Advanced Macro: Kiyotaki and Moore (1997, *JPE*)

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1 Introduction

As mentioned in class, there are two basic ways macroeconomists introduce financial frictions into models – either via the CSV approach (originally proposed by Townsend 1979, but incorporated into a macro model by Bernanke and Gertler 1989) – or the costly enforcement approach. Kiyotaki and Moore (1997) is the seminal paper in the latter approach, and we discuss it here.

The basic idea of the costly enforcement approach is that borrowers face a binding borrowing constraint, where the constraint is some function of the market-value of their assets. This constraint arises because of limited enforceability – lenders can seize borrower assets in default, but those assets are worth less to the lender than in the hands of the borrower (e.g. there is a bankruptcy cost or the borrower is more efficient at using the underlying asset). Because of this, the lender will limit how much credit he/she will extend to a borrower so that the borrower does not find it optimal to default. In equilibrium, provided certain assumptions (typically on discounting the future) are satisfied, the borrowing constraints will bind. This in turn gives rise to a financial accelerator type effect. Shocks that raise asset prices will ease borrowing constraints. This will allow borrowers more access to credit, which will result in more investment and aggregate demand, and hence even higher asset prices.

This is very similar to, for example, the Bernanke, Gertler, and Gilchrist (1999) story. But it turns out to be "easier" to work with, as you don't have the heterogeneity of the CSV framework. The drawback is that, in equilibrium, there is no default/bankruptcy in the costly enforcement approach, whereas there is in the CSV approach.

In what follows, I am going to work through only the "simple" model of Kiyotaki and Moore (1997) (i.e. Section II). In Section III, they add reproducible capital which has some desirable properties, including generating more persistence. In Section IV, they talk about sectoral spillovers. But the key insights come from Section II.

2 The Simple Model

There are two types of agents in the model – farmers and gatherers, with gatherers denoted with a $'$ superscript. Time is discrete and lasts forever, starting in period t. There is a durable asset which is used as a factor of production. It is not reproducible. Think of it as land, and denote the fixed aggregate supply of it as $\bar{K}$. Denote consumption of farmers and gatherers as x_t and x'_t, respectively. Both types of agents are risk neutral. The farmers have discount factor β and the gathers have discount factor β', with $\beta < \beta'$. This means that farmers are relatively impatient, and in equilibrium will be borrowers.

2.1 Farmer

Farmers and gatherers have different production technologies. Let k_t be the land held by a farmer in period t. This can be turned into output available in period $t+1$ via the constant returns to scale production function:

$$y_{t+1} = (a + c)k_t \tag{1}$$

ak_t is the amount of output that is tradeable. ck_t is the amount of a farmer's output that is non-tradeable, but nevertheless still consumable, but only by the farmer himself. Think of this fruit as being "bruised" – a farmer can't sell it, but he can eat it.

Land trades in a competitive spot market at price q_t (measured in units of the consumption good, called fruit). Let R_t be the gross interest rate on bonds carried from t to $t+1$. Farmers are subject to a borrowing constraint:

$$R_t b_t \leq q_{t+1} k_t \tag{2}$$

What is the intuition for (2)? There are two underlying assumptions that give rise to this constraint. First, the farmer has to "work" to produce output (though we are not formally modeling labor at all), but can in principle choose not to work. Second, if the farmer doesn't "work," no one else can use his land to produce trees. If a lender makes a loan to a farmer, he is due back $R_t b_t$ in the subsequent period. If the borrow chooses to not payback, the lender can't force the borrower to work (hence, "limited enforcement"). Rather, the lender can just confiscate the farmer's land, k_t, which will be worth q_{t+1} in period $t+1$. The lender would never loan to the farmer if $R_t b_t > q_{t+1} k_t$. If this were the case, the farmer would choose to default – instead of paying back $R_t b_t$, he could just not work and selling his land for $q_{t+1} k_{t+1}$, which would allow him to enjoy more consumption. So $R_t b_t \leq q_{t+1} k_t$ ensures that the farmer never defaults on the interperiod loan.

