 + \mu) - \delta) \frac{K}{N}$$

From the labor demand condition, we also know that:

$$w = \frac{1}{1 + \mu} (1 - \alpha) \left(\frac{K}{N} \right)^\alpha$$

Then we have:

$$\frac{1}{1+\mu}(1-\alpha)\left(\frac{K}{N}\right)^\alpha = (\xi(1+\mu) - \delta)\frac{K}{N}$$

This can be written:

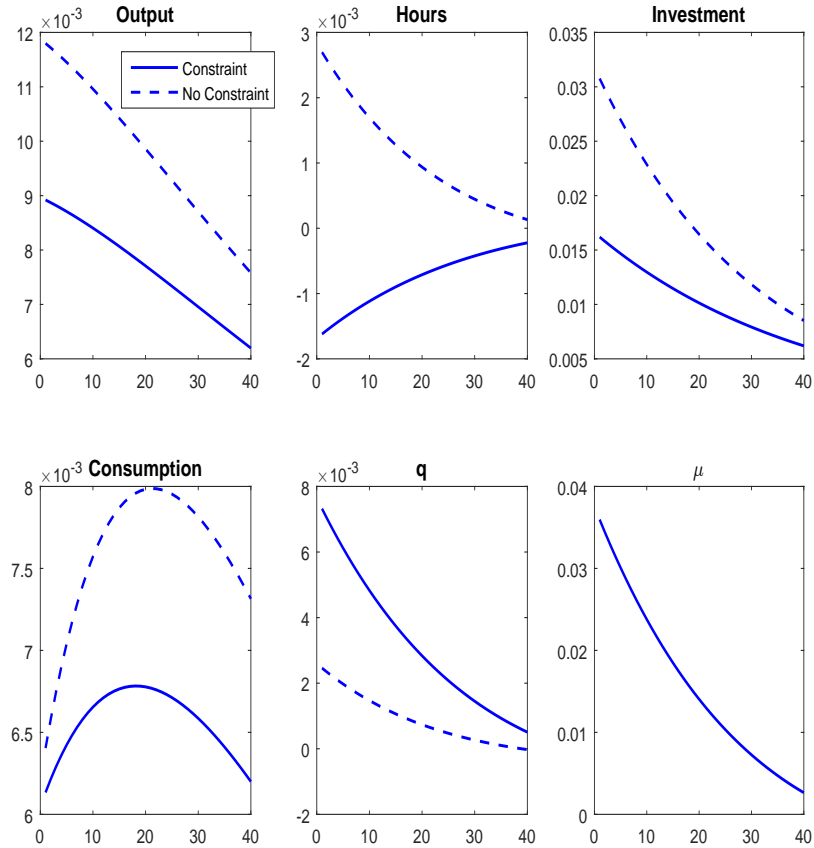
$$\frac{1}{1+\mu}(1-\alpha) = (\xi(1+\mu) - \delta)\left(\frac{K}{N}\right)^{1-\alpha}$$

Given that $\frac{K}{N}$ is now known as a function of μ , this expression can be solved for μ (it is albeit significantly more complicated than above).

3.1.2 Impulse Responses

I solve the model using the same parameterization as above in the case where the collateral constraint only applies to working capital for labor payments. Impulse responses to the productivity shock are shown in Figure 4.

