Advanced Macro: Jermann and Quadrini (2012, *American Economic Review*)

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1 Overview

This is a paper that incorporates novel collateral constraint for firms. It is in the Kiyotaki and Moore (1997) tradition. But in Jermann and Quadrini (2012), firms have to get an intraperiod loan to finance all their activities (i.e. there is a cash-flow mismatch). The amount of this working capital loan is constrained to equal a fraction of the liquidation value of the firm (end-of-period capital stock less lender's value of the firm's debt). The fraction is taken to be stochastic and interpreted as a financial shock. The fact that the borrowing constraint applies to working capital, and not intertemporal debt, generates an endogenous labor wedge that distorts the FOC for labor demand.

The paper mostly focused on a RBC model. Much of it is straightforward. There are two non-standard ingredients. First, there is a tax preference for firms to issue debt as opposed to paying (or reducing) dividends. This is essentially like making the firms impatient relative to the household in other models we've seen – it makes the firm want to issue debt, and insures that it ends up bumping up against its borrowing constraint. Second, there is a dividend adjustment cost. This is needed because without it, firms can adjust dividends seamlessly in such a way as to make the borrowing constraint not very important for cyclical dynamics.

The paper also includes a more involved model with sticky prices and wages and other frictions, based on Smets and Wouters (2007). This was likely included at the request of annoying referees and is not central to what they are doing. It is also not particularly well-spelled out by the authors. I'm only going to focus on the RBC part and will only ask you to do the same.

2 The RBC Model

2.1 Firms

There are a continuum of firms in the $[0, 1]$ interval. But there is no idiosyncratic uncertainty so it is as though there is just a representative firm. They produce output according to:

$$y_t = z_t k_t^\theta n_t^{1-\theta} \tag{1}$$

Capital evolves according to:

$$k_{t+1} = i_t + (1-\delta)k_t \tag{2}$$

Firms use equity, d_t, and debt, b_t. In addition, they use an intraperiod loan, l_t. The real interest rate is r_t. There is a tax advantage to debt. The effective gross interest rate available to firms is $R_t = 1 + r_t(1-\tau)$, where τ represents the tax benefit. This is kind of like assuming that the firms are impatient – you need a mechanism in the model to get the firms to hold debt so that their borrowing constraint will end up binding.

The intraperiod loan is used to finance all payments; the basic idea is that you have to pay workers, bond holders, and equity holders prior to producing. So we have:

$$l_t = w_t n_t + i_t + \varphi(d_t) + b_t - \frac{b_{t+1}}{R_t} \tag{3}$$

b_{t+1} are one-period discount bonds that pay out in $t+1$. They sell at price $\frac{1}{R_t}$. So b_t is the payment to existing bond holders, whereas b_{t+1}/R_t is the issuance of new debt (which reduces the amount of the working capital loan). Working capital must cover labor payments, new investment, and payouts to equity holders, $\varphi(d_t)$. Dividends are potentially subject to an adjustment cost – see below. Without the adjustment cost, $\varphi(d_t) = d_t$.

The firm's budget constraint is:

$$b_t + w_t n_t + i_t + \varphi(d_t) = y_t + \frac{b_{t+1}}{R_t} \tag{4}$$

On the "expenditure side," the firm pays off interperiod debt, pays workers, pays for new capital, and pays dividends, d_t, which are potentially subject to an adjustment cost. On the "income side," the firm earns revenue from output and issues new debt. New debt trades at price $1/R_t$.

Combining (3)-(4), we see that the intraperiod loan is equal to output, $l_t = y_t$.

The firm is subject to an enforcement constraint:

$$\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) \geq l_t \tag{5}$$

Stochastic variations in ξ_t will be considered financial shocks. Basically, the idea is that the firm's intraperiod loan is constrained by the liquidation value of its net assets – which is $k_{t+1} - b_{t+1}/(1+r_t)$ (it's $1+r_t$ because the lender doesn't get the tax benefit).

It is assumed that there is an adjustment cost to changing the equity payout. It is given by:

$$\varphi(d_t) = d_t + \kappa(d_t - \bar{d})^2 \tag{6}$$

$\kappa \geq 0$ will be important for the way the model performs. If $\kappa = 0$, then changes in financial conditions can basically be offset by changing dividends – in other words, tightening or loosening of the borrowing constraints on intraperiod debt can be undone through adjusting the dividend payout. So it's important to get dividends to be sticky in some sense.

Let $m_{t,t+s}$ be the stochastic discount factor. The value of the firm is:

$$V_t = \mathbb{E}_t \sum_{s=0}^{\infty} m_{t,t+s} d_{t+s}$$

Where with the adjustment cost we have:

$$\varphi(d_t) = d_t + \kappa(d_t - \bar{d})^2 = z_t k_t^{\theta} n_t^{1-\theta} - w_t n_t - k_{t+1} + (1-\delta)k_t - b_t + \frac{b_{t+1}}{R_t} \tag{7}$$

Effectively, the actual dividend payout is what it would normally be, minus $\kappa(d_t - \bar{d})^2$. The problem is therefore to pick d_t, b_{t+1}, k_{t+1}, and n_t subject to the enforcement constraint, (21), and the budget constraint, (7):

$$\max_{d_t, n_t, k_{t+1}, b_{t+1}} \quad \mathbb{E}_t \sum_{t=0}^{\infty} m_{0,t} d_t$$

s.t.

