Volatility and Welfare*

Robert Lester
University of Notre Dame

Michael Pries
University of Notre Dame

Eric Sims
University of Notre Dame, NBER, and ifo

August 27, 2013

Abstract

This paper explores the relationship between volatility and welfare. Even though households prefer smooth streams of consumption and leisure, welfare can be increasing in the volatility of an exogenous driving force if factor supply is sufficiently elastic. We provide some analytical results for a model without capital, and do some quantitative exercises in a model with capital and a variety of shocks. Welfare is greater in high shock volatility regimes under plausible parameter values. Augmenting the model with features that increase the elasticity of factor supply extends the range of parameters over which higher volatility results in greater welfare.

*Corresponding Author: Eric Sims, esims1@nd.edu. 710 Flanner Hall, Notre Dame, IN 46556. We are thankful for helpful comments and suggestions from Robert Flood, Julio Garín, Chris Otrok (the editor), an anonymous referee, and seminar participants at Notre Dame. We acknowledge Randall Wright for a comment on a different project which motivated interest in this topic. The usual disclaimer applies.

1 Introduction

There has been considerable recent interest in understanding the role of changes in volatility in macroeconomic models. This interest stems, in part, from a desire to better understand the causes and consequences of the "Great Moderation."[1] However, much less attention has been paid to the question of whether periods of lower volatility are actually preferable from a welfare perspective. Given risk-averse consumers, there is a natural inclination to assume that volatility is always welfare-reducing. However, if factors of production are in sufficiently elastic supply, they can be intertemporally allocated so as to take advantage of "good times" in a way that can boost well-being. In other words, elastic factor supply gives production the features of an option, and options are of course more valuable when volatility is higher. If this option effect dominates households' aversion to non-smooth streams of consumption and leisure, then more volatility in exogenous driving forces can actually be associated with higher levels of welfare. While this outcome may be conceptually possible, it is natural to ask whether it is a mere theoretical curiosity. Are there realistic parameter configurations, for a standard business cycle model, for which volatility is actually welfare-enhancing? And if so, how quantitatively significant are the effects?

When Lucas (1987) (see also Lucas (2003)) explored the welfare costs of business cycles, he did so by calculating the magnitude of the welfare gains that could be attained by completely eliminating consumption volatility. He famously concluded that those gains are very small. Thus, whether one interpreted economic fluctuations as the result of market imperfections that magnify the economy's response to shocks, or as an efficient response to those shocks, the potential benefit of dampened fluctuations, either from smaller shocks or from correcting the market imperfections that magnify shocks, was insignificant.

However, by focusing only on consumption volatility, and not specifying the equilibrium model in which that consumption volatility arises, Lucas's calculations by construction reflected an incomplete assessment of the relationship between volatility and welfare. In an equilibrium model, fluctuations in key aggregates are also affected by production decisions, and unlike households, to whom volatility by itself is unambiguously undesirable, firms may view volatility as a source of opportunity. Since at least Hartman (1972) (see also the subsequent elaboration by Abel (1983)), we have understood that when firms face greater volatility—either in the prices of their products, factor prices, or in productivity—they respond to that greater volatility with a higher average level of investment (and, similarly, output). A key ingredient of that result is that firms have the flex-

[1] Kim and Nelson (1999) and Perez-Quiros and McConnell (2000) were the first to document the decline in volatility since dubbed the "Great Moderation." Subsequent contributions include Stock and Watson (2003), Sims and Zha (2006), Davis and Kahn (2008), Fernández-Villaverde et al. (2010), and many others.

ibility to adjust inputs—increasing them when conditions are favorable and reducing them when conditions deteriorate. Thus, the potential benefits of greater volatility are closely linked to the degree of elasticity in factor supplies. A comprehensive assessment of the relationship between volatility and welfare must weigh those benefits against the undesirable aspects of consumption volatility emphasized by Lucas. Cho et al. (2012) analyze the welfare consequences of greater TFP volatility in an RBC context, which takes account of these benefits and costs. While our model shares many similarties with Cho et al. (2012), we extend the framework along many important directions that Cho et al. (2012) do not consider. These additional directions are elaborated on below.

We examine this question quantitatively within the benchmark laboratory used by macroeconomists to study fluctuations, i.e. a standard real business cycle model. We take this approach, as opposed to one in which we incorporate a variety of extensions common in the literature—such as sticky prices or labor market frictions—because the competing effects of volatility are easier to identify and assess in the simpler benchmark model. Moreover, the basic insights obtained from this benchmark model would naturally carry over to the more elaborate models that build upon it. For much of the paper we focus on the welfare implications of volatility in an exogenous productivity process, though we extend our analysis to volatility in demand-side disturbances later in the paper. To gain analytical insights, in Section 2 we begin with a simpler version of that model in which there is no capital. As a benchmark we make the common assumption that preferences are iso-elastic and additively separable in consumption and leisure. We are able to show analytically that welfare is increasing in the volatility of shocks to productivity if the coefficient of relative risk aversion is sufficiently low and the Frisch elasticity of labor supply is sufficiently high. However, in this simplified model, even if the Frisch elasticity is infinite, for volatility to be welfare-enhancing the coefficient of relative risk aversion must be much lower (less than 1/2) than what is typically considered plausible.

One would expect that introducing capital into the model, and thus introducing a means by which consumers can substitute intertemporally as well as a second input that firms can adjust to changing conditions, might expand the range of parameters over which greater volatility is welfare-enhancing. To explore whether that is the case, in Section 3 we solve a fully dynamic model with capital. We solve the model using a perturbation method; given the focus on the effects of volatility, we must use a second-order approximation.[2] We solve the model for two values—high and low—of the standard deviation of productivity shocks. To measure the welfare differences

[2]We use the method of Schmitt-Grohe and Uribe (2004). See also Aruoba et al. (2006) for a comparison of the speed and accuracy of this solution method relative to alternative methods.

between the different shock volatilities, we calculate the compensating variation—the percentage by which consumption in the high-volatility environment would have to be changed in order to achieve the same welfare as the low-volatility environment.

These compensating variation calculations are complicated by the fact that the equilibria of the high- and low-volatility environments have different mean capital stocks. In particular, the high-volatility environment has a higher mean capital stock, since the opportunity to take advantage of larger positive shocks makes capital more productive on average. That will naturally make the high-volatility environment relatively more attractive if the transition costs of having acquired that additional capital (i.e. the foregone consumption) are not taken into account. To address this issue, we calculate a *conditional* compensating variation that measures welfare in the two environments as the value function at a common value of capital (we condition on the value of capital in the non-stochastic steady state), so that the two environments are compared on an equal footing. We then also calculate the *unconditional* compensating variation, which measures welfare as the unconditional expectation of the value function and thus gives a sense of the welfare difference in the long-run, once the costs of acquiring any additional capital have already been absorbed.

We find that the range of parameter values over which the high-volatility regime is preferred to the low-volatility regime does indeed expand relative to the simple model without capital. Moreover, the range now includes plausible parameter configurations that have been used in the literature—for example, with the preferences utilized in Hansen (1985)—with log utility over consumption and an infinite Frisch labor supply elasticity—both the conditional and unconditional compensating variations are negative, meaning that agents prefer the environment with more volatility in the exogenous productivity process. Nevertheless, the potential welfare impact from changes in shock volatility is small; over a range of parameters, the compensating variation associated with cutting the shock volatility in half never exceeds 0.5% of consumption.

We next explore some extensions of the baseline model. To better understand the role that different preference specifications play, we consider two specifications commonly used in the business cycles literature that depart from the conventional separability between consumption and leisure. First, we consider the King et al. (1988) specification that allows for balanced growth even when the coefficient of relative risk-aversion differs from 1. Second, we consider the preferences introduced in Greenwood et al. (1988), which eliminate the wealth effect of shocks on labor supply, and thus amplify the response to shocks by making labor supply effectively more elastic. While the results for the King et al. (1988) preference specification differ little from the baseline additively separable specification, we find that with Greenwood et al. (1988) preferences the high-volatility environment is more likely to be welfare-enhancing, as one would expect from the more elastic labor supply.

For parameterizations in which volatility is welfare-improving, the welfare gains from moving from low- to high-volatility environments are much larger than in either the additively separable or King et al. (1988) preference specifications.

Given the important role played by factor supply elasticities, we also explore, in Section 4, the inclusion of variable capital utilization in the baseline RBC model, which is often incorporated in models to achieve greater amplification of shocks. With variable capital utilization, firms can respond more easily to changing economic conditions—raising utilization when conditions are good and reducing it when conditions deteriorate—and hence the benefits of higher volatility are potentially greater. Indeed, we find that this extension further extends the range of parameters over which the high-volatility regime is preferred. Moreover, the welfare differences become quantitatively more significant. For example, for the Hansen (1985) specification with log preferences and indivisible labor, the conditional compensating variation is -0.61 (consumption in the high-volatility environment must be reduced by 0.61% in order to achieve the same welfare as the low-volatility environment) and the unconditional compensating variation is -1.47.

In Section 5 we extend our analysis to look at the welfare effects of volatility in shocks other than neutral productivity shocks. We focus on investment-specific technology shocks and preference shocks, both of which are common features of modern DSGE models. Investment-specific shocks affect the efficiency of transforming non-consumed output into new capital goods and have been shown to be an important business cycle shock and driver of longer run trends. Unless households are extremely unwilling to substitute consumption intertemporally and labor for leisure, welfare is higher when the standard deviation of investment-specific technology shocks take on the larger value. Not only is the range of parameters over which welfare is increasing in volatility wider than in the case of neutral shocks, the welfare gains from more volatility are also quantitatively larger.

Preference shocks are a common way of modeling demand-side disturbances in recent DSGE models. In the case of preference shocks, we find that the high-volatility environment is preferred to the low-volatility environment over all preference parameters. The intuition for the desirability of volatility in preference shocks is similar to the intuition for both neutral and investment-specific productivity shocks. Episodes of "good times" and "bad times"—positive and negative shocks to the utility from consumption—present opportunities, and households can take advantage of those opportunities by substituting economic activity toward the "good times," and achieve a higher average utility by doing so.