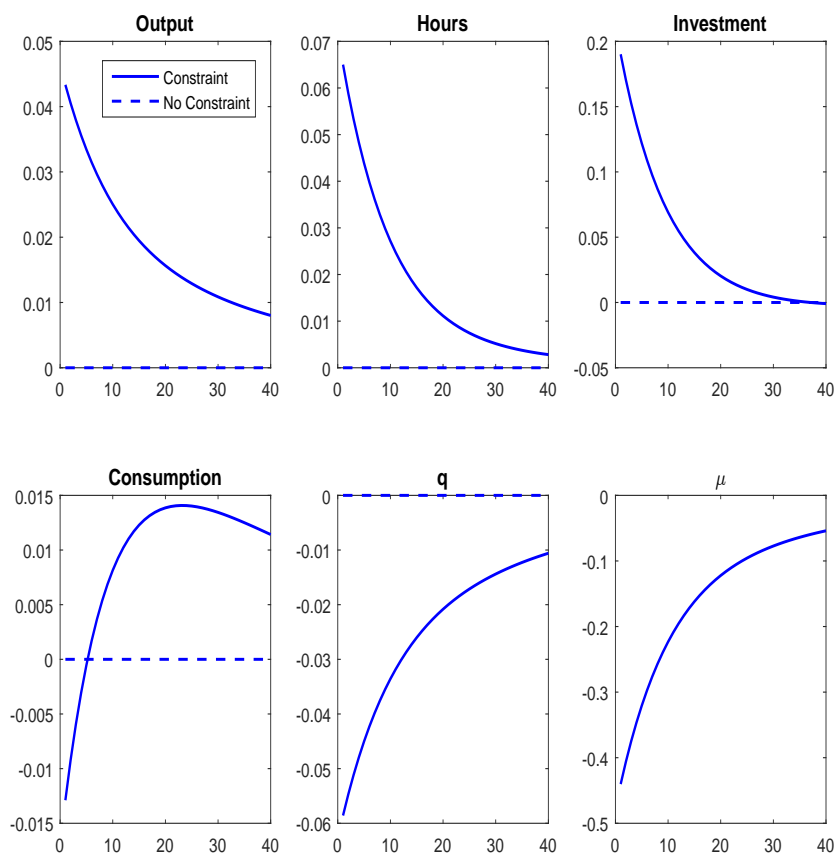
Figure 4: IRFs to Productivity Shock



When productivity increases, the firm would like to hire more labor and do more investment, so the constraint tightens. Consequently, μ_t increases. Similarly to the previous case, hours worked declines on impact. Differently than before, investment increases substantially less (relative to the unconstrained case). This is kind of mechanical given that investment now must be financed via working capital. The smaller increase in investment means that we get less propagation of the productivity shock relative to the case where working capital is only needed for labor payments. Hence, the output impulse response is not as hump-shaped.

Next, consider a financial shock. This eases the constraint, and hence results in μ_t falling. Differently than above, this results in q_t falling, not rising. Essentially, capital is less valuable. In spite of the decline in q_t , investment rises (because of the decline in μ_t). Consumption also declines, rather than rises, on impact here. The way to think about this is that prior to the financial shock, the economy was investing less (and hence consuming more) than it would like to in an unconstrained world. The easing of the constraint (when it applies to both payments to labor and investment) induces a substitution away from private consumption towards investment.

Figure 5: IRFs to Financial Shock



3.2 Collateral Constraint Applies to Intertemporal Debt

Suppose that, for unmodeled reasons, the firm finances all investment with new one period intertemporal debt (as opposed to the intratemporal debt considered in the working capital cases). That is, suppose that each period the firm issues new debt, D_{t+1} , to finance its current investment. In other words, we must have:

$$I_t = D_{t+1} \quad (32)$$

Each period, the firm is required to pay off principle plus interest on the debt it brought into the period. This amounts to paying off $(1 + r_{t-1})I_{t-1}$. Hence, the dividend the firm pays out is:

$$\Pi_t = Y_t - w_t N_t - (1 + r_{t-1})I_{t-1} \quad (33)$$

Now, the collateral constraint applies only to intertemporal debt. In particular:

$$D_{t+1} \leq \xi_t q_t K_t \quad (34)$$

Since $D_{t+1} = I_t$, this means that the firm's current investment is constrained by the value of its existing capital. The accumulation equation is the same as before. The firm problem now is:

$$\begin{aligned} \mathbb{L} = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ A_{t+j} K_{t+j}^\alpha N_{t+j}^{1-\alpha} - w_{t+j} N_{t+j} - (1 + r_{t-1}) I_{t+j-1} + \dots \right. \\ \left. q_{t+j} \left[I_{t+j} - \frac{\phi}{2} \left(\frac{I_{t+j}}{K_{t+j}} - \delta \right)^2 K_{t+j} + (1 - \delta) K_{t+j} - K_{t+j+1} \right] + \mu_{t+j} (\xi_{t+j} q_{t+j} K_{t+j} - I_{t+j}) \right\} \end{aligned}$$

The first order conditions are:

$$\frac{\partial \mathbb{L}}{\partial N_t} = 0 \Leftrightarrow w_t = (1 - \alpha) A_t K_t^\alpha N_t^{-\alpha} \quad (35)$$

$$\frac{\partial \mathbb{L}}{\partial I_t} = 0 \Leftrightarrow \mu_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 + r_t) = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] \quad (36)$$

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial K_{t+1}} = 0 \Leftrightarrow q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + \dots \right. \\ \left. q_{t+1} \left((1 - \delta) + \mu_{t+1} \xi_{t+1} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right] \quad (37) \end{aligned}$$

The first order condition with respect to K_t is the same as before. The first order condition for N_t now looks standard, without any wedge between the wage and the marginal product of labor. What is slightly different than before is the first order condition with respect to investment. From

the household's first order condition, we know that $\beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 + r_t) = 1$. Hence, this condition can be written:

$$1 + \mu_t = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] \quad (38)$$

This looks identical to the case where investment must be financed via working capital (as opposed to an intertemporal loan). If the collateral constraint does not bind, then $\mu_t = 0$, and this is the same first order condition as before (or as what would emerge in the standard RBC model). This is simply Modigliani-Miller – if there is no financial constraint, the equilibrium conditions will be identical if we force the firm to finance investment with debt (this case) or equity (the earlier case considered when there was a working capital constraint on labor).

