Growth

ECON 30020: Intermediate Macroeconomics

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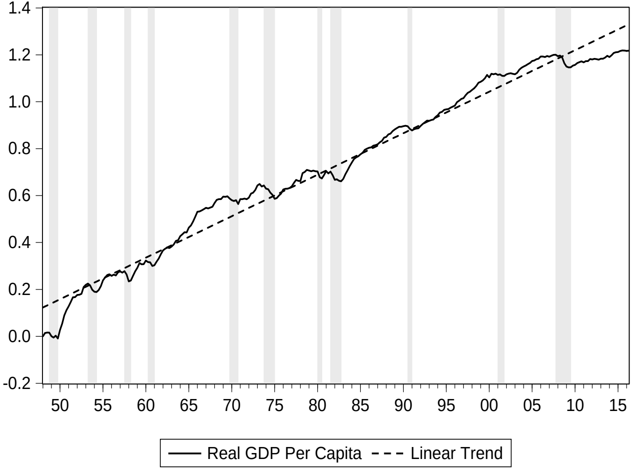
Readings

- GLS Ch. 4 (facts)
- GLS Ch. 5-6 (Solow Growth Model)
- GLS Ch. 7 (cross-country income differences)

Economic Growth

- When economists say "growth," typically mean average rate of growth in real GDP per capita over long horizons
- Long run: frequencies of time measured in decades
- *Not* period-to-period fluctuations in the growth rate
- "Once one begins to think about growth, it is difficult to think about anything else" – Robert Lucas, 1995 Nobel Prize winner

US Real GDP per capita



Summary Stats

- Average (annualized) *growth rate* of per capita real GDP: 1.8%
- Implies that the *level* of GDP doubles roughly once every 40 years
- Growing just 0.2 percentage points faster (2% growth rate): level doubles every 35 years
- Rule of 70: number of years it takes a variable to double is approximately 70 divided by the growth rate
- Consider two countries that start with same GDP, but country A grows 2% per year and country B grows 1% per year. After 100 years, A will be 165% richer!
- Small differences in growth rates really matter over long horizons

Key Question

- What accounts for this growth?
- In a mechanical sense, can only be two things:
  - Growth in productivity: we produce more output given the same inputs
  - Factor accumulation: more factors of production help us produce more stuff
    - Two key factors of production on which we focus are capital and labor
    - Labor input per capita is roughly trendless – empirically not a source of growth in per capita output
    - The key factor of production over the long run is *capital*: physical stuff that must itself be produced that in turn helps us produce more stuff (e.g. machines).
- Which is it? Productivity or capital accumulation? What are policy implications?
- What accounts for differences in standards of living across countries? Productivity or factor accumulation?

Stylized Facts: Time Series

1. Output per worker grows at an approximately constant rate over long periods of time ▸ picture
2. Capital per worker grows at an approximately constant rate over long periods of time ▸ picture
3. The capital to output ratio is roughly constant over long periods of time ▸ picture
4. Labor's share of income is roughly constant over long periods of time ▸ picture
5. The return to capital is roughly constant over long periods of time ▸ picture
6. The real wage grows at approximately the same rate as output per worker over long periods of time ▸ picture

Stylized Facts: Cross-Section

1. There are large differences in income per capita across countries ▸ table
2. There are some examples where poor countries catch up (growth miracles), otherwise where they do not (growth disasters) ▸ table
3. Human capital (e.g. education) strongly correlated with income per capita ▸ table

Solow Model

- Model we use to study long run growth and cross-country income differences is the Solow model, after Solow (1956)
- Model capable of capturing stylized facts
- Main implication of model: productivity is key
  - Productivity key to sustained growth (not factor accumulation)
  - Productivity key to understanding cross-country income differences (not level of capital)
- Important implications for policy
- Downside of model: takes productivity to be exogenous. What is it? How to increase it?