The literature that this paper connects with most directly is the "cost of business cycles" literature that followed Lucas (1987). Recognizing that the exercise in Lucas (1987) was a first pass at the question, a large body of subsequent research attempted to understand what changes, relative to the benchmark in Lucas (1987), might result in business cycles being more costly. For example, Alvarez and Jermann (2004) used fluctuations in asset prices to infer the costs of business cycles, in way that bypasses the need to specify a utility function. Like Lucas (1987) they find that eliminating the consumption volatility due to business cycles has little impact on welfare. Tallarini (2000) also uses asset price data to discipline his exercise, but considers an alternative preference specification—Epstein and Zin (1989) preferences—and finds that the costs of business cycles can be large. Otrok (2001) shows, however, that if the preference parameters are chosen so as to also match key business cycle statistics, then the costs are again very small.

Krusell and Smith (1999) and Krusell et al. (2009) consider standard preferences, but depart from the representative agent assumption. In particular, they explore the possibility that uninsurable idiosyncratic risk might substantially raise the welfare costs of business cycles, since the adverse effects of business cycles might be particularly concentrated among a subset of the population. Krusell et al. (2009) find that if a large portion of idiosyncratic income risk is associated with the business cycle, then the welfare costs are considerably larger than in the Lucas benchmark—on the order of 1% of average consumption. Schulhofer-Wohl (2008) also departs from the representative agent framework, but instead of assuming idiosyncratic risk and incomplete markets he considers a complete markets environment with heterogeneity in risk-aversion. In such an environment, agents with low risk aversion effectively insure the more risk averse agents, and as a result the less risk averse agents can actually be made better off by economic fluctuations. Moreover, the ability to buy insurance from the less risk averse also reduces the welfare costs of fluctuations for the more risk averse agents.

Our approach in this paper is not to seek out alternative assumptions that might raise the welfare costs of business cycles, but rather to draw attention, within the context of a baseline business cycle model, to a countervailing force that, by itself, works to make fluctuations potentially welfare-improving. We concede that other factors not considered here, such as the non-time-separability of preferences or uninsured idiosyncratic risk, may in fact make fluctuations quite costly. We also do not want to claim that a reduction in volatility such as the Great Moderation is necessarily welfare-reducing. Rather, our exercises show that such a decline in volatility *may* be welfare-reducing, and, in any event, that standard cost of business cycle accounting approaches will tend to overstate the benefits (if there are any) of a volatility decline by ignoring the beneficial aspects of volatility on mean utility.

Within the "costs of business cycles" literature, the paper closest to ours is the aforementioned Cho et al. (2012). While the overall focus of the two papers is in many ways similar, there are several important distinctions. First, our welfare calculations consider both the conditional and unconditional metrics described above, which highlights the significance of the transitional costs, in terms of capital accumulation, associated with reaping the benefits of greater volatility. Second, we explore more extensively the importance of the elasticity of factor supply by considering the extension in which there is variable capital utilization. Third, we consider a broader class of preference specifications. Finally, whereas Cho et al. (2012) focus only on neutral productivity shocks, we show that the potential benefits of greater volatility also arise in an environment in which fluctuations are the result of investment-specific productivity shocks and preference shocks.

In addition to its obvious connection with the "costs of business cycles" literature, this paper is related to a couple of other areas of research. First, there is a growing literature on the role of time-varying volatility. While the approach of our paper is to investigate the impact of more secular changes in volatility—i.e. from a high-volatility regime to a low-volatility regime—the idea that higher frequency movements in uncertainty or volatility can be a driver of economic fluctuations has also received considerable attention. For example, Bloom (2009) explores the idea that an increase in uncertainty can cause firms to delay investment and hiring, due to a"real option effect," and thus can precipitate an economic downturn. Similarly, Fernández-Villaverde et al. (2011) examine the impact that uncertainty about fiscal policy has on economic activity.

The paper is also related to the vast literature on the Great Moderation. One of the debates in that literature focuses on whether the moderation arose due to a period of diminished shocks (the "good luck hypothesis") or due to better economic policy or a change in the structure of the economy. Stock and Watson (2003) survey the evidence and tentatively conclude that "most of the moderation seems to be attributable to reductions in the volatility of structural shocks." Sims and Zha (2006) utilize a structural VAR to examine whether improved monetary policy could account for the reduction in economic volatility, but they find that the model that best fits the data is the one in which only the variances of the structural disturbances change, and policy coefficients do not change. Fernández-Villaverde et al. (2010) explore the same question using different methods—Bayesian estimation of a fairly rich DSGE model—but again find that most of the change in the economy can be attributed to changes in the magnitude of shocks.

This evidence provides support for our approach of considering a reduction in the magnitude of shocks as the source of reduced economic volatility. Nevertheless, it may be worthwhile to explore the relationship between volatility and welfare in a model in which different policies or policy rules account for the changing volatility. The relationship between welfare and volatility in that context is potentially much more complicated. In the approach that we take here, the First Welfare theorem applies and so any policy intervention designed to reduce volatility can only reduce welfare. To talk about policy interventions that potentially increase welfare by reducing volatility, one would have to take a stand on the market imperfections that policy-makers are trying to counteract. Furthermore, market imperfections do not always increase volatility—e.g. in sticky-price models the response to a positive productivity shock is generally too small—and thus optimal policy will in some cases seek to increase volatility in order to move closer to the first-best allocation. As such, it is at least conceivable that a reduction in volatility, such as the Great Moderation, could in fact reflect a worsening of policy.

We leave these questions for future work and instead explore the relationship between economic volatility and welfare by focusing on changes in the volatility of shocks. In the next section, we begin by considering a simple analytical model without capital.

2 A Simple Analytical Model

In order to gain intuition, this section considers a model without capital in which exogenous productivity is the only state variable. The simple structure allows us to solve for the policy functions analytically. With these analytical policy functions we can construct an indirect utility function from which we can characterize the set of parameters for which higher volatility in the exogenous productivity process is welfare-improving.

The economy has a representative agent with time separable flow utility over consumption, C_t, and labor, N_t, that discounts the future by the discount factor β, with $0 < \beta < 1$. The flow utility function, $U(C_t, N_t)$, is assumed to be twice continuously differentiable, increasing and concave in the first argument, and decreasing and convex in labor (equivalently increasing and concave in leisure). For the remainder of this section we assume that utility is separable in its arguments and takes on the common iso-elastic form, though we will consider alternative specifications of preferences in later sections:

$$u(C_t, N_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{N_t^{1+\phi}}{1+\phi}, \qquad \psi, \gamma, \phi \geq 0 \tag{1}$$

The parameter $\psi > 0$ is a scaling parameter that is of little interest for the dynamic solution of the model. As such, it is convenient to normalize it to $\psi = 1$, which we do for the remainder of this section. γ is the coefficient of relative risk aversion and ϕ is the inverse Frisch labor supply elasticity.

A representative firm produces output using a constant returns to scale technology, with labor hired from households as the only input. The production function is subject to an exogenous productivity disturbance, Z_t, which obeys some known, stationary stochastic process:[3]

$$Y_t = Z_t N_t \tag{2}$$

There are no frictions that would distort the competitive equilibrium. As such, the equilibrium of this economy can be characterized as the solution to a social planner's problem, with the expected present discounted value of flow utility as the objective function, subject to the constraint that consumption not exceed production. Because there is no endogenous state variable, the dynamic planner's problem is equivalent to a sequence of static one-period problems:

$$\max_{C_t, N_t} \quad \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi}$$

$$\text{s.t.}$$

$$C_t \leq Z_t N_t$$

The policy functions for the optimal choices of C_t and N_t in terms of Z_t are:

$$C_t = Z_t^{\frac{1+\phi}{\gamma+\phi}} \tag{3}$$

$$N_t = Z_t^{\frac{1-\gamma}{\gamma+\phi}} \tag{4}$$

After substituting the policy functions into the objective function and defining $\widetilde{U}(Z_t)$ as the indirect utility function, some algebraic manipulations yield:

$$\widetilde{U}(Z_t) = \frac{\gamma+\phi}{(1+\phi)(1-\gamma)} Z_t^{\frac{(1+\phi)(1-\gamma)}{\gamma+\phi}} - \frac{1}{1-\gamma} \tag{5}$$

The first and second derivatives are:

$$\widetilde{U}'(Z_t) = Z_t^{\frac{(1+\phi)(1-\gamma)}{\gamma+\phi} - 1} > 0$$

$$\widetilde{U}''(Z_t) = \left(\frac{(1+\phi)(1-\gamma)}{\gamma+\phi} - 1 \right) Z_t^{\frac{(1+\phi)(1-\gamma)}{\gamma+\phi} - 2}$$

The second derivative is greater than zero if and only if:

$$1 > 2\gamma + \gamma\phi \tag{6}$$

[3] For this section, we do not need to formally specify the process for Z_t. With no endogenous state variable in the model, the policy functions depend only on the level of Z_t, not on the properties of the stochastic process it obeys. In subsequent sections, we focus on the commonplace AR(1) specification.

If $\widetilde{U}''(Z_t) > 0$, then, by Jensen's inequality, $\widetilde{U}(E(Z_t)) < E(\widetilde{U}(Z_t))$. If this condition is satisfied, it means that a mean-preserving spread on Z_t will increase welfare. In other words, in expectation the household prefers more volatility in Z_t.

Figure 1 plots frontier of parameters, (γ, ϕ), along which the household is indifferent between more or less volatility. In the shaded region below the curve the household prefers more volatility; above the curve more volatility reduces welfare. As one moves towards the origin, the household is increasingly less risk averse and more willing to substitute labor for leisure. That is, at points near the origin, the coefficient of relative risk aversion, γ, is small, while the Frisch labor supply elasticity, $1/\phi$, is large.

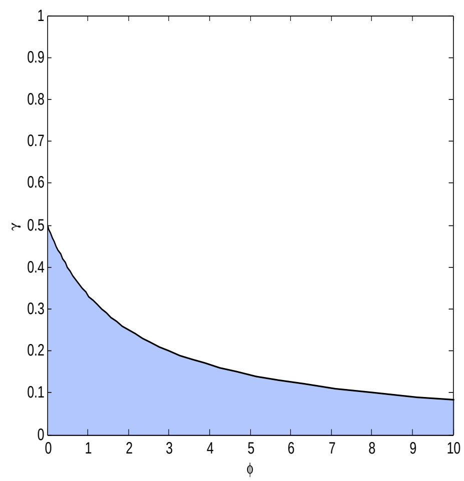

Figure 1: This figure plots the set of (γ, ϕ) pairs for which the household is indifferent between more or less volatility in Z_t in the model without capital described in this section. In the shaded region, the household prefers more volatility in Z_t; above the curve welfare is decreasing in volatility.