The farmer's budget constraint looks as follows:

$$q_t(k_t - k_{t-1}) + R_{t-1} b_{t-1} + x_t = (a + c)k_{t-1} + b_t \tag{3}$$

On the "expenditure side" of (3), the farmer (i) purchases new land, $q_t(k_t - k_{t-1})$; (ii) pays off interest plus principle on any loans, $R_{t-1} b_{t-1}$; and (iii) chooses how much to eat. On the "income side," the farmer produces output using last period's capital, $(a + c)k_{t-1}$, and can issue more intertemporal debt, b_t.

A Lagrangian for the farmer is as follows, with λ_t and μ_t denoting the multipliers on the budget and borrowing constraints. We have a third constraint, which is that $x_t \geq ck_{t-1}$ – since ck_{t-1} is not tradeable, the farmer must eat at least this much. Let φ_t be the multiplier on this constraint. We will consider a world with no aggregate uncertainty (though will consider perfect foresight or "MIT" shocks later), so we can drop expectations operators.

$$\mathbb{L} = \sum_{s=0}^{\infty} \beta^s \Bigg\{ x_{t+s} + \lambda_{t+s} \left[(a+c)k_{t+s-1} + b_{t+s} - q_{t+s}(k_{t+s} - k_{t+s-1}) - R_{t+s-1}b_{t+s-1} - x_{t+s} \right] + \mu_{t+s}[q_{t+s+1}k_{t+s} - R_{t+s}b_{t+s}] + \varphi_{t+s}[x_{t+s} - ck_{t+s-1}] \Bigg\}$$

Take derivatives of the Lagrangian:

$$\frac{\partial \mathbb{L}}{\partial x_t} = 1 - \lambda_t + \varphi_t$$

$$\frac{\partial \mathbb{L}}{\partial b_t} = \lambda_t - \mu_t R_t + \beta \lambda_{t+1} R_t$$

$$\frac{\partial \mathbb{L}}{\partial k_t} = -q_t \lambda_t + \mu_t q_{t+1} + \beta \lambda_{t+1} \left[(a+c) + q_{t+1} \right] - \beta c \varphi_{t+1}$$

Set these equal to zero and eliminate the multiplier on the budget constraint:

$$1 + \varphi_t = (\beta(1 + \varphi_{t+1}) + \mu_t) R_t \tag{4}$$

$$q_t(1 + \varphi_t) + \beta c \varphi_{t+1} = \beta(1 + \varphi_{t+1})[a + c + q_{t+1}] + \mu_t q_{t+1} \tag{5}$$

(4)-(5) would be standard asset pricing conditions in the absence of the constraints. The price of the bond (normalized to 1) would just equal the product of stochastic discount factor (just β with linear preferences) with the bond payout, R_t. The price of the land would equal the product of the stochastic discount factor (again, just β) with the sum of the flow benefit of the land, $a + c$, with the continuation value of land, q_{t+1}. $\mu_t \geq 0$ and $\varphi_t > 0$ throw "wedges" into both first order conditions.

2.2 Gatherer

Gather's produce output via:

$$y'_{t+1} = G(k'_t) \tag{6}$$

Where $G'(\cdot) > 0$, $G''(\cdot) < 0$, $G'(0) > 0$. There are two other auxiliary assumptions that ensure that, in equilibrium, both farmers and gatherers will produce. All gatherer output is tradeable.

A gatherer's budget constraint is:

$$q_t(k'_t - k'_{t-1}) + R_{t-1}b'_{t-1} + x'_t = G(k'_{t-1}) + b'_t \tag{7}$$

In equilibrium, we will have $b'_t < 0$, so that gathers are actually saving (positive values would denote borrowing the way the constraint has been written). (7) is the same idea as the constraint for the farmer.