The household holds all debt issued by the firm. The full set of equilibrium conditions can therefore be written:

$$\frac{1}{C_t} = \lambda_t \quad (39)$$

$$\theta N_t^\chi = \lambda_t w_t \quad (40)$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + r_t) \quad (41)$$

$$w_t = (1 - \alpha) A_t K_t^\alpha N_t^{-\alpha} \quad (42)$$

$$1 + \mu_t = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] \quad (43)$$

$$q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + \dots \right. \\ \left. q_{t+1} \left((1 - \delta) + \mu_{t+1} \xi_{t+1} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \right] \quad (44)$$

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} \quad (45)$$

$$Y_t = C_t + I_t \quad (46)$$

$$K_{t+1} = I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t + (1 - \delta) K_t \quad (47)$$

$$I_t \leq \xi_t q_t K_t \quad (48)$$

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \epsilon_{A,t} \quad (49)$$

$$\ln \xi_t = (1 - \rho_\xi) \ln \xi + \rho_\xi \ln \xi_{t-1} + s_\xi \epsilon_{\xi,t} \quad (50)$$

What we see here is that there really isn't a difference between requiring the firm to finance its investment with an intratemporal loan or intertemporal debt – for investment, the first order

conditions are the same.

3.2.1 The Steady State

We need to solve for the non-stochastic steady state. The capital-labor ratio in steady state is the same as in the working capital constraint model:

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1 - \delta) - \mu\xi} \right)^{\frac{1}{1-\alpha}} \quad (51)$$

The wage can be determined directly from this. From the accumulation equation, we also know that $I = \delta K$. From the first order condition for investment, we then have:

$$1 + \mu = q \quad (52)$$

In other words, if the constraint binds, $q > 1$. Further, q_t could fluctuate outside of steady state even without an adjustment cost (in a sense, similar to the discussion above, the collateral constraint on intertemporal debt functions like an adjustment cost). From the collateral constraint at equality, using $I = \delta K$, we get:

$$\delta = \xi(1 + \mu) \quad (53)$$

This means that:

$$\mu = \frac{\delta}{\xi} - 1 \quad (54)$$

Hence, for the collateral constraint to bind, we must have $\xi < \delta$ (so that $\mu > 0$). Once we have μ , we can then have $\frac{K}{N}$, and we can use that to solve for N as:

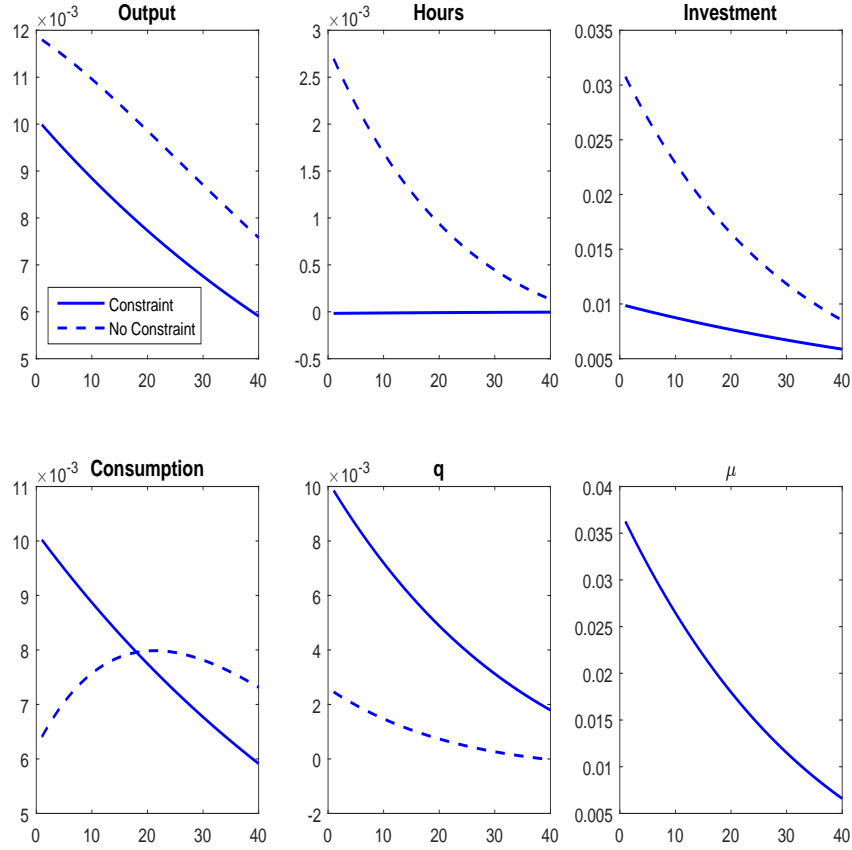
$$N = \left[\frac{1(1-\alpha) \left(\frac{K}{N}\right)^\alpha}{\theta \left(\frac{K}{N}\right)^\alpha - \delta \frac{K}{N}} \right]^{\frac{1}{1+\alpha}} \quad (55)$$

This is similar to the case where the collateral constraint applies to working capital, but there is no explicit μ term present.