The mechanism through which more volatility can lead to higher welfare is through its impact on the stochastic mean of consumption. For a given mean of consumption and leisure, welfare decreases in volatility given the assumptions that $\gamma \geq 0$ and $\phi \geq 0$ (in other words, that utility is concave in both consumption and leisure). If this were an endowment economy, for example, then more volatility in the exogenous driving force would be strictly welfare-reducing. But with the ability to respond to shocks via endogenous factor supply, the mean of consumption may be higher when volatility is higher. Consider two probability distributions for Z_t, $f(\cdot)$ and $g(\cdot)$, with $g(\cdot)$ a mean-preserving spread of $f(\cdot)$. The "good times" in an economy subject to the distribution $g(\cdot)$ are "better" (in terms of Z_t) than in an economy facing $f(\cdot)$, but the bad times are also worse. By substituting labor across time, households may be able to achieve a higher mean value of consumption by allocating more work effort to periods when Z_t is high and less when the realization of Z_t is low. If factor supply is sufficiently elastic, this effect may dominate the concavity of preferences, resulting in higher welfare from higher volatility.[4]

For this particular specification of the model, the set of parameters in which welfare is increasing in volatility is small. Even if $\phi = 0$, so that preferences over labor are linear, $\gamma < 0.5$ is a necessary condition for volatility to be welfare-improving. This is below values of γ typically employed in the macro literature, which are typically around 1.[5]

3 A Model with Capital

In the previous section we presented a stylized model in which it was possible for welfare to be increasing in volatility, even though the representative household dislikes risk. For that model, the range of parameters over which volatility is welfare-improving was small and outside the bounds of what most economists would consider empirically plausible. As such, one might be tempted to view these results as a mere theoretical curiosity with little real world relevance.

In this section we extend the simple model of the previous section to include capital. Qualitatively, the basic conclusions from the previous section hold—if factor supply is sufficiently elastic, welfare can be increasing in the volatility of the exogenous productivity process. The availability of an asset to allow for intertemporal substitution of consumption, however, considerably expands the set of parameters over which welfare is increasing in volatility.

3.1 The Model

The production function is a constant returns to scale technology in capital and labor, with an exogenous productivity disturbance, Z_t:

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha}, \;\; 0 \leq \alpha \leq 1 \tag{7}$$

[4] From (3), it is clear that C_t is convex in Z_t if $\gamma < 1$. If this restriction is satisfied, it means that the household achieves a higher mean level of consumption when the variance of Z_t increases. $\gamma < 1$ is a much weaker condition than in (6), which requires $\gamma < 0.5$ when $\phi = 0$.

[5] There does not seem to be strong compelling evidence pointing to any particular value of γ. Though Hall (1988) and Dynan (1993) find values of γ of 10 or greater, Mulligan (2002) and Gruber (2006) report values of this parameter less than 1, while Blundell et al. (1994) find that γ likely ranges from 0.75-1.75. It is common in macro models to assume log utility over consumption, implying a value of γ of 1.

In equilibrium, α will be equivalent to capital's share of income. The law of motion for capital is given by the standard accumulation equation, where δ is an exogenous depreciation rate:

$$K_{t+1} = I_t + (1-\delta)K_t \tag{8}$$

As before, there are no distortions, and so the competitive equilibrium can be characterized by the solution to a social planner's problem. Given the possibility to substitute resources intertemporally through investment in physical capital, the problem cannot be reduced to a sequence of one-period problems. The problem of the planner is to choose allocations of consumption, labor, and future capital to maximize the present discounted value of flow utility, subject to the constraint that consumption plus investment not exceed production. The problem can be written recursively as a Bellman equation:

$$V(Z_t, K_t) = \max_{C_t, N_t, K_{t+1}} U(C_t, N_t) + \beta E_t V(Z_{t+1}, K_{t+1}) \tag{9}$$

$$\text{s.t.:} \tag{10}$$

$$C_t + K_{t+1} - (1-\delta)K_t \leq Z_t K_t^{\alpha} N_t^{1-\alpha}$$

The conditions characterizing an optimal interior solution to this problem are given by:

$$U_C(C_t, N_t) = \beta E_t \left(U_C(C_{t+1}, N_{t+1}) \left(\alpha Z_{t+1} \left(\frac{K_{t+1}}{N_{t+1}} \right)^{\alpha-1} + (1-\delta) \right) \right) \tag{11}$$

$$-U_N(C_t, N_t) = U_C(C_t, N_t)(1-\alpha) Z_t \left(\frac{K_t}{N_t} \right)^{\alpha} \tag{12}$$

(11) is the standard Euler equation describing the tradeoff between current and future consumption, while (12) characterizes the intratemporal consumption-labor tradeoff.

Because of the dynamics introduced by endogenous capital accumulation, to close the model we need to make an assumption on the stochastic process for Z_t. We assume that it obeys a stationary AR(1) process with an unconditional mean of unity:

$$Z_t = (1-\rho) + \rho Z_{t-1} + \sigma_i \varepsilon_t, \;\; \varepsilon \sim N(0,1) \tag{13}$$

The autoregressive parameter ρ governs the persistence of the process and satisfies $0 \leq \rho < 1$. The shock is scaled by σ_i, which can take on two values, $i = h$ or l, with $\sigma_h > \sigma_l$. The variance of Z_t is $\frac{\sigma_i^2}{1-\rho^2}$, and is increasing in both the innovation variance, σ_i^2, and in the persistence, ρ. We focus on the welfare implications of different values of the innovation variance holding the persistence parameter fixed.[6] Finally, note that the exogenous process for Z_t is specified in levels, not log-levels. The reasons for this is to focus exclusively on the implications of volatility. If the process for Z_t were written as an AR(1) in the log, then an increase in the innovation variance would mechanically translate into an increase in the mean of the level of Z_t by Jensen's inequality.[7]

3.2 Parameterization and Numerical Approximation

Because analytic solutions for the policy functions are generally not available for this model, we must resort to numerical approximations, which requires picking values for the parameters of the model.

We consider two values of the standard deviation of the technology shock: $\sigma_h = 0.02$ and $\sigma_l = 0.01$.[8] Most other parameters are chosen to match longer run moments and are held fixed at conventional values. The unit of time is taken to be a quarter. We set $\beta = 0.995$ to match an annualized real risk-free rate of return of two percent. $\alpha = 1/3$, in line with the average labor share in post-war US data. $\delta = 0.02$ to match the investment to capital ratio in the data. We fix $\rho = 0.95$ and abstract from trend growth. For the remainder of this section we assume that preferences are given by the additively separable iso-elastic specification in (1). We consider a range of values of γ and ϕ, the coefficient of relative risk aversion and the inverse Frisch labor supply elasticity. For given values of γ and ϕ, the scaling parameter, ψ, is set to ensure that labor hours are one-third in the non-stochastic steady state.[9]

[6] The reason for this is that ρ affects not only the variance of Z_t but also the effective elasticity of factor supply. Since the mechanism by which volatility may increase welfare is endogenous factor supply, the exercise is not very clean when one considers different values of ρ. When ρ is small, for example, the wealth effect of an increase in Z_t is small, and hence hours react strongly. In contrast, when ρ is large, there is a large positive wealth effect when Z_t increases, leading to a much more muted hours response. Another justification for holding ρ fixed is given in Stock and Watson (2003), who find that the Great Moderation is better understood as a result of lower shock variances than as a result of lower persistence of shocks.

[7] Since we specify the process for Z_t in levels centered around a mean of 1, its mean deviations units can nevertheless by interpreted as log or percentage deviations. Cho et al. (2012) specify a productivity process which follows a mean zero AR(1) in the log. They employ a mean correction to avoid the problem that an increase in innovation variance leads to an increase in the mean of the level of Z_t via exponentiation of the log. While this procedure does correct for the mean, it does introduce right skewness into the distribution of the level of Z_t, which tends to be desirable from a welfare perspective. By specifying our process for Z_t in the levels centered around one, we avoid this issue – changes in the innovation variance do not lead to left or right skewness.

[8] Because of the non-linear nature of the solution technique, in principle the compensating variations we calculate depend on both the difference between the high- and low-volatility regimes (e.g. $\sigma_h - \sigma_l$), as well as on the base volatility, e.g. σ_l. We have numerically verified that our results are not very dependent on the base volatility, σ_l, and that the compensating variations are approximately linear in the difference in the variances of the high- and low-volatility regimes.

[9] In the model considered here, the non-stochastic steady-state capital to labor ratio is independent of ψ, γ, and ϕ. If ψ is held fixed as γ and ϕ vary, non-stochastic steady-state hours will vary, which means that the non-stochastic steady-state value of the capital stock will vary proportionally. This difference is immaterial for making welfare comparisons for different volatilities for a given set of parameters, but for comparing compensating variations across

The solution of the model is approximated using perturbation methods. Specifically, we take a second-order approximation of the first-order and market-clearing conditions around the model's non-stochastic steady state. The general method is described in detail in Schmitt-Grohe and Uribe (2004); further detail is provided in Appendix A. A first-order approximation of the model implies that the approximated mean of discounted utility is equal to the discounted utility evaluated at the non-stochastic steady state. Since the non-stochastic steady state is invariant to the variance of the exogenous shock process, a first-order approximation would therefore result in no welfare consequences from higher or lower volatility. In a second-order approximation, this certainty equivalence property no longer holds and changes in volatility will affect welfare. In addition to finding approximated policy functions for consumption and labor, we obtain a second-order approximation of the value function, which can be used for welfare calculations.