A Lagrangian is:

$$\mathbb{L} = \sum_{s=0}^{\infty} (\beta')^s \left\{ x'_{t+s} + \lambda'_t \left[G(k'_{t-1}) + b'_t - q_t(k'_t - k'_{t-1}) - R_{t-1}b'_{t-1} - x'_t \right] \right\}$$

The derivatives are:

$$\frac{\partial \mathbb{L}}{\partial x'_t} = 1 - \lambda'_t$$

$$\frac{\partial \mathbb{L}}{\partial b'_t} = \lambda'_t - \beta' R_t \lambda'_{t+1}$$

$$\frac{\partial \mathbb{L}}{\partial k'_t} = -\lambda'_t q_t + \beta \lambda'_{t+1} G'(k'_t) + \beta q_{t+1}$$

Eliminating the multiplier and we get:

$$1 = \beta' R_t \tag{8}$$

$$q_t = \beta' \left[G'(k'_t) + q_{t+1} \right] \tag{9}$$

(8) implies that the gross interest rate is constant at $R = 1/\beta'$. (10) is a standard asset pricing condition.

2.3 Equilibrium

The population size of farmers is 1; the population size of gatherers is m. Within type, everyone is identical. So for aggregate market-clearing, we can just sum across types.

Market-clearing requires:

$$b_t + mb'_t = 0 \tag{10}$$

$$k_t + mk'_t = \bar{K} \tag{11}$$

(10) just requires that one type's saving equals the other type's borrowing. The m just scales the gatherer population relative to the farmers. (11) reflects market-clearing for the fixed quantity of land. Now sum the budget constraints across type, imposing that R is fixed as shown above:

$$q_t k_t - q_t k_{t-1} + R b_{t-1} + x + q_t m k'_t - q_t m k'_{t-1} + R m b'_{t-1} + m x'_t = (a+c)k_{t-1} + b_t + mG(k'_{t-1}) + mb'_t$$

Which may be written:

$$q_t(k_t+mk_t')-q_t(k_{t-1}+mk_{t-1}')+R(b_{t-1}+mb_{t-1}')+x_t+mx_t'=(a+c)k_{t-1}+mG(k_{t-1}')+(b_t+mb_t')$$

But then using the market-clearing conditions (zero total debt, fixed supply of capital), we get:

$$x_t+mx_t'=(a+c)k_{t-1}+mG(k_{t-1}')=y_t+my_{t-1}=Y_t \tag{12}$$

(12) is a standard aggregate resource constraint.

The full set of equilibrium conditions can then be written:

$$1+\varphi_t=(\beta(1+\varphi_{t+1})+\mu_t)\frac{1}{\beta'} \tag{13}$$

$$q_t(1+\varphi_t)+\beta c\varphi_{t+1}=\beta(1+\varphi_{t+1})[a+c+q_{t+1}]+\mu_t q_{t+1} \tag{14}$$

$$q_t(k_t-k_{t-1})+\frac{1}{\beta'}b_{t-1}+x_t=(a+c)k_{t-1}+b_t \tag{15}$$

$$b_t\leq\beta' q_{t+1}k_t \tag{16}$$

$$q_t=\beta'\left[G'(k_t')+q_{t+1}\right] \tag{17}$$

$$x_t+mx_t'=(a+c)k_{t-1}+mG(k_{t-1}') \tag{18}$$

$$k_t+mk_t'=\bar{K} \tag{19}$$

$$x_t\geq ck_{t-1} \tag{20}$$

I have eliminated R_t as a variable, instead treating it as a parameter, $R=\frac{1}{\beta'}$. I have also eliminated b_t' using the bond market-clearing condition and the resource constraint, (18), subsumes the budget constraint for gatherers. This leaves eight equations in eight variables – $\left\{x_t,x_t',k_t,k_t',b_t,q_t,\mu_t,\varphi_t\right\}$.

2.4 Steady State

Let's suppose that both constraints bind in the steady state. After solving for the steady state using this assumption, we can then check whether $\varphi>0$ and $\mu>0$, thus confirming (or not) our guess.