Model Basics

- Time runs from t (the present) onwards into infinite future
- Representative household and representative firm
- Everything real, one kind of good

Production Function

- Production function:

$$Y_t = A_t F(K_t, N_t)$$

- K_t: capital. Must be itself produced, used to produce other stuff, does not get completely used up in production process
- N_t: labor
- Y_t: output
- A_t: productivity (exogenous)
- Think about output as units of fruit. Capital is stock of fruit trees. Labor is time spent picking from the trees

Properties of Production Function

- Both inputs necessary: $F(0, N_t) = F(K_t, 0) = 0$
- Increasing in both inputs: $F_K(K_t, N_t) > 0$ and $F_N(K_t, N_t) > 0$
- Concave in both inputs: $F_{KK}(K_t, N_t) < 0$ and $F_{NN}(K_t, N_t) < 0$
- Constant returns to scale: $F(qK_t, qN_t) = qF(K_t, N_t)$
- Capital and labor are paid marginal products:

$$w_t = A_t F_N(K_t, N_t)$$
$$R_t = A_t F_K(K_t, N_t)$$

- Example production function: Cobb-Douglas:

$$F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1$$

Consumption, Investment, Labor Supply

- Fruit can either be eaten (consumption) or re-planted in the ground (investment), the latter of which yields another tree (capital) with a one period delay
- Assume that a constant fraction of output is invested, $0 \leq s \leq 1$. "Saving rate" or "investment rate"
- Means $1 - s$ of output is consumed
- Resource constraint:

$$Y_t = C_t + I_t$$

- Abstract from endogenous labor supply – labor supplied inelastically and constant
- Current capital stock is exogenous – depends on past decisions
- Capital accumulation, $0 < \delta < 1$ depreciation rate:

$$K_{t+1} = I_t + (1 - \delta) K_t$$

Equations of Model

$$Y_t = A_t F(K_t, N_t)$$

$$Y_t = C_t + I_t$$

$$K_{t+1} = I_t + (1 - \delta) K_t$$

$$C_t = (1 - s) Y_t$$

$$I_t = s Y_t$$

$$w_t = A_t F_N(K_t, N_t)$$

$$R_t = A_t F_K(K_t, N_t)$$

- Six endogenous variables $(Y_t, C_t, K_{t+1}, I_t, w_t, R_t)$ and three exogenous variables $(A_t,\ K_t,\ N_t)$

Central Equation

- First four equations can be combined into one:

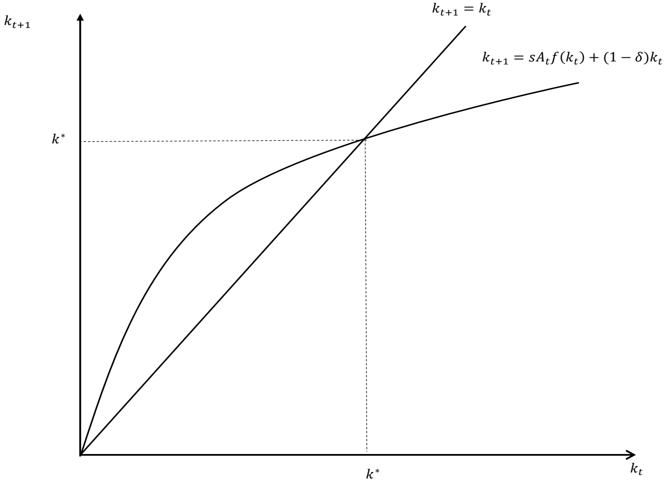
$$K_{t+1} = sA_t F(K_t, N_t) + (1-\delta)K_t$$

- Define lowercase variables as "per worker." $k_t = \frac{K_t}{N_t}$. In per-worker terms:

$$k_{t+1} = sA_t f(k_t) + (1-\delta)k_t$$

- One equation describing dynamics of k_t. Once you know dynamic path of capital, you can recover everything else

Plot of the Central Equation of the Solow Model



The Steady State

- The steady state capital stock is the value of capital at which $k_{t+1} = k_t$
- We call this k^*
- Graphically, this is where the curve (the plot of k_{t+1} against k_t) crosses the 45 degree line (a plot of $k_{t+1} = k_t$)
- Via assumptions of the production function along with auxiliary assumptions (the Inada conditions), there exists one non-zero steady state capital stock
- The steady state is "stable" in the sense that for any initial $k_t \neq 0$, the capital stock will converge to this point
- "Once you get there, you sit there"
- Since capital governs everything else, all other variables go to a steady state determined by k^*
- Work through dynamics