Let $\widetilde{C}_{i,t}$ and $\widetilde{N}_{i,t}$ denote the optimal choices of consumption and labor hours for both the high ($i = h$) and low ($i = l$) volatility regimes. Given the regime, $i = h$ or l, the value of being in a particular state at time t, $V_i(Z_t, K_t)$ is the expected present discounted value of flow utility evaluated at the optimal choices of consumption and labor:

$$V_i(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j U\left(\widetilde{C}_{i,t+j}, \widetilde{N}_{i,t+j}\right), \;\; i = h, l \tag{14}$$

From this we can compute a compensating variation welfare metric for the two different volatility regimes. In particular, let λ be the fraction of consumption that the household would need each period in the high-volatility regime to yield the same welfare as would be achieved in the low-volatility regime. A positive value for λ means that the household prefers the low-volatility regime—it would need extra consumption when volatility is high to be indifferent between the two regimes. In contrast, a negative value of λ means that the household prefers the high-volatility regime. λ is the solution to the following expression:

$$V_l(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j U\left((1+\lambda)\widetilde{C}_{h,t+j}, \widetilde{N}_{h,t+j}\right) \tag{15}$$

Further details on the calculation of compensating variations, with different preference specifications, are provided in Appendix B. It is important to note that above we conditioned on the same state vector in computing the compensating variation for different volatility regimes. In other words, the value functions in both the high- and low- volatility regime are evaluated at the same

different parameter configurations, which we do below, it is important that the steady-state capital stocks be the same.

point in the state space; hence the expectations operator on the right hand side of (15) is conditional on the same (Z_t, K_t) at which the value function in the low-volatility regime is evaluated on the left hand side. The point at which these value functions are evaluated will in general affect the magnitude of λ. We evaluate the value functions at the non-stochastic steady-state values of Z_t and K_t, denoted by Z^* and K^*.

Another welfare metric that one could compute is based on an unconditional value function. In this exercise one compares mean welfare across the two regimes, rather than conditioning on the same initial point in the state space. The expected welfare of a particular regime is:

$$E\left(V_i(Z_t, K_t)\right) = E\sum_{j=0}^{\infty}\beta^j U\left(\widetilde{C}_{i,t+j}, \widetilde{N}_{i,t+j}\right), \quad i = h, l \tag{16}$$

In contrast with (14), in this formulation E is an unconditional expectations operator and appears on both sides; the unconditional expectation replaces the conditional operator on the right hand side via application of the Law of Iterated Expectations. We can define an unconditional compensating variation, λ^u as follows:

$$E\left(V_l(Z_t, K_t)\right) = E\sum_{j=0}^{\infty}\beta^j U\left((1+\lambda^u)\widetilde{C}_{h,t+j}, \widetilde{N}_{h,t+j}\right) \tag{17}$$

The conditional and unconditional exercises will in general yield different compensating variation measures. The reason for this is that the mean of the capital stock depends on the volatility of the productivity process. For most of the exercises we consider, the mean capital stock is increasing in the innovation variance of Z_t. The unconditional welfare comparison is then essentially endowing the high-volatility economy with more capital than an economy subject to the low-volatility regime, and therefore ignores the cost of the transition from a low to a high mean capital stock. Which welfare metric one prefers is contingent on the question being asked. If one wants to account for the transitional effects of changing policies, the conditional welfare metric is preferred because the unconditional one ignores the sacrifices of transitioning to a higher mean capital stock. On the other hand, if one wants to know the longer term consequences of operating under different regimes, the unconditional metric is preferred because it takes into account one of the key benefits of higher volatility—a higher mean capital stock.

3.3 Results

As a benchmark we assume that preferences are given by the additively separable iso-elastic specification in (1). Table 1 shows both the conditional, λ, and unconditional, λ^u, compensating variations in the baseline model for different values of γ and ϕ. The compensating variations are multiplied by 100 and therefore can be interpreted as percentages of consumption. We consider five values of each of the two preference parameters: $\gamma = (0.5, 1, 1.5, 3, 5)$ and $\phi = (0, 0.4, 1, 3, 10)$. For comparison, we also report the compensating variations of an exercise similar in spirit to that of Lucas (1987), in which the labor/leisure margin is ignored so that the calculation is independent of ϕ. For this exercise we reduce consumption volatility from its level in the high productivity shock volatility production economy to the low-volatility regime, treating consumption as an exogenous endowment stream rather than an equilibrium outcome. This exercise is slightly different from Lucas in that he considered reducing consumption volatility all the way to 0.[10]

λ	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-0.2186	-0.0134	0.0734	0.1744	0.2203
$\phi = 0.4$	-0.0981	0.0331	0.1049	0.2054	0.2585
$\phi = 1$	-0.0455	0.0554	0.1203	0.2263	0.2909
$\phi = 3$	-0.0036	0.0743	0.1338	0.2534	0.3451
$\phi = 10$	0.0162	0.0837	0.1408	0.2753	0.4034
λ^u	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-0.2915	-0.0493	0.0405	0.1376	0.1794
$\phi = 0.4$	-0.1275	0.0149	0.0854	0.1767	0.2218
$\phi = 1$	-0.0599	0.0452	0.1075	0.2008	0.2530
$\phi = 3$	-0.0083	0.0706	0.1268	0.2297	0.3004
$\phi = 10$	0.0156	0.0832	0.1369	0.2522	0.3484
λ^{lucas}	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
	0.0938	0.1867	0.2877	0.5544	0.9488

Table 1: The numbers in this table show both the conditional (upper panel) and unconditional (lower panel) compensating variations of moving from a high-volatility regime, $\sigma_h = 0.02$, to a low-volatility regime, $\sigma_l = 0.01$, for different values of γ and ϕ in the basic RBC model. All numbers are multiplied by 100, and are interpreted as percentages of consumption.

[10] The numbers presented in the table are larger than those which Lucas presented. He argued that reducing the standard deviation of consumption from 3.5 percent to 0 percent would result in a welfare gain with log utility of only about 0.05 percent of steady-state consumption. In the experiment we run, unconditional log consumption volatility declines from about 7 percent to 3.5 percent. While this is the same change in standard deviation, our experiment is a three times larger reduction in the variance of consumption than Lucas considered. As noted in footnote 7, the compensating variations are roughly linear in the variance of the shock, and so our number is approximately three times larger than Lucas's.

Positive values of the compensating variations mean that the household prefers lower volatility. Negative values, in contrast, mean that the household prefers more volatility. For both the conditional and unconditional compensating variations, the numbers are declining in the household's risk aversion parameter, γ, and increasing in its Frisch labor supply elasticity, $1/\phi$. This finding accords with the analytical results in Section 2—the more willing the household is to substitute consumption across time (the smaller is γ) and the more willing it is to substitute leisure for labor (the smaller is ϕ), the more likely it is to prefer higher volatility. In contrast to the previous section, however, the range of parameters over which welfare is increasing in volatility is substantially larger. For $\gamma = 0.5$, for example, the household prefers higher volatility so long as its Frisch labor supply elasticity is greater than about 0.2 (ϕ smaller than 5). For log utility over consumption ($\gamma = 1$), welfare is increasing in both the conditional and unconditional sense when $\phi = 0$, so that the labor supply elasticity is infinite. This parameterization of preferences is quite common in macroeconomics—it is isomorphic to the Hansen (1985) indivisible labor model.

Note that the λ^{lucas} compensating variations are always greater than either λ or λ^u, usually be a factor of two to three. By taking consumption to be exogenous, the Lucas exercise focuses only on the direct implication of volatility, which is a less smooth stream of consumption. By ignoring how firms and households can respond in an optimal way to shocks to the economy, the Lucas (1987) exercise ignores the effects of volatility on means, and thus overstates the costs of business cycles. Whereas in his exercise volatility is always welfare-reducing, by taking into account production decisions there exist reasonable parameter configurations under which more volatility is actually preferred.

Figure 2 plots the frontier of (γ, ϕ) values along which the household is indifferent between the two volatility levels. The parameter frontier for the model without capital, shown in Figure 1, is reproduced here for comparison. While the set of values of (γ, ϕ) over which welfare is increasing in volatility is still modest, it is substantially wider than in the model without capital.

An important takeaway from Table 1 is that the unconditional compensating variations, λ^u, are always smaller (less positive or more negative) than their conditional counterparts, λ. The reason for this difference points to the key mechanism by which volatility may increase welfare, and thus merits more discussion. In the model, the means of the capital stock, output, and consumption are all increasing in the standard deviation of productivity shocks. By intertemporally substituting consumption and intratemporally substituting between labor and leisure, the household can take advantage of good times by saving and working more when productivity is high, and the opposite when productivity is low. By allocating relatively more inputs to the "good times" than the "bad," the economy can achieve higher average productivity. Thus, in equilibrium there will also be a

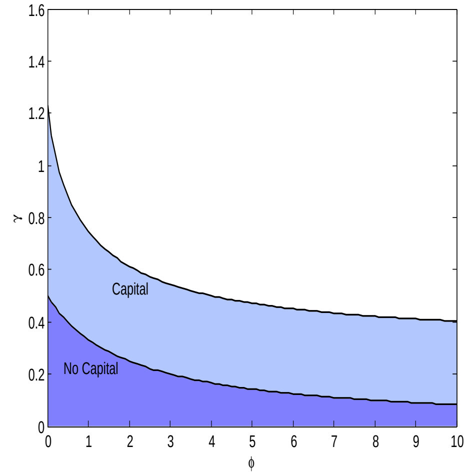

Figure 2: This figure plots the set of (γ, ϕ) pairs for which the household is indifferent between higher or lower innovation variance in the process for Z_t in the baseline model with capital that is described in this section. This figure is based on the unconditional compensating variations. In either of the shaded regions welfare is increasing in the innovation variance of Z_t. For point of comparison, the set of (γ, ϕ) pairs for which the household is indifferent between more or less volatility in Z_t in the model with no capital from Section 2 is also plotted, with areas in the darker shaded region those in which welfare is increasing in volatility.

higher mean capital stock. The larger the difference between "good times" and "bad" (e.g. the more volatile the productivity process is), the greater the opportunities for the household to achieve a higher mean capital stock.

Figure 3 plots the mean capital stock and the steady-state capital stock as a function of the standard deviation of the productivity shock for two different specifications of preference parameters: $\gamma = 1$ and $\phi = 0$, which corresponds to the indivisible labor model, and $\gamma = 1$ and $\phi = 1$, with a Frisch elasticity of unity. The steady-state capital stock is independent of these two parameters. Under both specifications the mean capital stock is increasing in volatility, more so for the specification with $\phi = 0$. This follows from the fact that when labor supply is more elastic, the ability of the household to take advantage of volatility is greater. When the standard deviation of the productivity shock is 0.01, the mean capital stock is 0.3 percent higher than the steady state when $\phi = 0$ and 0.15 percent higher when $\phi = 1$. When the standard deviation of the shock is 0.02, these differences relative to the steady state grow to 1.2 percent and 0.6 percent, respectively. When considering an increase in volatility from 0.01 to 0.02, the mean capital stock increases by 0.9 percent when $\phi = 0$ and 0.45 percent when $\phi = 1$.