$$\varphi(d_t) = z_t k_t^{\theta} n_t^{1-\theta} - w_t n_t - k_{t+1} + (1-\delta)k_t - b_t + \frac{b_{t+1}}{R_t}$$

$$\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) \geq z_t k_t^{\theta} n_t^{1-\theta}$$

Let λ_t be the multiplier on the budget constraint, and μ_t be the multiplier on the enforcement constraint. A Lagrangian is:

$$\mathbb{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \Bigg\{ d_t + \lambda_t \left[z_t k_t^{\theta} n_t^{1-\theta} - w_t n_t - k_{t+1} + (1-\delta)k_t - b_t + \frac{b_{t+1}}{R_t} - \varphi(d_t) \right] + \mu_t \left[\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) - z_t k_t^{\theta} n_t^{1-\theta} \right] \Bigg\}$$

The FOC are:

$$\frac{\partial \mathbb{L}}{\partial d_t} = 1 - \lambda_t \varphi'(d_t)$$

$$\frac{\partial \mathbb{L}}{\partial n_t} = \lambda_t \left((1-\theta) z_t k_t^{\theta} n_t^{-\theta} - w_t \right) - \mu_t (1-\theta) z_t k_t^{\theta} n_t^{-\theta}$$

$$\frac{\partial \mathbb{L}}{\partial b_{t+1}} = \lambda_t \frac{1}{R_t} - \frac{\mu_t \xi_t}{1+r_t} - m_{t,t+1} \lambda_{t+1}$$

$$\frac{\partial \mathbb{L}}{\partial k_{t+1}} = -\lambda_t + \mu_t \xi_t + \mathbb{E}_t\, m_{t,t+1} \lambda_{t+1} \left(\theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} + (1-\delta) \right) - \mathbb{E}_t\, m_{t,t+1} \mu_{t+1} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta}$$

Setting these equal to zero, we can solve out for λ_t:

$$\lambda_t = \frac{1}{\varphi'(d_t)} \tag{8}$$

Now sub this into the different FOC. We can write the labor supply condition as:

$$(1-\theta) z_t k_t^{\theta} n_t^{-\theta} - w_t = \varphi'(d_t) \mu_t (1-\theta) z_t k_t^{\theta} n_t^{-\theta}$$

Or:

$$w_t = \left(1 - \mu_t \varphi'(d_t)\right)(1-\theta) z_t k_t^{\theta} n_t^{-\theta} \tag{9}$$

Note that (9) is equivalent to (4) in their paper.

The FOC for bonds can be written:

$$\frac{\lambda_t}{R_t} = \frac{\mu_t \xi_t}{1+r_t} + \mathbb{E}_t \, m_{t,t+1} \lambda_{t+1}$$

After eliminating λ_t, we can write this as:

$$1 = \varphi'(d_t) \mu_t \xi_t \frac{R_t}{1+r_t} + \mathbb{E}_t \, m_{t,t+1} R_t \frac{\varphi'(d_t)}{\varphi'(d_{t+1})} \tag{10}$$

Note that (10) is the same as (6) in their paper.

Now go to the FOC for capital. Setting it equal to zero, we have:

$$\lambda_t = \mu_t \xi_t + \mathbb{E}_t \, m_{t,t+1} \lambda_{t+1}(1-\delta) + \mathbb{E}_t \, m_{t,t+1} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} (\lambda_{t+1} - \mu_{t+1})$$

Which can be written:

$$1 = \frac{\mu_t \xi_t}{\lambda_t} + \mathbb{E}_t \, m_{t,t+1} \frac{\lambda_{t+1}}{\lambda_t}(1-\delta) + \mathbb{E}_t \, m_{t,t+1} \frac{1}{\lambda_t} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} (\lambda_{t+1} - \mu_{t+1})$$

Which can be simplified further:

$$1 = \frac{\mu_t \xi_t}{\lambda_t} + \mathbb{E}_t \, m_{t,t+1} \frac{\lambda_{t+1}}{\lambda_t}(1-\delta) + \mathbb{E}_t \, m_{t,t+1} \frac{\lambda_{t+1}}{\lambda_t} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} \left(1 - \frac{\mu_{t+1}}{\lambda_{t+1}}\right)$$

But then given the definition of λ_t, this becomes:

$$1 = \mu_t \xi_t \varphi'(d_t) + \mathbb{E}_t \, m_{t,t+1} \frac{\varphi'(d_t)}{\varphi'(d_{t+1})} \left[1 - \delta + \left(1 - \mu_{t+1} \varphi'(d_{t+1})\right) \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta}\right] \tag{11}$$

(11) is the same as (5) in their paper.

2.2 Household

There is a representative household. Its utility function is:

$$U(c_t, n_t) = \ln c_t + \alpha \ln(1 - n_t)$$

Its budget constraint is:

$$c_t + p_t s_{t+1} + \frac{b_{t+1}}{1 + r_t} = w_t n_t + b_t + s_t d_t + s_t p_t - T_t \tag{12}$$

Here s_t is the number of shares of equity the household enters a period with; d_t is the equity payout and p_t is the price. The household can consume, buy more shares, or issue more bonds. Its income comes from working, payouts on existing bonds, dividend plus capital gains on shares, less a lump sum tax.