Go to (15) evaluated in the steady state. We have:

$$\frac{b}{\beta'}=ak+b$$

This makes use of assuming that (20) binds, so $x=ck$. But from (16) we can then eliminate b:

$$qk=ak+\beta' qk$$

But then the ks cancel, and we can solve for q:

$$q = \frac{a}{1-\beta'} \tag{21}$$

Note (21) is the same as (13a) in the paper (albeit written somewhat differently). But once we know q, we can get k' from (17):

$$q = \beta'\alpha(z+k')^{\alpha-1} + \beta' q$$

$$(1-\beta')q = \beta'\alpha(z+k')^{\alpha-1}$$

So:

$$k' = \left(\frac{\beta'\alpha}{a}\right)^{\frac{1}{1-\alpha}} - z \tag{22}$$

But then we have k from the market-clearing condition for capital:

$$k = \bar{K} - mk \tag{23}$$

But then we have b:

$$b = \beta' qk \tag{24}$$

Similarly, we now have x':

$$x' = \frac{ak}{m} + (z+k')^{\alpha} \tag{25}$$

Now we need to check the multipliers. From (13), we have:

$$\mu = (\beta' - \beta)(1+\varphi)$$

From (14), we have:

$$q(1+\varphi) + c\beta\varphi = \beta(1+\varphi)(a+c) + \beta(1+\varphi)q + \mu q$$

Plug in for μ:

$$q(1+\varphi) + c\beta\varphi = \beta(1+\varphi)(a+c) + \beta(1+\varphi)q + q(\beta'-\beta)(1+\varphi)$$

Distribute terms:

$$q + q\varphi + c\beta\varphi = \beta(a+c) + \beta(a+c)\varphi + \beta q + \beta q\varphi + q(\beta'-\beta) + q(\beta'-\beta)\varphi$$

Isolate terms involving φ on the LHS:

$$\left[q + c\beta - \beta(a+c) - \beta q - q(\beta'-\beta)\right]\varphi = \beta(a+c) + \beta q + q(\beta'-\beta) - q$$

This can be written:

$$\left[q(1-\beta')-\beta a\right]\varphi=\beta(a+c)-q(1-\beta')$$

But recall from above that $q=a/(1-\beta')$. Hence:

$$a(1-\beta)\varphi=\beta(a-1)+\beta c$$

Or:

$$\varphi=\frac{a(\beta-1)+\beta c}{a(1-\beta)} \tag{26}$$

Note, this ties into Assumption 2 in the paper. That assumption requires that $\beta c>(1-\beta)a$. That assumption assures that the numerator in (26) is positive, which means that the farmer wants to just eat the non-tradeable fruit. Now that we have this, we can solve for μ:

$$\mu=(\beta'-\beta)\left(1+\frac{a(\beta-1)+\beta c}{a(1-\beta)}\right)$$

Which simplifies to:

$$\mu=(\beta'-\beta)\frac{\beta c}{a(1-\beta)}$$

The sign of μ is simply determined by $\beta'-\beta$. As long as gatherers are more patient than borrowers, i.e. $\beta'>\beta$, we will have $\mu>0$ so that the farmers will be up against their borrowing constraint in the steady state.

2.5 Efficient Solution

We can characterize the *efficient* solution to the model by thinking about a social planner picking the allocation of capital between farmers and gathers to maximize aggregate output in each period. Because of the linearity of preferences, we cannot find a planner's solution for the *allocation* of consumption across the two types of agents. But we can think about the optimal allocation of capital.

In period t, the planner would want to pick k_t and k_t' to maximize next period's output (current output is predetermined), subject to the constraint that capital sums up to the total available. We could impose the constraint that $k_t'=\frac{\bar{K}-k_t}{m}$ and write the problem as choosing k_t:

$$\max_{k_t^e}\quad (a+c)k_t^e+mG\left(\frac{\bar{K}-k_t^e}{m}\right)$$

The first order condition would be:

$$a+c=G'\left(\frac{\bar{K}-k_t^e}{m}\right) \tag{27}$$

This is pretty simple, really – the planner would like to allocate capital so as to equate the marginal products of capital across farmers and gathers. This is what maximizes total output; how that is split among the two types is not something we can solve for. With our particular production function, this would be:

$$a + c = \alpha \left(z + \frac{\bar{K} - k_t^e}{m} \right)^{\alpha - 1}$$

Which implies:

$$k_t^e = mz + \bar{K} - m \left(\frac{\alpha}{a + c} \right)^{\frac{1}{1-\alpha}} \tag{28}$$

There are couple of things to note from (28). First, the value of k_t is independent of anything with a t subscript. This means that, in the efficient allocation, the economy just sits in steady state in terms of how capital is allocated across the two types of producers. If there were productivity shocks, it would not affect the allocation of capital across farmers and gatherers. It would be constant.