This beneficial aspect of volatility—higher mean capital and higher mean output—may or may not be enough to overcome the utility cost of a more uneven stream of consumption and leisure—as

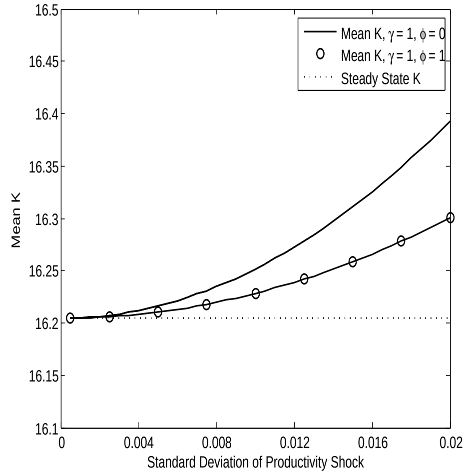

Figure 3: This figure plots the mean capital stocks for different values of the standard deviation of productivity shocks (ranging from 0 to 0.02) for two different sets of preference parameters in the baseline model: $\gamma = 1$ and $\phi = 0$, and $\gamma = 1$ and $\phi = 1$. The dotted line shows the non-stochastic steady-state value of the capital stock, which is independent of those parameters.

shown in Table 1, welfare is higher with more volatility for $\gamma = 1$ and $\phi = 0$, but is lower with higher volatility when $\gamma = 1$ and $\phi = 1$, even though the mean capital stock is increasing in volatility under both parameter specifications. The fact that the mean of capital is increasing in volatility is also the reason why the unconditional compensating variations are always smaller than the conditional ones. The unconditional compensating variations effectively endow the high-volatility economy with more capital than the low-volatility economy. This means that expected welfare in the high-volatility economy is greater than welfare evaluated at the non-stochastic steady state, and means, other things being equal, that the high-volatility economy is relatively more attractive. The conditional welfare metrics, which condition on the same capital stock in both the high- and low-volatility regimes, take into account the transitional dynamics of accumulating a larger capital stock, and thus produce smaller compensating variations.

It is worth noting that the numbers in the table are all fairly small in an absolute sense. Even when the household is extremely unwilling to substitute consumption intertemporally and when its labor is essentially in inelastic supply, the compensating variations are still less then 0.5 percent of consumption. With a compensating variation of 0.5 percent, for example, a household with an annual consumption stream of \$50,000 would only need to be compensated \$250 to be indifferent between low and high volatility.

3.4 Alternative Preferences

This subsection repeats the analysis of the previous subsection using two other popular preference specifications, both of which feature non-separability between consumption and labor: King et al. (1988) preferences, (hereafter KPR) and Greenwood et al. (1988) preferences (hereafter GHH).

In spite of its frequent use, a drawback of the additively separable, iso-elastic preference specification in (1) is that it is inconsistent with balanced growth unless $\gamma = 1$. King et al. (1988) show that the following specification is always consistent with balanced growth:

$$U(C_t, N_t) = \frac{1}{1-\gamma}\left(\left(C_t \times \exp\left(-\psi \frac{N_t^{1+\phi}}{1+\phi}\right)\right)^{1-\gamma} - 1\right) \quad \gamma, \phi \geq 0 \tag{18}$$

Application of L'Hopital's Rule reveals that the additively separable, iso-elastic specification in (1) is a special case of the KPR preferences when $\gamma = 1$.

The second class of preferences we consider is GHH:

$$U(C_t, N_t) = \frac{1}{1-\gamma}\left(C_t - \psi \frac{N_t^{1+\phi}}{1+\phi}\right)^{1-\gamma} \qquad \gamma, \phi \geq 0 \tag{19}$$

The defining feature of GHH preferences is that they effectively eliminate the wealth effect on labor supply. Rather than desiring smooth streams of both consumption and leisure, as in the additively separable and KPR specifications, with GHH preferences households only care about smoothing $C_t - \psi \frac{N_t^{1+\phi}}{1+\phi}$, not consumption and labor separately. By eliminating the wealth effect, these preferences tend to make labor hours much more volatile and can help resolve co-movement issues between consumption and leisure conditional on non-productivity shocks.[11] The differences between these three kinds of preferences are most easily seen by looking at the static labor supply conditions. For additively separable, KPR, and GHH preferences, the first-order conditions for

[11]Because of the wealth effect, for both additively separable and KPR preference specifications, consumption and labor cannot move in the same direction following any non-technology shocks in a standard competitive business cycle model. By eliminating the wealth effect, GHH preferences make it much easier to generate this co-movement. For instance, see the extensive discussion in Jaimovich and Rebelo (2009) concerning the ability of these preferences to generate co-movement following anticipated technology shocks.

labor supply are, respectively:

$$\psi N_t^{\phi} = C_t^{-\gamma}(1-\alpha)Z_t\left(\frac{K_t}{N_t}\right)^{\alpha} \tag{20}$$

$$\psi N_t^{\phi} = C_t^{-1}(1-\alpha)Z_t\left(\frac{K_t}{N_t}\right)^{\alpha} \tag{21}$$

$$\psi N_t^{\phi} = (1-\alpha)Z_t\left(\frac{K_t}{N_t}\right)^{\alpha} \tag{22}$$

When productivity improves, consumption increases with any of these preference specifications. The increase in consumption mitigates the hours response to an increase in productivity by effectively shifting the labor supply schedule in. This inward shift is absent in (22), the first-order condition associated with GHH preferences. Hence, hours will respond more to productivity shocks with the wealth effect absent. Given the greater elasticity of factor supply, we would expect the household to be more likely to prefer higher volatility with GHH preferences.

Table 2 shows both conditional and unconditional compensating variations for KPR and GHH preferences over a range of values of γ and ϕ. The table is constructed analogously to Table 1.[12] The compensating variations for KPR preferences are very similar to those with separable preferences in the baseline case. For GHH preferences there is a wider range of preferences over which the household prefers more volatility, which follows from the fact that labor supply is effectively much more elastic with no wealth effect. In some instances the welfare gains from going from low to high volatility are quite large—for example, when $\gamma = 1$ and $\phi = 0$, the unconditional compensating variation is -0.57, meaning that the household would be willing to forfeit more than a half of a percent of period-by-period consumption to keep volatility high.

4 Variable Capital Utilization

The general result from the previous two sections is that welfare may be increasing in volatility if factor supply is sufficiently elastic. The mechanism through which volatility can be welfare improving is that volatility presents households with an opportunity—by increasing production in "good times" and reducing it in "bad times," the economy can achieve a higher mean level of utility, which may be more than enough to compensate it for less smooth streams of consumption and leisure.

[12] There are two exceptions to this. First, for the GHH preference specification the problem lacks a unique steady state when $\phi = 0$, so we use $\phi = 0.1$ as a lower bound. Second, for KPR preferences, some restrictions in addition to $\gamma \geq 0$ and $\phi \geq 0$ are necessary to ensure that the utility function is concave. These restrictions are violated when both γ and ϕ are very low, so we do not report results for cases in which both $\gamma < 1$ and $\phi < 1$.

KPR Preferences					
λ	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	n/a	-0.0134	0.0724	0.2243	0.3829
$\phi = 0.4$	n/a	0.0331	0.0981	0.2439	0.4033
$\phi = 1$	-0.0401	0.0554	0.1142	0.2584	0.4189
$\phi = 3$	0.0071	0.0743	0.1304	0.2748	0.4376
$\phi = 10$	0.0205	0.0837	0.1396	0.2851	0.4497
λ^u	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	n/a	-0.0493	0.0466	0.1775	0.2867
$\phi = 0.4$	n/a	0.0149	0.0802	0.2035	0.3149
$\phi = 1$	-0.0689	0.0452	0.1016	0.2229	0.3369
$\phi = 3$	0.0020	0.0706	0.1233	0.2456	0.3637
$\phi = 10$	0.0200	0.0832	0.1356	0.2600	0.3814
GHH Preferences					
λ	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0^*$	-0.4046	-0.2401	-0.0902	0.3388	0.7896
$\phi = 0.4$	-0.2070	-0.0889	0.0118	0.2836	0.5871
$\phi = 1$	-0.0946	-0.0022	0.0764	0.2860	0.5213
$\phi = 3$	-0.0200	0.0549	0.1190	0.2900	0.4826
$\phi = 10$	0.0115	0.0788	0.1366	0.2909	0.4652
λ^u	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0^*$	-0.8267	-0.5650	-0.3819	-0.1675	-0.6817
$\phi = 0.4$	-0.2868	-0.1529	-0.0520	0.1608	0.2966
$\phi = 1$	-0.1200	-0.0233	0.0520	0.2248	0.3725
$\phi = 3$	-0.0264	0.0494	0.1100	0.2546	0.3903
$\phi = 10$	0.0105	0.0779	0.1324	0.2643	0.3921

Table 2: This table shows both the conditional (upper panel) and unconditional (lower panel) compensating variations of moving from a high-volatility regime, $\sigma_h = 0.02$, to a low-volatility regime, $\sigma_l = 0.01$, for different values of γ and ϕ in the model with KPR preferences (upper panel) and GHH preferences (lower panel). The numbers are multiplied by 100 and are interpreted as a percentage of consumption. For the KPR preference specification, overall concavity of preferences is violated when both ϕ and γ take on small values, and thus our solution strategy breaks down. Hence, for two combinations, $(\gamma, \phi) = (0.5, 0)$ and $(0.5, 0.4)$, we do not report compensating variations. This is represented by "n/a" in the table. For the GHH preference specification, there does not exist a well-defined steady state when $\phi = 0$. Hence, we use $\phi = 0.1$ instead of 0 as the lower bound on ϕ. We denote this change with a superscript * in the table.