A Lagrangian is:

$$\mathbb{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \alpha \ln(1 - n_t) + \nu_t \left(w_t n_t + b_t + s_t d_t + s_t p_t - T_t - c_t - p_t s_{t+1} - \frac{b_{t+1}}{1 + r_t} \right) \right\}$$

The FOC are:

$$\frac{\partial \mathbb{L}}{\partial c_t} = \frac{1}{c_t} - \nu_t$$

$$\frac{\partial \mathbb{L}}{\partial n_t} = -\frac{\alpha}{1 - n_t} + \nu_t w_t$$

$$\frac{\partial \mathbb{L}}{\partial b_{t+1}} = -\nu_t \frac{1}{1 + r_t} + \beta \, \mathbb{E}_t \, \nu_{t+1}$$

$$\frac{\partial \mathbb{L}}{\partial s_{t+1}} = -p_t \nu_t + \beta \, \mathbb{E}_t \, \nu_{t+1}(d_{t+1} + p_t)$$

Eliminating the multiplier, we get:

$$\frac{\alpha}{1 - n_t} = \frac{w_t}{c_t} \tag{13}$$

$$1 = \beta \, \mathbb{E}_t \, \frac{c_t}{c_{t+1}}(1 + r_t) \tag{14}$$

$$p_t = \beta \, \mathbb{E}_t \, \frac{c_t}{c_{t+1}} \left(d_{t+1} + p_{t+1}\right) \tag{15}$$

These are the same as (7)-(9) in the paper. (13) is just the labor supply condition and (14) is the Euler equation for bonds. (15) is how shares of equity are priced. If you solve this forward, the price of newly issued equity is the PDV of future dividends, where discounting is by the SDF. This is consistent with the firm's optimization problem, where it maximizes current dividends plus the price of new shares, which is equivalent to the present discounted value of dividends, where discounting is by the stochastic discount factor. The stochastic discount factor is just:

$$m_{t-1,t} = \beta \frac{c_{t-1}}{c_t} \tag{16}$$

2.3 Equilibrium Conditions

The competitive equilibrium conditions include the optimality conditions for the household and firm along with their budget constraints, the borrowing constraint, and the law of motion for capital. We have:

$$\frac{\alpha}{1-n_t} = \frac{w_t}{c_t} \tag{17}$$

$$1 = \beta\,\mathbb{E}_t \frac{c_t}{c_{t+1}}(1+r_t) \tag{18}$$

$$p_t = \beta\,\mathbb{E}_t \frac{c_t}{c_{t+1}}(d_{t+1}+p_{t+1}) \tag{19}$$

$$m_{t-1,t} = \beta \frac{c_{t-1}}{c_t} \tag{20}$$

$$w_t = \big(1-\mu_t\varphi'(d_t)\big)(1-\theta)z_t k_t^{\theta} n_t^{-\theta} \tag{21}$$

$$1 = \varphi'(d_t)\mu_t\xi_t \frac{R_t}{1+r_t} + \mathbb{E}_t\, m_{t,t+1} R_t \frac{\varphi'(d_t)}{\varphi'(d_{t+1})} \tag{22}$$

$$1 = \mu_t\xi_t\varphi'(d_t) + \mathbb{E}_t\, m_{t,t+1}\frac{\varphi'(d_t)}{\varphi'(d_{t+1})}\left[1-\delta+\big(1-\mu_{t+1}\varphi'(d_{t+1})\big)\theta z_{t+1}k_{t+1}^{\theta-1}n_{t+1}^{1-\theta}\right] \tag{23}$$

$$c_t + \frac{b_{t+1}}{1+r_t} = w_t n_t + b_t + d_t - T_t \tag{24}$$

$$\varphi(d_t) = z_t k_t^{\theta} n_t^{1-\theta} - w_t n_t - k_{t+1} + (1-\delta)k_t - b_t + \frac{b_{t+1}}{R_t} \tag{25}$$

$$\xi_t\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right) \geq z_t k_t^{\theta} n_t^{1-\theta} \tag{26}$$

$$k_{t+1} = i_t + (1-\delta)k_t \tag{27}$$

$$y_t = z_t k_t^{\theta} n_t^{1-\theta} \tag{28}$$

$$\ln z_t = \rho_z \ln z_{t-1} + s_z\varepsilon_{z,t} \tag{29}$$

$$\ln \xi_t = \rho_\xi \ln \xi_{t-1} + s_\xi\varepsilon_{\xi,t} \tag{30}$$

$$R_t = 1 + r_t(1-\tau) \tag{31}$$

The variables are $\Big\{w_t, c_t, n_t, k_t, i_t, y_t, R_t, r_t, \mu_t, b_t, d_t, \xi_t, p_t, m_{t-1,t}, T_t, z_t\Big\}$. This is sixteen variables, but we only have fifteen equations. We need some rule for T_t. We must have:

$$\frac{b_{t+1}}{1+r_t} = \frac{b_{t+1}}{R_t} - T_t$$

Or:

$$T_t = b_{t+1}\left[\frac{1}{R_t} - \frac{1}{1+r_t}\right] \tag{32}$$

As long as $\tau > 0$, we have $R_t < 1 + r_t$, so this term is positive. Effectively, the government is subsidizing firms issuing debt and funding this by taxing the household. Including (32) as an equilibrium condition makes sixteen variables and sixteen equations.