Note in the parameterization I'm using, we would have $k^e = 0.91$, whereas in the steady state of the competitive equilibrium we have $k = 0.84$. The borrowing constraint distorts the steady state by having too much capital allocated to gatherers.

3 Solving and Simulating the Simple Model

I solve the model via a first order approximation. To do so, I need to (i) specify parameter values and (ii) introduce a shock process.

I set $\beta' = 0.99$ and $\beta = 0.98$. I set $m = 0.5$. For the production technology of farmers, I set $a = 0.7$ and $c = 0.3$. For the production technology of the gatherers I assume $z = 0.01$ and $\alpha = 1/3$. I need z to be sufficiently small so that the conditions in (5) in their paper are satisfied and both types of households produce in the steady state.

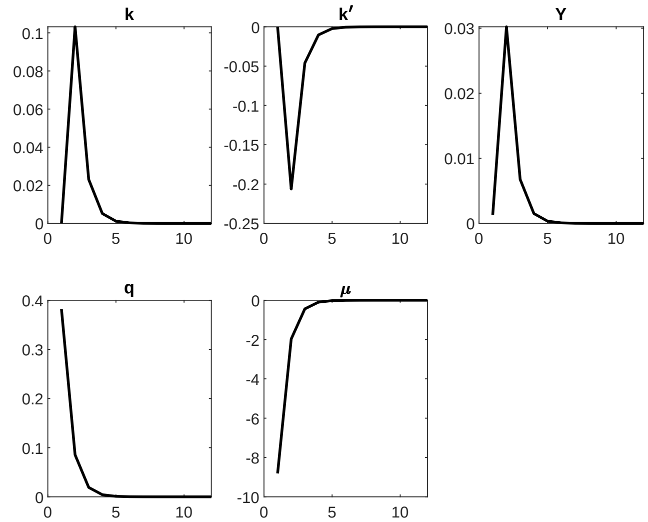
Then I need to introduce a shock. I'm going to introduce a one period iid mean zero technology shock. It affects the production technologies as follows:

$$y_t = (1 + \varepsilon_t)(a + c)k_{t-1} \tag{29}$$

$$y'_t = (1 + \varepsilon_t)(z + k'_{t-1})^{\alpha} \tag{30}$$

Since it is iid and mean zero, $\mathbb{E}_t \, \varepsilon_{t+j} = 0$ for $j > 0$. Thus, to first order, I don't need to worry about this in any of the dynamic Euler equations. It will only appear in (15) and (18) multiplying the relevant period t outputs.

Figure 1: IRFs to Productivity Shock



The figure above plots the impulse responses of k_t, k'_t, $Y_t = y_t + my'_t$, q_t, and μ_t. Focus first on output. Because the capital stock is predetermined, in the period of the iid shock output just reacts proportionally to the shock. But then starting in the next period, it jumps way up, and remains high for about four periods. What is going on? We can see in the first two graphs that capital is being reallocated to the farmers away from the gatherers. Why is this happening? The productivity shock is pushing up the price of land, q_t. This ends up easing the borrowing constraint facing the farmers, as evidenced by the decline in μ. This allows the farmers to borrow more and hence purchase more land. Because the steady state is distorted relative to the first-best (discussed above), reallocating capital to farmers is efficient and gets the economy closer to the efficient outcome. This results in output rising. This effect lasts more than just one period. With more capital, this further eases the borrowing constraint facing farmers in the future, even though productivity has gone back to where it started. But this easing of the constraint pushes up future land prices, and results in land still being allocated predominantly back to farmers. This effect eventually fades out. But the important point here is that a perfectly transitory productivity shock generates a persistent reallocation of capital that results in output rising.

It is worth noting, referencing back to (28), that in the efficient allocation there would be *no reallocation* of capital between farmers and gatherers. Since the shock is iid, this means there would be *no persistence* in response to the iid shock in the efficient allocation. But there is persistence here. So, in a sense, the borrowing constraint both *propagates* the iid shock through time, as we can see in the impulse response graph.