In the specifications considered thus far, the only way to alter production in the immediate short run is via labor supply. In this section we modify the production side of the economy to include variable capital utilization, which allows capital services (the product of the physical capital stock with utilization) to be elastic in the short run. This addition increases overall factor supply elasticity and works to significantly expand the set of parameters over which increased volatility is welfare improving. Not only is volatility welfare improving over a wide range of reasonable parameter values, the welfare gains from higher volatility are often quite large when variable utilization is included into the model.

4.1 Model

The only difference relative to the model of the previous section is the inclusion of capital utilization, which we denote by u_t. Capital services are the product of utilization and the physical capital stock. Output is produced using a Cobb-Douglas technology in capital services and labor hours:

$$Y_t = Z_t \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}, \quad 0 \leq \alpha \leq 1 \tag{23}$$

The cost of utilization is faster depreciation. In particular, we assume that the depreciation rate on capital is a convex function of utilization:

$$\delta_t = \delta_0 u_t^{\zeta}, \quad \zeta > 1 \tag{24}$$

The presence of variable utilization does not introduce any distortions that would lead to a divergence between the competitive equilibrium and the planner's solution. Written recursively, the planner's problem is:

$$V(Z_t, K_t) = \max_{C_t, N_t, u_t, K_{t+1}} \quad U(C_t, N_t) + \beta E_t V(Z_{t+1}, K_{t+1}) \tag{25}$$

s.t.:

$$C_t + K_{t+1} - (1 - \delta_0 u_t^{\zeta}) K_t \leq Z_t \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}$$

The first-order conditions associated with this problem are:

$$U_C(C_t, N_t) = \beta E_t \left(U_C(C_{t+1}, N_{t+1}) \left(\alpha Z_{t+1} u_{t+1}^{\alpha} \left(\frac{K_{t+1}}{N_{t+1}} \right)^{\alpha-1} + \left(1 - \delta_0 u_{t+1}^{\zeta}\right) \right) \right) \tag{26}$$

$$-U_N(C_t, N_t) = U_C(C_t, N_t)(1-\alpha) Z_t u_t^{\alpha} \left(\frac{K_t}{N_t} \right)^{\alpha} \tag{27}$$

$$\zeta \delta_0 u_t^{\zeta-1} K_t = \alpha Z_t u_t^{\alpha-1} K_t^{\alpha} N_t^{1-\alpha} \tag{28}$$

(26) and (27) are the standard Euler equation and static labor supply condition, amended to take account of the presence of variable capital utilization. (28) is the first-order condition for utilization.

4.2 Calibration and Results

For the numerical results in this section we revert to the additively separable iso-elastic specification of preferences given in (1). Most of the parameters are the same as in Section 3.3. The only new parameters to calibrate are those governing utilization, δ_0 and ζ. Following Burnside and Eichenbaum (1996), we normalize utilization to be unity in the non-stochastic steady state, $u^* = 1$. To match the long run investment to capital ratio we therefore set $\delta_0 = 0.02$, the same as the calibration of δ in the previous sections. Using the normalization, solving for the steady-state capital to labor ratio from (26), and evaluating (28) in steady state yields a restriction on ζ:

$$\zeta = \frac{\frac{1}{\beta} - 1 + \delta_0}{\delta_0} \tag{29}$$

Given our calibrations of β and δ_0, this implies that $\zeta = 1.2513$.

Our numerical experiments are the same as in the previous sections, as we consider the compensating variations needed to make the household indifferent between high ($\sigma_h = 0.02$) and low ($\sigma_l = 0.01$) volatility of productivity shocks. Table 3 presents these results:

λ	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-0.9161	-0.6117	-0.4970	-0.3751	-0.3239
$\phi = 0.4$	-0.3005	-0.1743	-0.1103	-0.0272	0.0134
$\phi = 1$	-0.1627	-0.0766	-0.0242	0.0561	0.1018
$\phi = 3$	-0.0761	-0.0147	0.0306	0.1178	0.1819
$\phi = 10$	-0.0401	0.0114	0.0538	0.1518	0.2440
λ^u	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-1.8228	-1.4704	-1.3501	-1.2238	-1.1652
$\phi = 0.4$	-0.5236	-0.4449	-0.4121	-0.3754	-0.3592
$\phi = 1$	-0.2817	-0.2450	-0.2285	-0.2108	-0.2041
$\phi = 3$	-0.1421	-0.1253	-0.1187	-0.1175	-0.1250
$\phi = 10$	-0.0873	-0.0768	-0.0744	-0.0861	-0.1163

Table 3: The numbers in this table show both the conditional (upper panel) and unconditional (lower panel) compensating variations of moving from a high-volatility regime, $\sigma_h = 0.02$, to a low-volatility regime, $\sigma_l = 0.01$, for different values of γ and ϕ in the model augmented with variable capital utilization. The numbers are multiplied by 100, and are interpreted as a percentage of consumption.

There are two important takeaways from this table. First, the range of parameters over which welfare is increasing in volatility is much wider than in the baseline model. In an unconditional sense households prefer more volatility for all of the parameter configurations considered. Conditional on the same capital stock, though volatility is welfare-reducing for very high values of γ and ϕ, volatility is now welfare-enhancing for a much wider range of parameters than in the baseline model. Second, for cases where households prefer more volatility, the compensating variations are quantitatively much larger here than in the baseline model without variable utilization. For example, in the Hansen (1985) specification with log utility and indivisible labor, the unconditional compensating variation is about -1.5 percent of consumption, compared with about -0.05 percent of consumption in the baseline model. Further, for cases in which households dislike more volatility (e.g. the conditional compensating variation for very high values of γ and ϕ), the compensating variations are smaller, meaning that the cost of volatility is lower. Variable utilization turns out to be a powerful force that greatly increases effective factor supply elasticity, which allows the economy to achieve even higher mean levels of output and capital when volatility increases than in the baseline specification.[13]

[13]Fernald (2012), building on the insights of Basu et al. (2006), imputes an empirical measure of capital utilization and finds it to be highly variable over the business cycle. This suggests that variable capital utilization is not merely an interesting theoretical addition to the model, but is an empirically relevant phenomenon.

5 Other Shocks

The previous sections have focused exclusively on the welfare consequences of more volatility in an exogenous neutral productivity process. In this section we extend our analysis to study the welfare consequences of more or less volatility in other kinds of shocks. In particular, we focus on investment-specific technology shocks and preference shocks. Investment-specific shocks have been shown to be an important contributor to secular trends (Greenwood et al. (1997)) and business cycle fluctuations (Fisher (2006) and Justiniano et al. (2010)). Preference shocks are an important feature of medium scale DSGE models and are motivated as sources of "demand shocks."

5.1 Investment-Specific Technology Shocks

Investment-specific technology shocks alter the marginal efficiency of transforming investment goods into new capital. Though one can motivate such shocks in a multi-sector framework, we follow much of the literature in incorporating these shocks into the standard one sector framework from Section 3.1. Because there are no distortions, we can again express the equilibrium allocations as the solutions to a planner's problem (for this exercise we abstract from the presence of a neutral productivity shock, implicitly fixing $Z_t = 1$):

$$V(A_t, K_t) = \max_{C_t, N_t, K_{t+1}} \quad U(C_t, N_t) + \beta E_t V(A_{t+1}, K_{t+1}) \tag{30}$$

s.t.

$$K_{t+1} \leq A_t \left(K_t^{\alpha} N_t^{1-\alpha} - C_t \right) + (1-\delta) K_t \tag{31}$$

Here A_t represents the efficiency of transforming non-consumed output into new capital, while Z_t is again a neutral technology shifter. Higher values of A_t mean that the economy is more productive at producing new capital. We assume that A_t follows a mean one stationary AR(1) in the level, with either a high ($\sigma_h = 0.02$) or low ($\sigma_l = 0.01$) innovation standard deviation:

$$A_t = (1-\rho_a) + \rho_a A_{t-1} + \sigma_i \varepsilon_{A,t}, \;\; \varepsilon_{A,t} \sim N(0,1) \tag{32}$$

The first order conditions of the problem are:

$$-U_N(C_t, N_t) = U_C(C_t, N_t)(1-\alpha) K_t^{\alpha} N_t^{-\alpha} \tag{33}$$

$$U_C(C_t, N_t) = \beta E_t U_C(C_{t+1}, N_{t+1}) \frac{A_t}{A_{t+1}} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1-\delta) \right) \tag{34}$$

We conduct the same quantitative experiments as in earlier sections: we consider the welfare effects of moving from a high-volatility investment-specific shock ($\sigma_h = 0.02$) to a low-volatility shock ($\sigma_l = 0.01$). As with the case of a neutral productivity shock, we set $\rho_a = 0.95$. We consider the additively separable preference specification and consider a range of values of γ and ϕ. All other parameters are set at their baseline values. The compensating variations, expressed as a percentage of consumption, are shown below in Table 4.

λ	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-0.2322	-0.1596	-0.1351	-0.1103	-0.1003
$\phi = 0.4$	-0.1716	-0.1132	-0.0904	-0.0654	-0.0546
$\phi = 1$	-0.1453	-0.0909	-0.0683	-0.0418	-0.0296
$\phi = 3$	-0.1245	-0.0720	-0.0489	-0.0198	-0.0050
$\phi = 10$	-0.1146	-0.0626	-0.0390	-0.0077	0.0103
λ^u	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-0.3172	-0.2030	-0.1679	-0.1343	-0.1212
$\phi = 0.4$	-0.2212	-0.1389	-0.1085	-0.0766	-0.0632
$\phi = 1$	-0.1812	-0.1086	-0.0796	-0.0470	-0.0327
$\phi = 3$	-0.1504	-0.0832	-0.0547	-0.0201	-0.0033
$\phi = 10$	-0.1361	-0.0707	-0.0420	-0.0053	0.0149

Table 4: The numbers in this table show both the conditional (upper panel) and unconditional (lower panel) compensating variations of moving from a high-volatility regime, $\sigma_h = 0.02$, to a low-volatility regime, $\sigma_l = 0.01$, for different values of γ and ϕ in the model with investment-specific technology shocks. All numbers are multiplied by 100, and are interpreted as percentages of consumption.