Algebraic Example

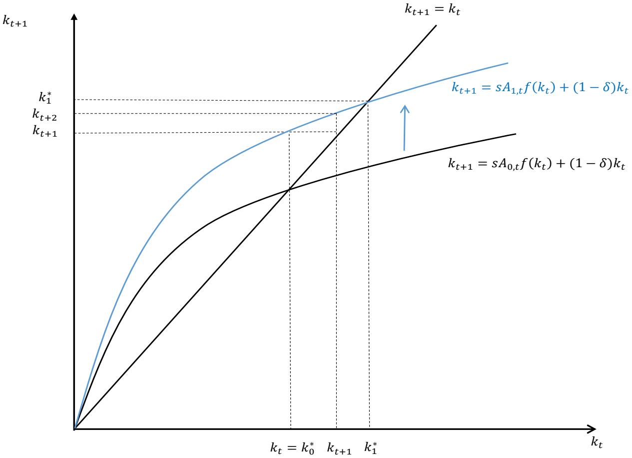
- Suppose $f(k_t) = k_t^{\alpha}$. Suppose A_t is constant at A^*. Then:

$$
\begin{aligned}
k^* &= \left(\frac{sA^*}{\delta}\right)^{\frac{1}{1-\alpha}} \\
y^* &= A^* k^{*\alpha} \\
c^* &= (1-s)A^* k^{*\alpha} \\
i^* &= sA^* k^{*\alpha} \\
R^* &= \alpha A^* k^{*\alpha-1} \\
w^* &= (1-\alpha)A^* k^{*\alpha}
\end{aligned}
$$

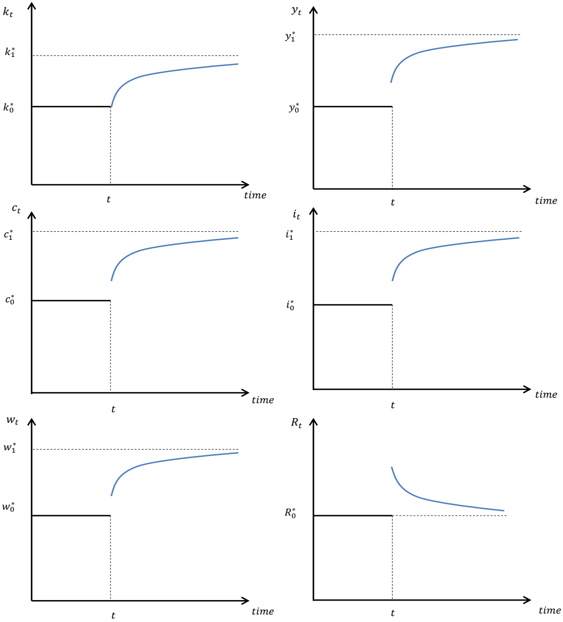
Dynamic Effects of Changes in Exogenous Variables

- Want to consider the following exercises:
  - What happens to endogenous variables in a dynamic sense after a permanent change in A^* (the constant value of A_t)?
  - What happens to endogenous variables in a dynamic sense after a permanent change in s (the saving rate)?
- For these exercises:
  1. Assume we start in a steady state
  2. Graphically see how the steady state changes after the change in productivity or the saving rate
  3. *Current* capital stock cannot change (it is predetermined/exogenous). But $k_t \neq k^*$. Use dynamic analysis of the graph to figure out how k_t reacts dynamically
  4. Once you have that, you can figure out what everything else is doing