Note that we can write the firm's budget constraint as:

$$\varphi(d_t) + w_t n_t + i_t + b_t - \frac{b_{t+1}}{R_t} = y_t$$

The household's budget constraint can be written:

$$w_t n_t = c_t + \frac{b_{t+1}}{1+r_t} - b_t - d_t + T_t$$

Now plug this into the firm's condition:

$$\varphi(d_t) + \left[c_t + \frac{b_{t+1}}{1+r_t} - b_t - d_t + T_t\right] + i_t + b_t - \frac{b_{t+1}}{R_t} = y_t$$

But this becomes:

$$\varphi(d_t) - d_t + c_t + i_t + T_t + \frac{b_{t+1}}{1+r_t} - \frac{b_{t+1}}{R_t} = y_t$$

But then using the definition of T_t, along with the fact that $\varphi(d_t) = d_t + \kappa(d_t - \bar{d})^2$, would yield:

$$y_t = c_t + i_t + \kappa(d_t - \bar{d})^2 \tag{33}$$

This is a standard looking resource constraint, just with a resource cost of adjusting dividend payments. In a more standard model we could get rid of d_t, but here we cannot.

2.4 Steady State and Parameterization

Several parameters are standard. $\beta = 0.9825$, $\theta = 0.36$, and $\delta = 0.025$. $z = 1$ in steady state. They set $\tau = 0.35$. Furthermore, they set $\xi = 0.1634$.

From all this, we can solve for several steady state values. Note that $\varphi'(d_t) = 1 + \kappa(d_t - \bar{d})$, so that in steady state $\varphi'(d) = 1$.

Let's solve for the steady state. First, note that:

$$r = \frac{1}{\beta} - 1 \tag{34}$$

But this then gives us R;

$$R = 1 + r(1-\tau) \tag{35}$$

But then we can solve for μ:

$$1 = \mu\xi\frac{R}{1+r} + \beta R$$

Or:

$$\mu = (1-\beta R)\frac{1+r}{\xi R} \tag{36}$$

But now we can solve for the capital-labor ratio via:

$$1 = \mu\xi + \beta\left[1-\delta+(1-\mu)\theta\left(\frac{k}{n}\right)^{\theta-1}\right]$$

So:

$$\frac{1-\mu\xi}{\beta} = 1-\delta+(1-\mu)\theta\left(\frac{k}{n}\right)^{\theta-1}$$

Which implies:

$$\frac{k}{n} = \left(\frac{(1-\mu)\theta}{\frac{1-\mu\xi}{\beta}-(1-\delta)}\right)^{\frac{1}{1-\theta}} \tag{37}$$

But then we have the wage:

$$w = (1-\mu)(1-\theta)\left(\frac{k}{n}\right)^{\theta} \tag{38}$$

Divide the resource constraint by n:

$$\left(\frac{k}{n}\right)^{\theta} = \frac{c}{n} + \delta\frac{k}{n}$$

This then gives us the consumption-hours ratio:

$$\frac{c}{n} = \left(\frac{k}{n}\right)^{\theta} - \delta\frac{k}{n}$$

But then we can solve for steady state n:

$$n = \left[1+\frac{\alpha\frac{c}{n}}{w}\right]^{-1}$$

But then we have everything else effectively. We can then solve for b as:

$$b = (1+r)\left[K-\frac{y}{\xi}\right] \tag{39}$$

But this then gives us d:

$$d = y - wn - i - b + b/R \tag{40}$$

The steady state share price can be written:

$$p = d\sum_{s=1}^{\infty}\beta^s$$

Which works out to:

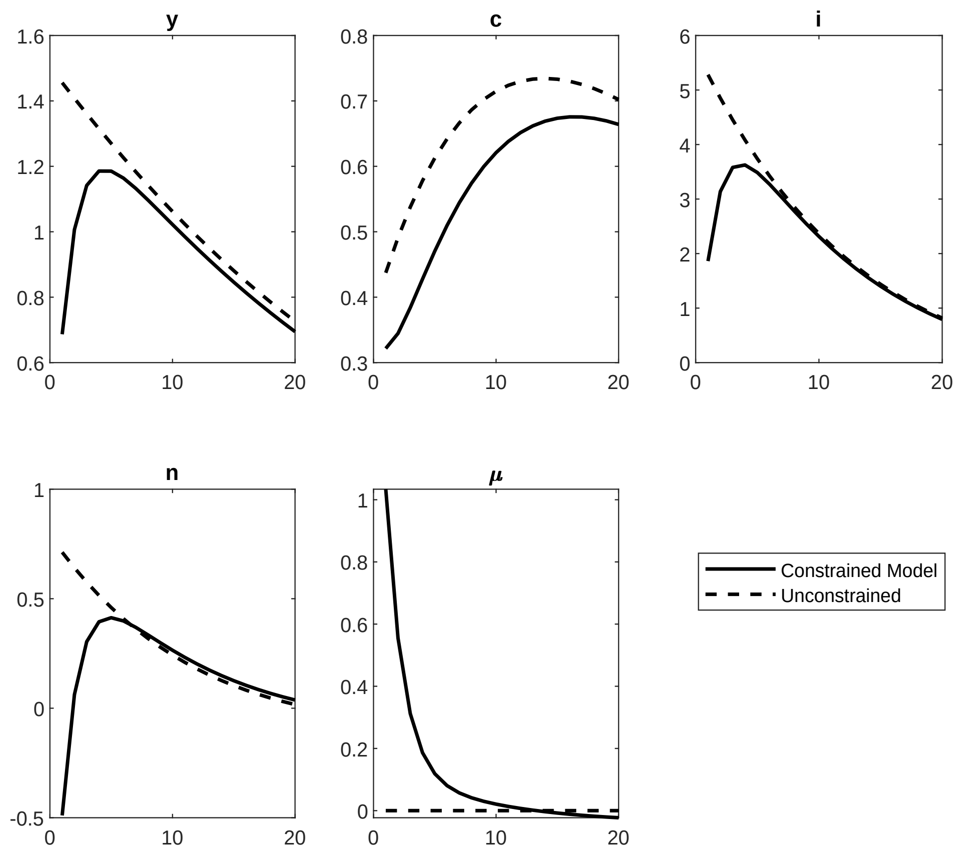
$$p = \frac{\beta}{1-\beta}d \tag{41}$$

2.5 Calibration and Impulse Responses

I calibrate the model following much of what Jermann and Quadrini do. I set $\beta = 0.9825$ and $\tau = 0.35$. $\alpha = 1.8834$ and $\theta = 0.36$, with $\delta = 0.025$. I assume a steady state value of the financial shock of $\xi = 0.163$, and a dividend adjustment cost of $\kappa = 0.1460$. They estimate a VAR to get the AR parameters of the shock processes, allowing for cross-correlations. I'm just going to do something simpler and set both AR(1) parameters to 0.95. This ends up being close to what they have, but makes comparisons to their IRFs slightly off because they estimate that there is a positive effect of lagged productivity on the financial condition, albeit a small one.

In the two figures below, I show impulse responses of selected variables to a productivity shock and a financial shock. I show responses in the baseline model as well as a version of the model without constraints on the firm (i.e. $\tau = 0$ and $\mu_t = 0$ with no dividend adjustment cost). That is just a straight-up RBC model, and in that model the financial structure of the firm is irrelevant, so I don't show impulse responses of financial variables. The constrained model responses are in solid lines; the vanilla RBC responses are dashed lines.

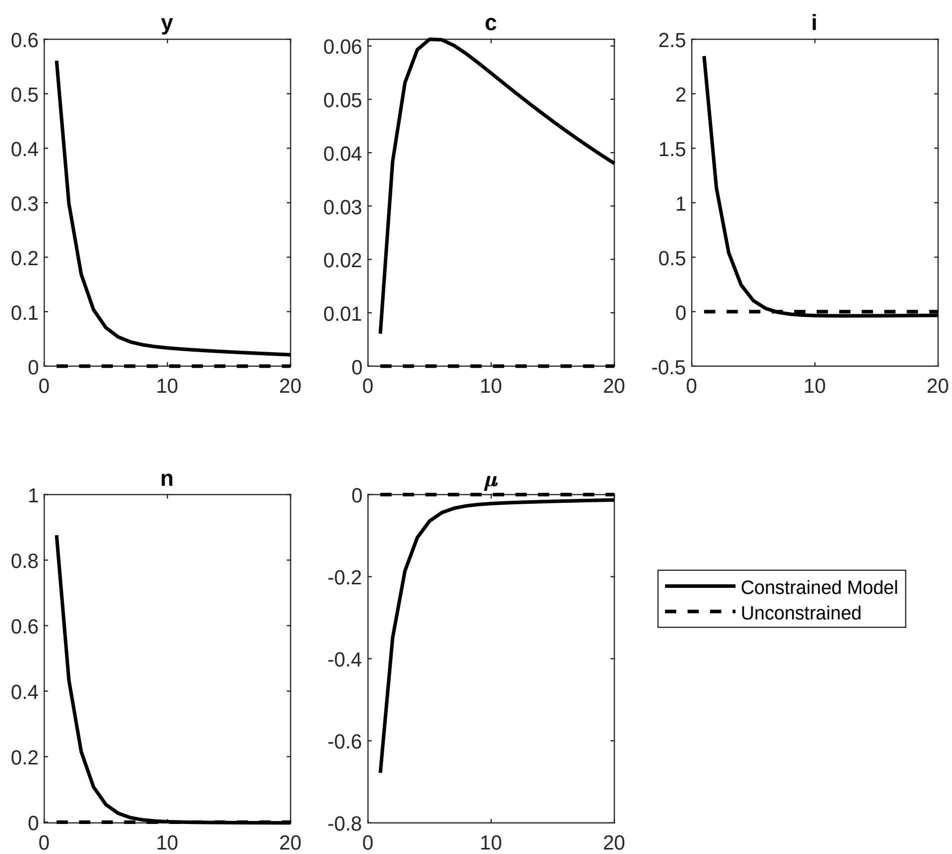
Figure 1: IRFs to Productivity Shock



Consider first the responses to a productivity shock. The straight up RBC model responses are familiar – output, consumption, investment, and hours rise. Output, investment, and hours jump immediately and return to steady state as the productivity shock dissipates; there are more interesting dynamics for consumption, but this is driven by the increase in the marginal product of capital / real rate, which causes the household to defer consumption. The responses in the model with the financial friction are noticeably different for the first several quarters. In particular, hours *declines* on impact, while output and investment follow *hump-shaped* patterns. The hump-shape responses are very similar to what one gets in the Carlstrom and Fuerst (1997) model. What is driving this is that the positive productivity shock *tightens* the firm's collateral constraint. Why is this? The firm wants to increases its intraperiod working capital loan to expand production, but is limited by its net worth, which reacts more slowly. Hence, we observe μ_t increasing. This multiplier shows up directly in the labor demand schedule, which acts like a labor wedge. This labor wedge reduces labor demand in the short run, causing hours to fall and output to underreact. After 6-8 periods, the effects of the constraint tightening have more or less worn off, and the responses look like the standard RBC model.