3.2.2 Impulse Responses

The model is parameterized as above. I set $\xi = 0.015$, which ensures that the collateral constraint binds in the steady state. Impulse responses to the productivity shock (Figure 6) and financial shock (Figure 7) are shown below:

Figure 6: IRFs to Productivity Shock

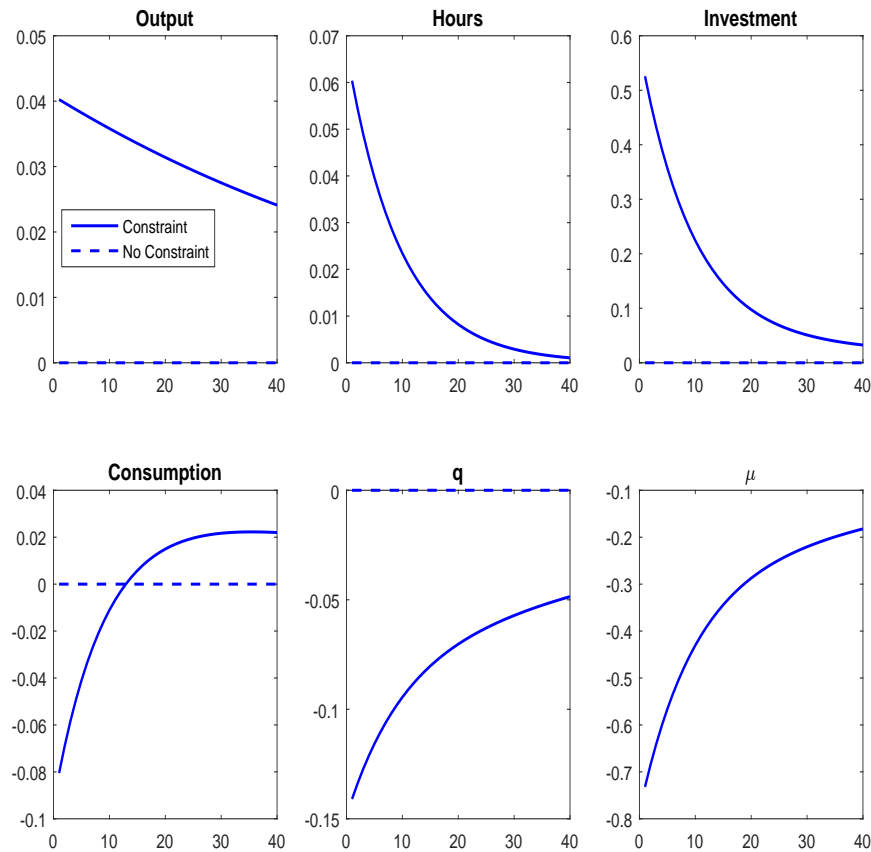


We can see here that the intertemporal collateral constraint dampens the output response to a productivity shock. When productivity increases, μ_t increases (essentially, the collateral constraint becomes tighter). The firm would like to do more investment, but can only increase its investment by the amount that its capital increases in value. Labor hours essentially do not react at any horizon, meaning that the output response is close to equal to the increase in exogenous productivity. In contrast, consumption increases by more in the economy with the constraint than without. What is going on? When productivity improves, the firm would like to increase its investment, but is unable to do so by much because of the constraint binding. Because investment can't go up by much, consumption goes up by more. But consumption going up by more means that hours go up by less (there is a bigger offsetting wealth effect on labor supply), leaving labor hours roughly unchanged.

Next, consider the responses to the financial shock. When ξ_t increases, μ_t falls (the constraint becomes looser). This results in a decline in q_t – capital is less valuable because it's not as necessary to ease the collateral constraint. Investment increases, in a rather mechanical way – you can borrow

more, so you invest more. Note that investment and q_t no longer move in the same direction here. Consumption declines. The reason consumption declines is that you want to do more investment – essentially, prior to the shock the economy is constrained from doing as much investment as it wants, and the easing of the constraint means that you substitute away from consumption and into investment. The fall in consumption results in hours going up (and the wage going down, though this is not shown). As a result of the increase in hours, output goes up.

Figure 7: IRFs to Financial Shock



You will note that these responses look very similar to the case where both labor and investment must be financed via working capital.