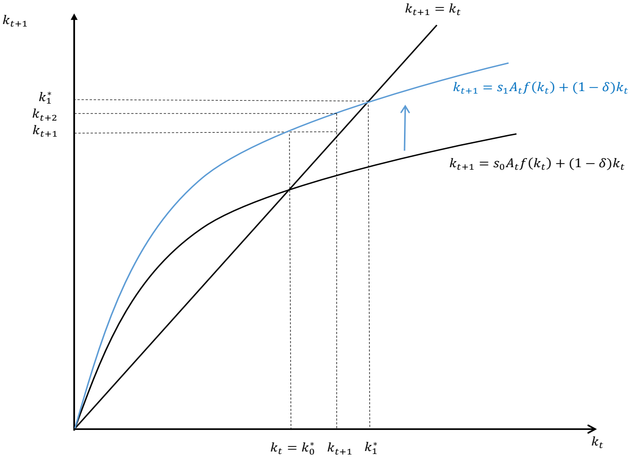
Permanent Increase in A^*



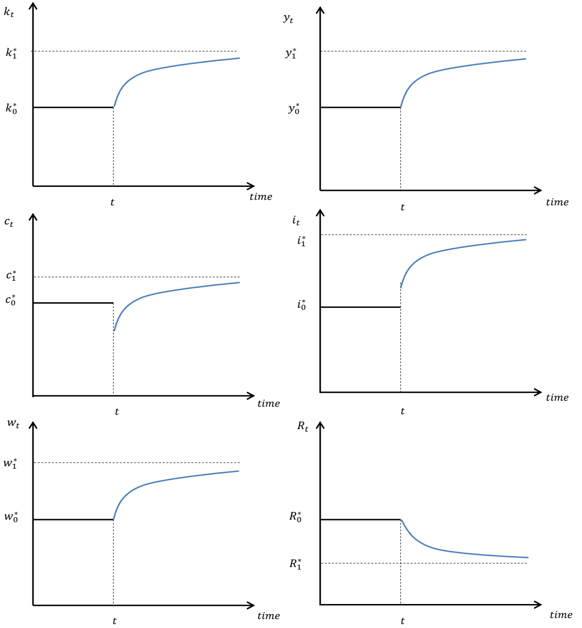
Impulse Response Functions: Permanent Increase in A^*



Permanent Increase in s



Impulse Response Functions: Permanent Increase in s



Discussion

- Neither changes in A^* nor s trigger sustained increases in *growth*
- Each triggers faster growth *for a while* while the economy accumulates more capital and transitions to a new steady state
- Aren't we supposed to be studying growth? In the long run, there is *no growth* in this model – it goes to a steady state!
- We'll fix that. You can kind of see, however, that sustained growth must come from increases in productivity. Why?
  - No limit on how high A can get – it can just keep increasing. Upper bound on s
  - Repeated increases in s would trigger continual decline in R_t, inconsistent with stylized facts
- Bottom line: sustained growth must be due to productivity growth, not factor accumulation. You can't save your way to more growth
- Key assumption: diminishing returns to capital

Golden Rule

- What is the "optimal" saving rate, s?
- Utility from consumption, not output
- Higher s has two effects – the "size of the pie" and the "fraction of the pie":
  - More capital $\rightarrow$ more output $\rightarrow$ more consumption (bigger size of the pie)
  - Consume a smaller fraction of output $\rightarrow$ less consumption (eat a smaller fraction of the pie)
- Golden Rule: value of s which maximizes steady state consumption, c^*
  - $s = 0$: $c^* = 0$
  - $s = 1$: $c^* = 0$
- Implicity characterized by $A^* f'(k^*) = \delta$. Graphical intuition.

Growth

- Wrote down a model to study growth
- But model converges to a steady state with no growth
- Isn't that a silly model?
- It turns out, no
- Can modify it

Augmented Solow Model

- Production function is:

$$Y_t = A_t F(K_t, Z_t N_t)$$

- Z_t: labor-augmenting productivity
- $Z_t N_t$: efficiency units of labor
- Assume Z_t and N_t both grow over time (initial values in period 0 normalized to 1):

$$Z_t = (1+z)^t$$
$$N_t = (1+n)^t$$

- $z = n = 0$: case we just did
- Z_t not fundamentally different from A_t. Mathematically convenient to use Z_t to control *growth* while A_t controls *level* of productivity

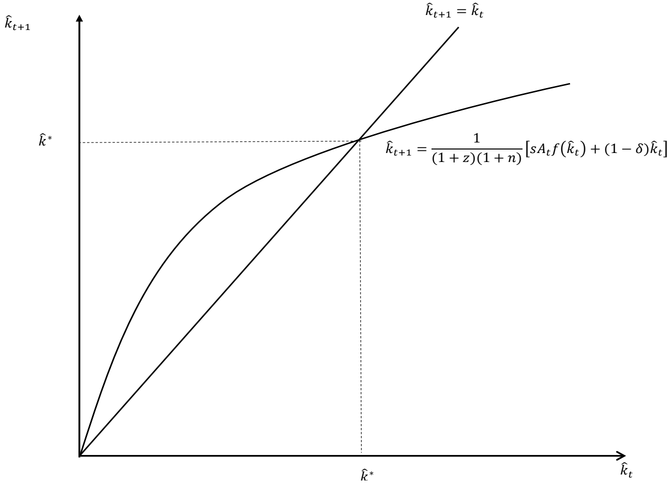
Per Efficiency Unit Variables

- Define $\widehat{k}_t = \frac{K_t}{Z_t N_t}$ and similarly for other variables. Lower case variables: per-capita. Lower case variables with "hats": per efficiency unit variables
- Can show that modified central equation of model is:

$$\widehat{k}_{t+1} = \frac{1}{(1+z)(1+n)} \left[sA_t f(\widehat{k}_t) + (1-\delta)\widehat{k}_t \right].$$