Next, consider impulse responses to the financial shock. The direct effect is to ease the firm's borrowing constraint, manifested in a smaller μ. This allows the firm to hire more labor (i.e. the labor wedge declines, and consequently hours and output rise) and allows the firm to issue more debt, which permits investment rising. Consumption actually goes up but this effect is very muted. In the unconstrained model there is no effect of the financial shock – a shock to ξ_t having effects requires $\mu_t > 0$.

Figure 2: IRFs to Financial Shock



Before turning to the more involved model, it is useful to explore the role of the dividend adjustment cost, measured by κ. When $\kappa = 0$, it is almost as though the model is unconstrained. Basically, what is going on is that, in response to shocks, if $\kappa = 0$ the firm can adjust its dividends so as to more or less neutralize the borrowing constraint. It can't completely neutralize the constraint given the built-in tax preference for debt over equity. But the ability to adjust dividends freely makes the constraint not that big of an issue in a dynamic sense (it still binds in the steady state, which distorts the steady state relative to the vanilla RBC model), which results in the responses in the constrained model looking overall very similar to if there were no constraints facing the firm. This is easy to see for the productivity shock. The differences look starker for the financial shock, but this is largely an illusion driven by scale. The output response to a financial shock when $\kappa = 0$ is about two orders of magnitude smaller (i.e. 20 times smaller) than when $\kappa = 0.15$.

Figure 3: IRFs to Productivity Shock, $\kappa = 0$

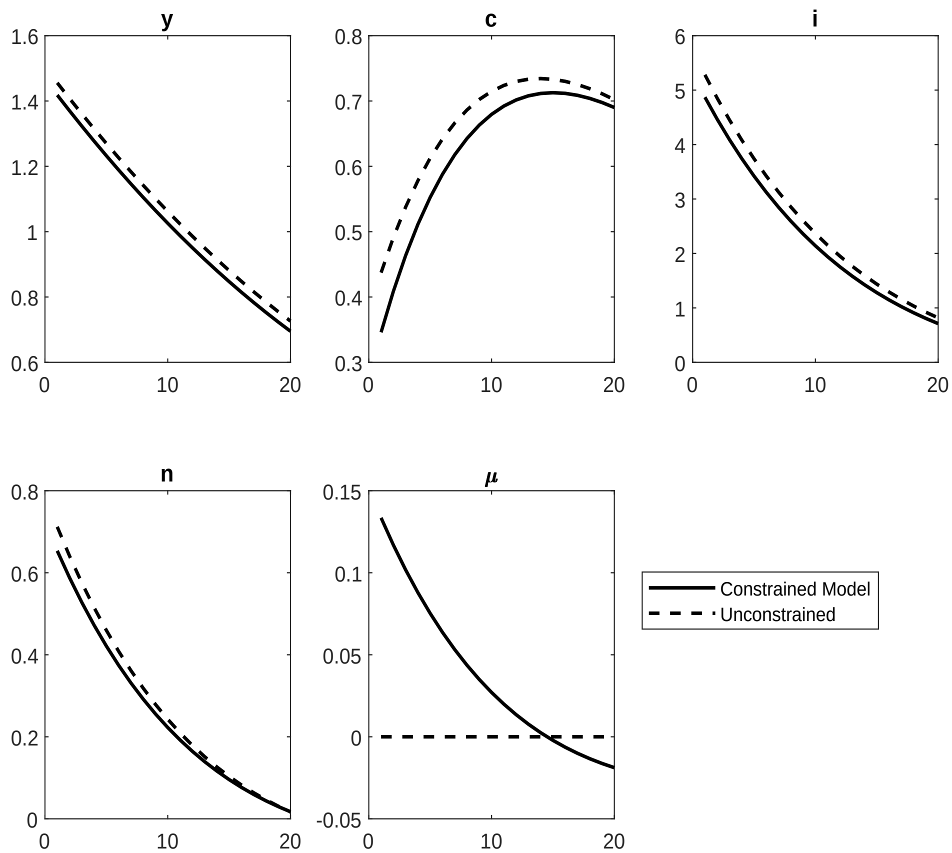
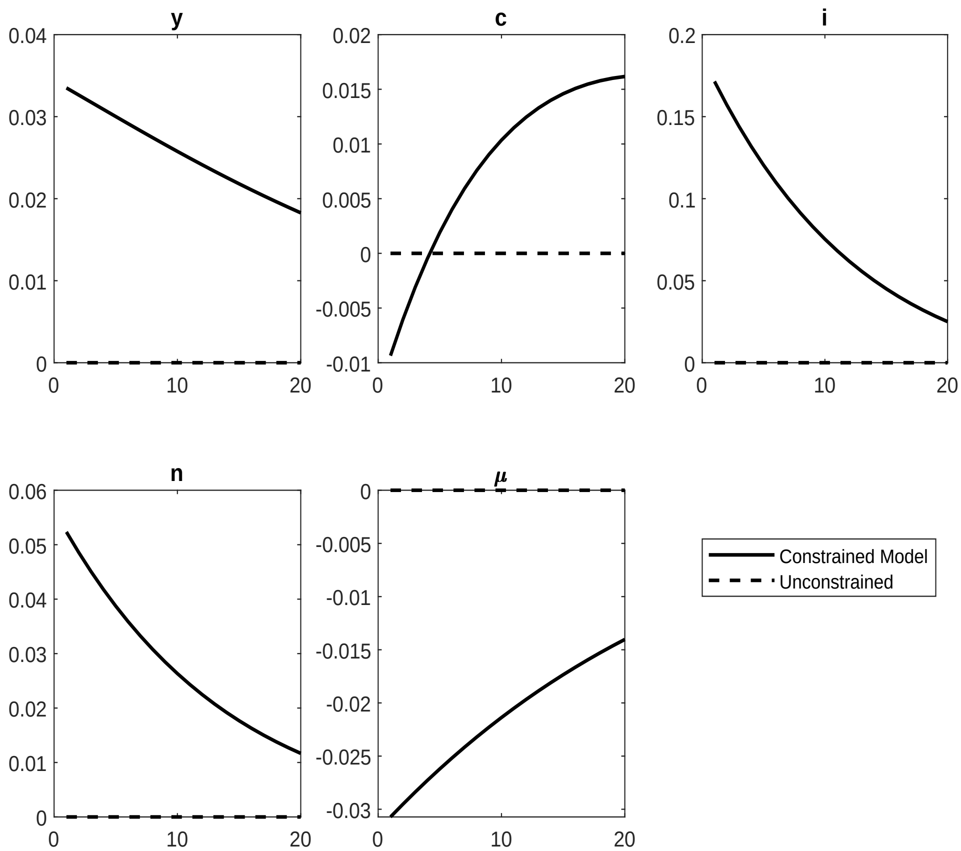