As in earlier tables, negative compensating variations mean that households prefer higher volatility. With the exception of the case of highly inelastic labor supply ($\phi = 10$) and high risk aversion ($\gamma = 5$), welfare is increasing in the volatility of the investment-specific technology shock. Relative to neutral productivity shocks, these numbers are quantitatively larger and the range of parameters over which volatility is welfare-improving is wider. Naturally, the unconditional compensating variations are larger in absolute value (more negative), reflecting the fact that welfare benefit of volatility is reflected in a higher mean capital stock. Also, the welfare benefits of volatility depend on the parameterizations in exactly the same way in the case of neutral productivity shocks: households are more likely to prefer higher volatility the more elastic is labor supply and the more willing households are to substitute consumption intertemporally.

5.2 Preference Shocks

We next study the welfare consequences of volatility in a demand-side disturbance. We do so by including a preference shock that affects flow utility of consumption, thereby leading to increased demand for consumption goods.[14] This is the same specification as in Ireland (2004). In the text we focus on the model with capital described in Section 3.1. In Appendix C we show that welfare is everywhere increasing in the volatility of the preference shock in the simpler model without capital.

For this exercise we focus on the additively separable preference specification. Within period preferences are given by:

$$U(C_t, N_t) = \nu_t \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{N_t^{1+\phi}}{1+\phi} \tag{35}$$

The stochastic random variable ν_t is a multiplicative shock to utility from consumption, and thereby also a shock to the marginal utility of consumption.[15] We assume that it follows a stationary AR(1) process in the level with mean 1:

$$\nu_t = (1-\rho_\nu) + \rho_\nu \nu_{t-1} + \sigma_i \varepsilon_{\nu,t}, \quad \varepsilon \sim N(0,1) \tag{36}$$

The production side of the economy is the same as in our benchmark specification without variable capital utilization or investment-specific shocks. For this exercise we again fix $Z_t = 1$ so that the preference shock is the only disturbance. The first-order conditions characterizing an optimal solution to the problem are:

$$\nu_t C_t^{-\gamma} = \beta E_t \left(\nu_{t+1} C_{t+1}^{-\gamma} \left(\alpha \left(\frac{K_{t+1}}{N_{t+1}} \right)^{\alpha-1} + (1-\delta) \right) \right) \tag{37}$$

$$\psi N_t^{\phi} = \nu_t C_t^{-\gamma} (1-\alpha) \left(\frac{K_{t+1}}{N_{t+1}} \right)^{\alpha} \tag{38}$$

We set $\rho_\nu = 0.95$. The other parameters of the model are chosen as in previous sections. Table 5 below presents both conditional and unconditional compensating variations from moving from a low-volatility ($\sigma_l = 0.01$) to a high-volatility ($\sigma_h = 0.02$) regime. Other than the fact that this exercise concerns volatility in a preference shock, it is otherwise identical to the previous exercises.[16]

[14] We also experimented with shocks that affect both the marginal utility of consumption and labor simultaneously, as well as preference shocks only to labor, and shocks to the discount factor. We find very similar results in these alternative specifications.

[15] An alternative way to model this preference shock would be to write flow utility from consumption as $\frac{(C_t - \nu_t)^{1-\gamma} - 1}{1-\gamma}$, as in Wen (2006). These specifications are the same to a first-order approximation. An advantage of our specification is that the coefficient of relative risk aversion and the elasticity of substitution are independent of ν_t, which is not the case under the alternative specification.

[16] We also experimented with augmenting this model to include variable capital utilization. As in the case of

λ	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-0.1684	-0.1070	-0.0784	-0.0434	-0.0273
$\phi = 0.4$	-0.1266	-0.0870	-0.0667	-0.0395	-0.0256
$\phi = 1$	-0.1080	-0.0753	-0.0588	-0.0362	-0.0241
$\phi = 3$	-0.0931	-0.0640	-0.0501	-0.0316	-0.0216
$\phi = 10$	-0.0859	-0.0577	-0.0447	-0.0278	-0.0191
λ^u	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3.0$	$\gamma = 5.0$
$\phi = 0$	-0.2020	-0.1157	-0.0816	-0.0434	-0.0268
$\phi = 0.4$	-0.1566	-0.0987	-0.0726	-0.0407	-0.0257
$\phi = 1$	-0.1377	-0.0893	-0.0669	-0.0386	-0.0248
$\phi = 3$	-0.1232	-0.0804	-0.0609	-0.0361	-0.0236
$\phi = 10$	-0.1165	-0.0757	-0.0573	-0.0342	-0.0227

Table 5: The numbers in this table show both the conditional (upper panel) and unconditional (lower panel) compensating variations of moving from a high-volatility regime, $\sigma_h = 0.02$, to a low-volatility regime, $\sigma_l = 0.01$, for different values of γ and ϕ in the model with preference shocks. The numbers are multiplied by 100, and are interpreted as percentages of consumption.

All of the numbers in this table are negative, meaning that, regardless of preference parameters, households prefer higher to lower volatility in the preference shock. The intuition for this result is essentially the same as in the case of productivity shocks: with more volatility in ν_t, households can intertemporally substitute consumption to periods where the marginal utility of consumption is higher in such a way as to achieve higher utility on average. It is therefore to be expected that the compensating variations are most negative when households are most willing to substitute intertemporally (i.e. when γ and ϕ are small). It is also worth noting that in contrast with the case of productivity shocks, the unconditional compensating variations are not uniformly less than their conditional counterparts. Moreover, the differences between the conditional and unconditional variations are frequently quite small. This reflects that fact that changing the volatility of preference shocks has a smaller impact on the mean capital stock than does changing the volatility of productivity shocks.

6 Conclusion

Lucas (1987) found that the welfare costs of business cycles were small. We have shown that in fact his calculations *overstated* these costs. In addition to the adverse consequences of fluctuations—consumption volatility—that were the sole focus of Lucas's attention, fluctuations also present

productivity shocks, this addition to the model makes higher volatility more appealing from a welfare perspective (e.g. the compensating variations are more negative), though the differences between the models with and without variable utilization are not quite as dramatic as in the case of productivity shocks.

opportunities. In an equilibrium model with endogenous factor supply, production can be increased in good times and reduced in bad times. When these welfare-enhancing aspects of fluctuations are taken into account, it is possible that business cycles are not costly at all, but are on net actually desirable.

Several results emerge from our analysis. First, as households become more willing to substitute consumption intertemporally and labor intratemporally, it is more likely that they prefer higher volatility in an exogenous productivity process. This finding is robust to all types of preferences considered. Second, extending the model to include additional mechanisms of substitution, most notably variable capital utilization, expands the range of parameters for which more volatile productivity shocks is welfare-enhancing and always reduces the compensating variation required to make the household indifferent between regimes. Third, when investment-specific and preference shocks are added to the model, households almost always prefer higher volatility in these exogenous driving forces.

Despite the robustness of our results to different preference specifications and model extensions, there is substantial room for future research. Once market imperfections are allowed, the relationship between volatility and welfare becomes significantly more complex, as does the potential role for welfare-enhancing policy. Additionally, heterogeneity of firms or consumers adds a dimension of interest, as well as complexity, since the welfare consequences of greater uncertainty vary across individuals.

References

Abel, A. B. (1983), "Optimal Investment under Uncertainty", *American Economic Review*, **73**, 228–33.

Alvarez, F. and Jermann, U. J. (2004), "Using Asset Prices to Measure the Cost of Business Cycles", *Journal of Political Economy*, **112**, 1223–1256.

Aruoba, S. B., Fernández-Villaverde, J., and Rubio-Ramírez, J. F. (2006), "Comparing solution methods for dynamic equilibrium economies", *Journal of Economic Dynamics and Control*, **30**, 2477–2508.

Basu, S., Kimball, M. S., and Fernald, J. G. (2006), "Are Technology Improvements Contractionary?", *American Economic Review*, **96**, 1418–1448.

Bloom, N. (2009), "The Impact of Uncertainty Shocks", *Econometrica*, **77**, 623–685.

Blundell, R., Browning, M., and Meghir, C. (1994), "Consumer Demand and the Life-Cycle Allocation of Household Expenditures", *Review of Economic Studies*, **61**, 57–80.

Burnside, C. and Eichenbaum, M. (1996), "Factor-Hoarding and the Propagation of Business-Cycle Shocks", *American Economic Review*, **86**, 1154–74.

Cho, J.-O., Cooley, T. F., and Kim, H. S. (2012), "Business Cycle Uncertainty and Economic Welfare", Working paper.

Davis, S. J. and Kahn, J. A. (2008), "Interpreting the Great Moderation: Changes in the Volatility of Economic Activity at the Macro and Micro Levels", *Journal of Economic Perspectives*, **22**, 155–80.

Dynan, K. E. (1993), "How Prudent Are Consumers?", *Journal of Political Economy*, **101**, 1104–13.

Epstein, L. G. and Zin, S. E. (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework", *Econometrica*, **57**, 937–69.

Fernald, J. (2012), "A quarterly, utilization-adjusted series on total factor productivity", Working Paper Series 2012-19, Federal Reserve Bank of San Francisco.

Fernández-Villaverde, J., Guerrón-Quintana, P., and Rubio-Ramírez, J. F. (2010), "Fortune or Virtue: Time-Variant Volatilities Versus Parameter Drifting in U.S. Data", NBER

Working Papers 15928, National Bureau of Economic Research, Inc.

Fernández-Villaverde, J., Guerrón-Quintana, P. A., Kuester, K., and Rubio-Ramírez, J. (2011), "Fiscal Volatility Shocks and Economic Activity", NBER Working Papers 17317, National Bureau of Economic Research, Inc.

Fisher, J. D. M. (2006), "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks", *Journal of Political Economy*, **114**, 413–451.

Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988), "Investment, Capacity Utilization, and the Real Business Cycle", *American Economic Review*, **78**, 402–17.

Greenwood, J., Hercowitz, Z., and Krusell, P. (1997), "Long-Run Implications of Investment-Specific Technological Change", *American Economic Review*, **87**, 342–62.

Gruber, J. (2006), "A Tax-Based Estimate of the Elasticity of Intertemporal Substitution", NBER Working Papers 11945, National Bureau of Economic Research, Inc.

Hall, R. E. (1988), "Intertemporal Substitution in Consumption", *Journal of Political Economy*, **96**, 339–57.

Hansen, G. D. (1985), "Indivisible labor and the business cycle", *Journal of Monetary Economics*, **16**, 309–327.

Hartman, R. (1972), "The Effects of Price and Cost Uncertainty on Investment", *Journal of Economic Theory*, **5**, 258–66.