- Practically the same as before

Plot of Modified Central Equation



Steady State Growth

- Via similar arguments to earlier, there exists a steady state $\widehat{k}^*$ at which $\widehat{k}_{t+1} = \widehat{k}_t$. Economy converges to this point from any non-zero initial value of $\widehat{k}_t$
- Economy converges to a steady state in which per efficiency unit variables do not grow. What about actual and per capita variables? If $\widehat{k}_{t+1} = \widehat{k}_t$, then:

$$\frac{K_{t+1}}{Z_{t+1}N_{t+1}} = \frac{K_t}{Z_t N_t}$$

$$\frac{K_{t+1}}{K_t} = \frac{Z_{t+1}N_{t+1}}{Z_t N_t} = (1+z)(1+n)$$

$$\frac{k_{t+1}}{k_t} = \frac{Z_{t+1}}{Z_t} = 1+z$$

- Level of capital stock grows at approximately sum of growth rates of Z_t and N_t. Per capita capital stock grows at rate of growth in Z_t
- This growth is manifested in output and the real wage, but not the return on capital

Steady State Growth and Stylized Facts

- Once in steady state, we have:

$$\frac{y_{t+1}}{y_t} = 1 + z$$

$$\frac{k_{t+1}}{k_t} = 1 + z$$

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{K_t}{Y_t}$$

$$\frac{w_{t+1}N_{t+1}}{Y_{t+1}} = \frac{w_t N_t}{Y_t}$$

$$R_{t+1} = R_t$$

$$\frac{w_{t+1}}{w_t} = 1 + z$$

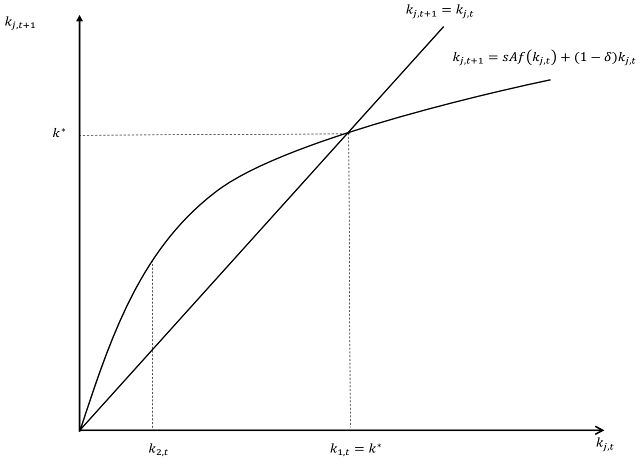
- These are the six time series stylized facts!

Understanding Cross-Country Income Differences

- Solow model can reproduce time series stylized facts if it is assumed that productivity grows over time
- Let's now use the model to think about cross-country income differences
- What explains these differences? Three hypotheses for why cross-country income differences exist:
  1. Countries initially endowed with different levels of capital
  2. Countries have different saving rates
  3. Countries have different productivity levels
- Like sustained growth, most plausible explanation for cross-country income differences is productivity
- Consider standard Solow model and two countries, 1 and 2. Suppose that 2 is poor relative to 1

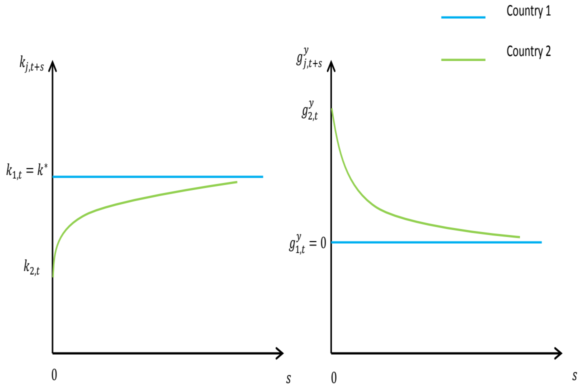
Convergence

- Suppose two countries are otherwise identical, and hence have the same steady state.
- But suppose that country 2 is initially endowed with less capital – $k_{2,t} < k_{1,t} = k^*$

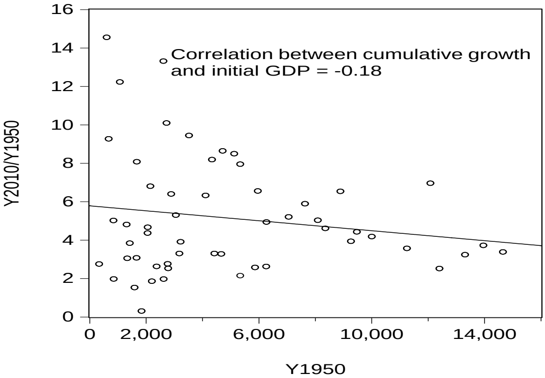


Catch Up

- If country 2 is initially endowed with less capital, it should grow faster than country 1, eventually catching up with country 1