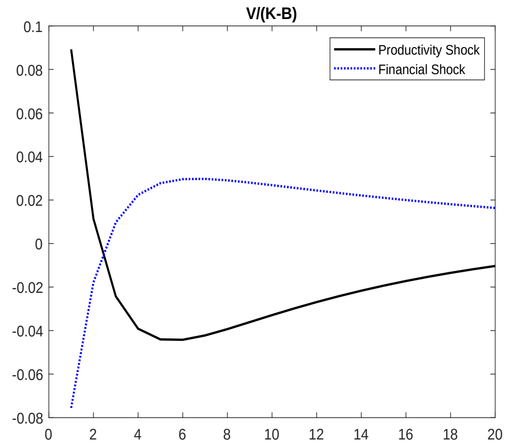
Figure 4: IRFs to Financial Shock, $\kappa = 0$



2.6 Augmenting the Model with an Adjustment Cost

In the model with constraints and $\kappa > 0$, the financial shock drives a significant fraction of output succeeds in generating macro comovement, which is a key requirement of any candidate business cycle shock – i.e. in the data consumption, investment, and hours worked are highly correlated, so we need a model that produces this pattern. This model does so conditional on the financial shock. But what about other financial variables, such as the stock price of the firm? The next figure plots the impulse response of the ratio of market to book value to the two kinds of shocks. Market value is $V = d_t + p_t$, what is sometimes the "cum-dividend" value (i.e. share price plus dividend payout in current period). The book value of the firm is simply $k_{t+1} - b_{t+1}$ (i.e. end-of-period capital less end-of-period debt). This is what Jermann and Quadrini plot in their paper. I only look at this conditional on the model with frictions.

Figure 5: Market/Book Value to Financial Shock



This is very close to what they report in Figure 6 (although they are considering *negative* shocks whereas I'm doing positive shocks). The equity value of the firm is procylical conditional on the productivity shock (good), but countercyclical conditional on the financial shock (bad, from the perspective of matching the data). Jermann and Quadrini discuss this problem and show how adding in an investment adjustment cost helps make the this equity value procyclical procyclical.

They use a capital accumulation equation given by:

$$k_{t+1} = \left[\frac{\varrho_1 \left(\frac{i_t}{k_t} \right)^{1-\nu}}{1-\nu} + \varrho_2 \right] k_t + (1-\delta) k_t \tag{42}$$

This is a bit of a non-standard adjustment cost specification. It will be easiest to write the firm problem out not substituting out i_t. Its problem is

$$\max_{d_t, n_t, i_t, k_{t+1}, b_{t+1}} \quad \mathbb{E}_t \sum_{t=0}^{\infty} m_{0,t} d_t$$

s.t.

$$\varphi(d_t) = z_t k_t^{\theta} n_t^{1-\theta} - w_t n_t - i_t - b_t + \frac{b_{t+1}}{R_t}$$

$$\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) \geq z_t k_t^{\theta} n_t^{1-\theta}$$

$$k_{t+1} = \left[\frac{\varrho_1 \left(\frac{i_t}{k_t} \right)^{1-\nu}}{1-\nu} + \varrho_2 \right] k_t + (1-\delta) k_t$$

Let ϕ_t be the multiplier on the accumulation equation. A Lagrangian is:

$$\mathbb{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \Bigg\{ d_t + \lambda_t \left[z_t k_t^{\theta} n_t^{1-\theta} - w_t n_t - i_t - b_t + \frac{b_{t+1}}{R_t} - \varphi(d_t) \right]$$
$$+ \mu_t \left[\xi_t \left(k_{t+1} - \frac{b_{t+1}}{1+r_t} \right) - z_t k_t^{\theta} n_t^{1-\theta} \right] + \phi_t \left[\left[\frac{\varrho_1 \left(\frac{i_t}{k_t}\right)^{1-\nu}}{1-\nu} + \varrho_2 \right] k_t + (1-\delta) k_t - k_{t+1} \right] \Bigg\}$$