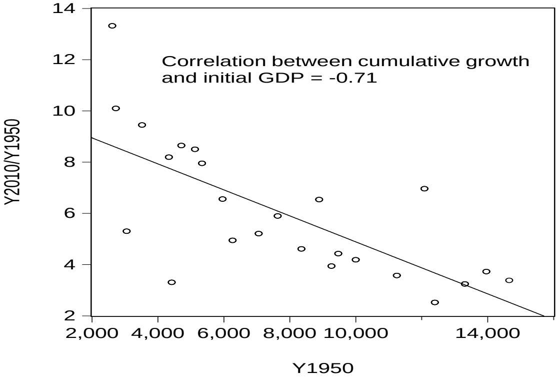


Is There Convergence in the Data?



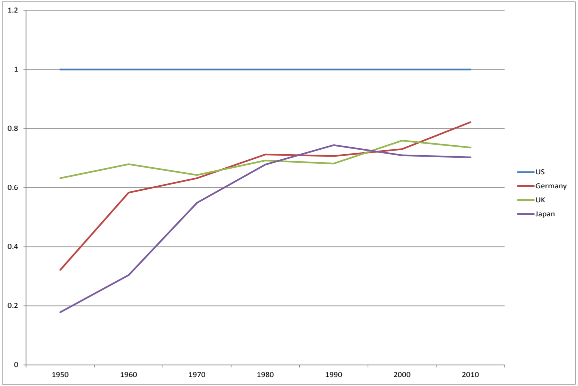
- Correlation between growth and initial GDP is weakly negative when focusing on all countries

Focusing on a More Select Group of Countries



- Focusing only on OECD countries (more similar) story looks more promising for convergence
- Still, catch up seems too slow for initial low levels of capital to be the main story

Pseudo Natural Experiment: WWII



- WWII losers (Germany and Japan) grew faster for 20-30 years than the winners (US and UK)
- But don't seem to be catching up all the way to the US: *conditional convergence*. Countries have different steady states

Differences in s and A^*

- Most countries seem to have different steady states
- For simple model with Cobb-Douglas production function, relative outputs:

$$\frac{y_1^*}{y_2^*} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{1-\alpha}} \left(\frac{s_1}{s_2}\right)^{\frac{\alpha}{1-\alpha}} .$$

- Question: can differences in s plausibly account for large income differences?
- Answer: no

Differences in s

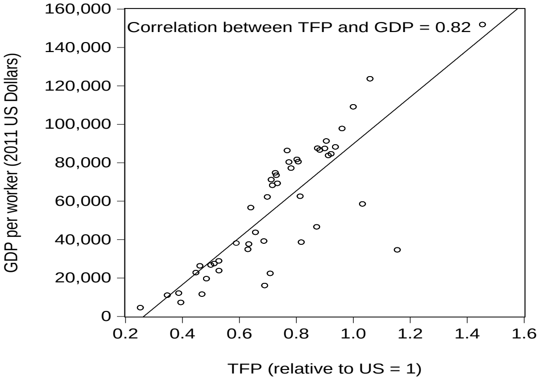
- Suppose A^* the same in both countries. Suppose country 1 is US, and country 2 is Mexico: $\frac{y_1^*}{y_2^*} = 4$. We have:

$$s_2 = 4^{\frac{\alpha-1}{\alpha}} s_1$$

- Based on data, a plausible value of $\alpha = 1/3$. Means $\frac{\alpha-1}{\alpha} = -2$
- Mexican saving rate would have to be 0.0625 times US saving rate
- This would be something like a saving rate of one percent (or less)
- Not plausible
- Becomes more plausible if α is much bigger

What Could It Be?

- If countries have different steady states and differences in s cannot plausibly account for this, must be differences in productivity
- Seems to be backed up in data: rich countries are highly productive



Productivity is King

- Productivity is what drives everything in the Solow model
- Sustained growth must come from productivity
- Large income differences must come from productivity
- But what is productivity? Solow model doesn't say

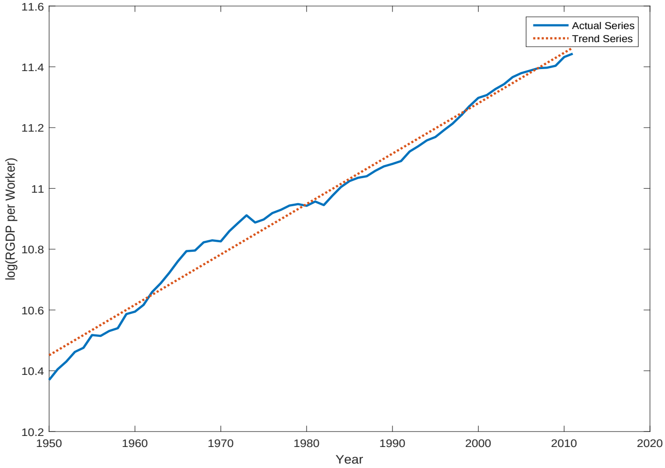
Factors Influencing Productivity

- Including but not limited to:
  1. Knowledge and education
  2. Climate
  3. Geography
  4. Institutions
  5. Finance
  6. Degree of openness
  7. Infrastructure

Policy Implications

- If a country wants to become richer, need to focus on policies which promote productivity
- Example: would giving computers (capital) to people in sub-Saharan Africa help them get rich? Not without the infrastructure to connect to the internet, the knowledge of how to use the computer, and the institutions to protect property rights
- Also has implications when thinking about poverty within a country

Output Per Worker over Time



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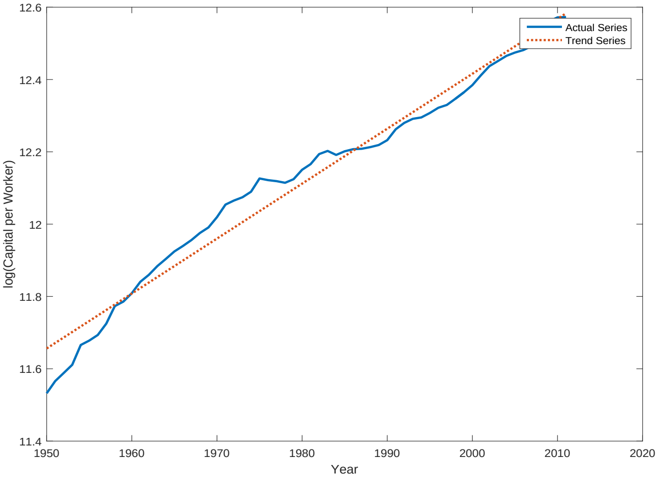
Capital Per Worker over Time



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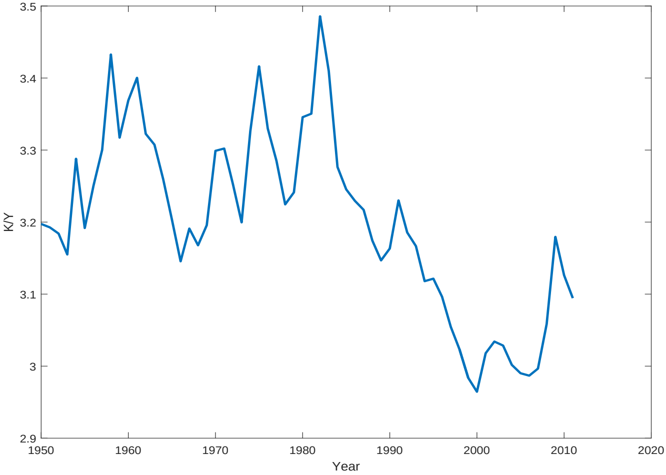
Capital to Output Ratio over Time



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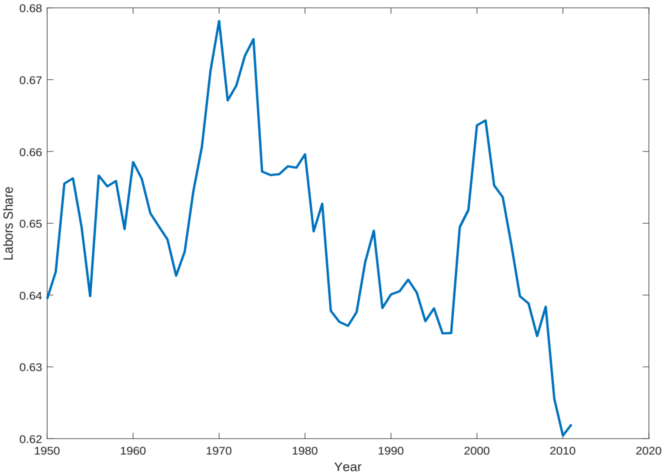
Labor Share over Time