The FOC are:

$$\frac{\partial \mathbb{L}}{\partial d_t} = 1 - \lambda_t \varphi'(d_t)$$
$$\frac{\partial \mathbb{L}}{\partial n_t} = \lambda_t \left((1-\theta) z_t k_t^{\theta} n_t^{-\theta} - w_t \right) - \mu_t (1-\theta) z_t k_t^{\theta} n_t^{-\theta}$$
$$\frac{\partial \mathbb{L}}{\partial b_{t+1}} = \frac{\lambda_t}{R_t} - \frac{\mu_t \xi_t}{1+r_t} + \mathbb{E}_t\, m_{t,t+1} \lambda_{t+1}$$
$$\frac{\partial \mathbb{L}}{\partial i_t} = -\lambda_t + \phi_t \varrho_1 \left(\frac{i_t}{k_t} \right)^{-\nu}$$

$$\frac{\partial \mathbb{L}}{\partial k_{t+1}} = \mu_t \xi_t - \phi_t + \mathbb{E}_t\, m_{t,t+1} \lambda_{t+1} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} - \mathbb{E}_t\, m_{t,t+1} \mu_{t+1} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta}$$
$$+ \mathbb{E}_t\, m_{t,t+1} \phi_{t+1} \left(1 - \delta + \varrho_1 \frac{\left(\frac{i_{t+1}}{k_{t+1}}\right)^{1-\nu}}{1-\nu} + \varrho_2 - \varrho_1 \left(\frac{i_{t+1}}{k_{t+1}} \right)^{1-\nu} \right)$$

Eliminating λ_t and setting equal to zero gives exactly the same FOC for labor and bonds that we had in the model without the adjustment cost:

$$w_t = (1 - \mu_t \varphi'(d_t))(1-\theta) z_t k_t^{\theta} n_t^{-\theta} \tag{43}$$
$$1 = \varphi'(d_t) \mu_t \xi_t \frac{R_t}{1+r_t} + \mathbb{E}_t\, m_{t,t+1} R_t \frac{\varphi'(d_t)}{\varphi'(d_{t+1})} \tag{44}$$

The FOC for investment implies:

$$\phi_t \varphi'(d_t) \varrho_1 = \left(\frac{i_t}{k_t} \right)^{\nu} \tag{45}$$

We need to specify some properties of this adjustment cost function. First, we need the term in brackets to equal δ at steady state (i.e. when $i/k = \delta$). This requires:

$$\frac{\varrho_1 \delta^{1-\nu}}{1-\nu} + \varrho_2 = \delta$$

Second, we need $\partial k_{t+1} / \partial i_t = 1$ evaluated in the steady state. This requires:

$$\varrho_1 \delta^{-\nu} = 1$$

This requires that:

$$\varrho_1 = \delta^{\nu}$$

And hence:

$$\varrho_2 = -\frac{\nu\delta}{1-\nu}$$

We can now say something about (45). We can interpret the ratio of the Lagrange multipliers, ϕ_t/λ_t, as Tobin's marginal q_t. But since $1/\lambda_t = \varphi'(d_t)$, (45) says:

$$q_t = \left(\frac{i_t}{k_t} \middle/ \delta\right)^{\nu} \tag{46}$$

In steady state, we will have $q_t = 1$. If $q_t > 1$, we will have $i_t/k_t > \delta$, so the capital stock will be growing.

Now let's go back to the FOC for capital accumulation. We can write this as:

$$\phi_t = \mu_t \xi_t + \mathbb{E}_t\, m_{t,t+1} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} (\lambda_{t+1} - \mu_{t+1}) \\ + \mathbb{E}_t\, m_{t,t+1} \phi_{t+1} \left(1 - \delta + \varrho_1 \frac{\left(\frac{i_{t+1}}{k_{t+1}}\right)^{1-\nu}}{1-\nu} + \varrho_2 - \varrho_1 \left(\frac{i_{t+1}}{k_{t+1}}\right)^{1-\nu}\right)$$

Which then using the FOC to eliminate λ_{t+1} and ϕ_t we can write:

$$\left(\frac{i_t}{k_t} \middle/ \delta\right)^{\nu} = \mu_t \xi_t \varphi'(d_t) + \mathbb{E}_t\, m_{t,t+1} \frac{\varphi'(d_t)}{\varphi'(d_{t+1})} \theta z_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} (1 - \varphi'(d_{t+1}) \mu_{t+1}) + \\ \mathbb{E}_t\, m_{t,t+1} \frac{\varphi'(d_t)}{\varphi'(d_{t+1})} \left(\frac{i_{t+1}}{k_{t+1}} \middle/ \delta\right)^{\nu} \left(1 - \delta + \varrho_1 \frac{\left(\frac{i_{t+1}}{k_{t+1}}\right)^{1-\nu}}{1-\nu} + \varrho_2 - \varrho_1 \left(\frac{i_{t+1}}{k_{t+1}}\right)^{1-\nu}\right) \tag{47}$$

The rest of the model is the same. I solve the model assuming $\nu = 0.5$, as they do. Below I report the IRFs of the value of the firm relative to book value. Now both shocks induce positive co-movement here, as they report in Figure 8.

Figure 6: Market/Book Value to Financial Shock