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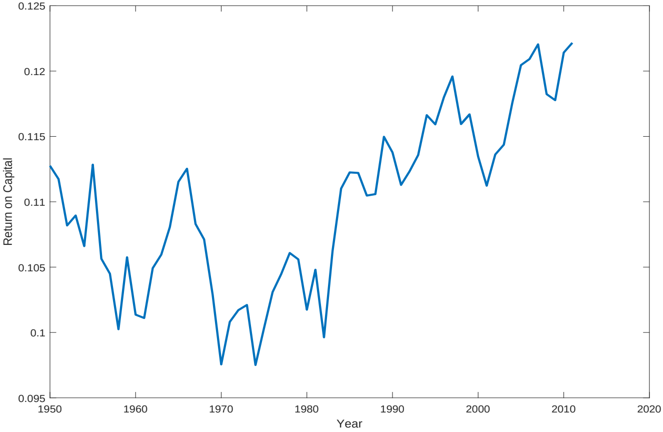
Return on Capital over Time



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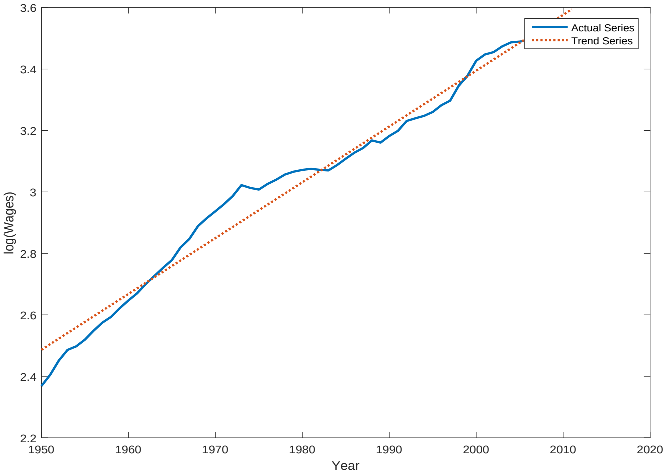
Real Wage over Time



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Income Differences

		GDP per Person
High income countries		
	Canada	$35,180
	Germany	$34,383
	Japan	$30,232
	Singapore	$59,149
	United Kingdom	$32,116
	United States	$42,426
Middle income countries		
	China	$8,640
	Dominican Republic	$8,694
	Mexico	$12,648
	South Africa	$10,831
	Thailand	$9,567
	Uruguay	$13,388
Low income countries		
	Cambodia	$2,607
	Chad	$2,350
	India	$3,719
	Kenya	$1636
	Mali	$1,157
	Nepal	$1,281

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Growth Miracles and Disasters

	1970 Income	2011 Income	% change
	Growth Miracles		
South Korea	$1918	$27,870	1353
Taiwan	$4,484	$33,187	640
China	$1,107	$8,851	700
Botswana	$721	$14,787	1951
	Growth Disasters		
Madagascar	$1,321	$937	-29
Niger	$1,304	$651	-50
Burundi	$712	$612	-14
Central African Republic	$1,148	$762	-34

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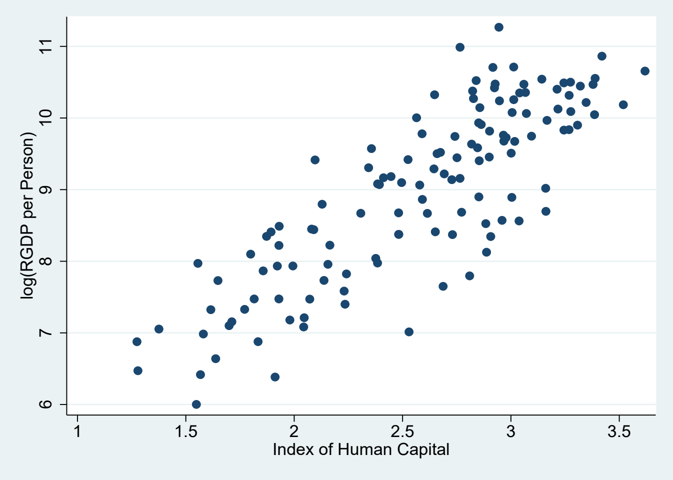
Education and Income Per Capita



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