Ireland, P. N. (2004), "Technology Shocks in the New Keynesian Model", *The Review of Economics and Statistics*, **86**, 923–936.

Jaimovich, N. and Rebelo, S. (2009), "Can News about the Future Drive the Business Cycle?", *American Economic Review*, **99**, 1097–1118.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2010), "Investment shocks and business cycles", *Journal of Monetary Economics*, **57**, 132–145.

Kim, C.-J. and Nelson, C. R. (1999), "Has The U.S. Economy Become More Stable? A Bayesian Approach Based On A Markov-Switching Model Of The Business Cycle", *The Review of Economics and Statistics*, **81**, 608–616.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988), "Production, growth and business cycles : I. The basic neoclassical model", *Journal of Monetary Economics*, **21**, 195–232.

KRUSELL, P., MUKOYAMA, T., SAHIN, A., AND ANTHONY A. SMITH, J. (2009), "Revisiting the Welfare Effects of Eliminating Business Cycles", *Review of Economic Dynamics*, **12**, 393–402.

KRUSELL, P. AND SMITH, A. A. (1999), "On the Welfare Effects of Eliminating Business Cycles", *Review of Economic Dynamics*, **2**, 245–272.

LUCAS, R. E. (1987), *Models of Business Cycles* (Basil Blackwell).

LUCAS, R. E. (2003), "Macroeconomic Priorities", *American Economic Review*, **93**, 1–14.

MULLIGAN, C. B. (2002), "Capital, Interest, and Aggregate Intertemporal Substitution", NBER Working Papers 9373, National Bureau of Economic Research, Inc.

OTROK, C. (2001), "On measuring the welfare cost of business cycles", *Journal of Monetary Economics*, **47**, 61–92.

PEREZ-QUIROS, G. AND MCCONNELL, M. M. (2000), "Output Fluctuations in the United States: What Has Changed since the Early 1980's?", *American Economic Review*, **90**, 1464–1476.

SCHMITT-GROHE, S. AND URIBE, M. (2004), "Solving dynamic general equilibrium models using a second-order approximation to the policy function", *Journal of Economic Dynamics and Control*, **28**, 755–775.

SCHULHOFER-WOHL, S. (2008), "Heterogeneous Risk Preferences and the Welfare Cost of Business Cycles", *Review of Economic Dynamics*, **11**, 761–780.

SIMS, C. A. AND ZHA, T. (2006), "Were There Regime Switches in U.S. Monetary Policy?", *American Economic Review*, **96**, 54–81.

STOCK, J. H. AND WATSON, M. W. (2003), "Has the Business Cycle Changed and Why?", in "NBER Macroeconomics Annual 2002, Volume 17", NBER Chapters, pages 159–230 (National Bureau of Economic Research, Inc).

TALLARINI, T. D. (2000), "Risk-sensitive real business cycles", *Journal of Monetary Economics*, **45**, 507–532.

WEN, Y. (2006), "Demand shocks and economic fluctuations", *Economics Letters*, **90**, 378–383.

Appendix

A Solution via Second-order Approximation

In the baseline model of Section 3, we can solve directly for an approximated value function by including the recursive representation of the value function as an equilibrium condition. We then use the general method for the second-order approximation as laid out in Schmitt-Grohe and Uribe (2004).

For a general class of preferences, $U(C_t, N_t)$, the conditions characterizing the solution to the planner's problem are:

$$U_C(C_t, N_t) = \beta E_t \left(U_C(C_{t+1}, N_{t+1}) \left(\alpha Z_{t+1} \left(\frac{K_{t+1}}{N_{t+1}} \right)^{\alpha-1} + (1-\delta) \right) \right) \tag{A.1}$$

$$U_N(C_t, N_t) = U_C(C_t, N_t)(1-\alpha) Z_t \left(\frac{K_t}{N_t} \right)^{\alpha} \tag{A.2}$$

$$V_t = U(C_t, N_t) + \beta E_t V_{t+1} \tag{A.3}$$

$$K_{t+1} = Z_t K_t^{\alpha} N_t^{1-\alpha} - C_t + (1-\delta) K_t \tag{A.4}$$

$$Z_t = (1-\rho) + \rho Z_{t-1} + \sigma_i \varepsilon_t \tag{A.5}$$

Equation A.3 is a representation of the value function, treating V_t is a variable. We can compute a direct second order approximation to this equation as in Schmitt-Grohe and Uribe (2004).

B Calculating Compensating Variations

Here we describe the calculation of compensating variations for several different classes of preferences.

B.1 Additively Separable, Log over Consumption

As in the main text, let $\widetilde{C}_{i,t}$ and $\widetilde{N}_{i,t}$ denote the optimal sequences of consumption and labor hours in a volatility regime $i = h$ or l. For the case additively separable preferences and log utility over consumption, the value function evaluated at a particular point in the state space, (Z_t, K_t), can be written:

$$V_i(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \left(\ln \widetilde{C}_{i,t+j} - \psi \frac{\widetilde{N}_{i,t+j}^{1+\phi}}{1+\phi} \right) \tag{B.1}$$

Given separability, it is helpful to define two auxiliary value functions, V^c and V^N:

$$V_i(Z_t, K_t) = V_i^C(Z_t, K_t) + V_i^N(Z_t, K_t) \tag{B.2}$$

$$V_i^C(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \ln \widetilde{C}_{i,t+j} \tag{B.3}$$

$$V_i^N(Z_t, K_t) = -E_t \sum_{j=0}^{\infty} \beta^j \frac{\psi \widetilde{N}_{i,t+j}^{1+\phi}}{1+\phi} \tag{B.4}$$

The conditional compensating variation for the two volatility regimes is defined by:

$$V_l(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \ln(1+\lambda)\widetilde{C}_{h,t+j} - E_t \sum_{j=0}^{\infty} \beta^j \frac{\psi \widetilde{N}_{h,t+j}^{1+\phi}}{1+\phi}$$

Using the definitions above, this reduces to:

$$\begin{aligned} V_l(Z_t, K_t) &= \sum_{j=0}^{\infty} \beta^j \ln(1+\lambda) + V_h^C(Z_t, K_t) + V_h^N(Z_t, K_t) \\ &= \sum_{j=0}^{\infty} \beta^j \ln(1+\lambda) + V_h(Z_t, K_t) \end{aligned}$$

Solving and simplifying yields an expression for λ:

$$\lambda = \exp\left((1-\beta)\left(V_l(Z_t, K_t) - V_h(Z_t, K_t)\right)\right) - 1 \tag{B.5}$$

The sign of λ is determined by whether the expression inside the exponential function is positive or negative. If $V_l(Z_t, K_t) > V_h(Z_t, K_t)$, so that the household would prefer to be in the low-volatility regime, then $\lambda > 0$. In contrast, if the household would prefer the high-volatility regime, then $\lambda < 0$. The calculation of the unconditional compensating variation is the same as above, but uses the unconditional expectations of the two value functions instead of the two value functions conditional on the same state vector:

$$\lambda^u = \exp\left((1-\beta)\left(E\left(V_l(Z_t, K_t)\right) - E\left(V_h(Z_t, K_t)\right)\right)\right) - 1 \tag{B.6}$$

B.2 Additively Separable

Conditional on the volatility regime, the value function evaluated at a particular point in the state space, (Z_t, K_t), can be written:

$$V_i(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\widetilde{C}_{i,t+j}^{1-\gamma} - 1}{1-\gamma} - \psi \frac{\widetilde{N}_{i,t+j}^{1+\phi}}{1+\phi} \right) \tag{B.7}$$

As in the case of log utility over consumption, we can define two auxiliary value functions:

$$V_i(Z_t, K_t) = V_i^C(Z_t, K_t) + V_i^N(Z_t, K_t) \tag{B.8}$$

$$V_i^C(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \frac{\widetilde{C}_{i,t+j}^{1-\gamma} - 1}{1-\gamma} \tag{B.9}$$

$$V_i^N(Z_t, K_t) = -E_t \sum_{j=0}^{\infty} \beta^j \frac{\psi \widetilde{N}_{i,t+j}^{1+\phi}}{1+\phi} \tag{B.10}$$

The conditional compensating variation for the two volatility regimes is defined by:

$$V_l(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \frac{\left((1+\lambda)\widetilde{C}_{h,t+j}\right)^{1-\gamma} - 1}{1-\gamma} - E_t \sum_{j=0}^{\infty} \beta^j \frac{\psi \widetilde{N}_{h,t+j}^{1+\phi}}{1+\phi}$$

Using the definitions above and simplifying:

$$V_l(Z_t, K_t) = (1+\lambda)^{1-\gamma} \left(V_h^C(Z_t, K_t) + \frac{1}{(1-\gamma)(1-\beta)} \right) - \frac{1}{(1-\gamma)(1-\beta)} + V_h^N(Z_t, K_t)$$

Solving for λ:

$$\lambda = \left(\frac{V_l(Z_t, K_t) - V_h^N(Z_t, K_t) + \frac{1}{(1-\gamma)(1-\beta)}}{V_h^C(Z_t, K_t) + \frac{1}{(1-\gamma)(1-\beta)}} \right)^{\frac{1}{1-\gamma}} - 1 \tag{B.11}$$

The unconditional compensating variation is defined similarly, but using unconditional expectation of the value functions instead of evaluating them at a particular point in the state space:

$$\lambda^u = \left(\frac{E\left(V_l(Z_t, K_t)\right) - E\left(V_h^N(Z_t, K_t)\right) + \frac{1}{(1-\gamma)(1-\beta)}}{E\left(V_h^C(Z_t, K_t)\right) + \frac{1}{(1-\gamma)(1-\beta)}} \right)^{\frac{1}{1-\gamma}} - 1 \tag{B.12}$$

B.3 KPR (1988) Preferences

Under King et al. (1988) preferences, the value of being in a particular state under a particular volatility regime is:

$$V_i(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{1}{1-\gamma} \left(\left(\widetilde{C}_{i,t+j} \times \exp\left(-\theta \frac{\widetilde{N}_{i,t+j}^{1+\phi}}{1+\phi} \right) \right)^{1-\gamma} - 1 \right) \right) \tag{B.13}$$

Given non-separability forming auxiliary value functions is of no use here. The conditional compensating variation for the two volatility regimes is given by:

$$V_l(K_t, Z_t) = E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{1}{1-\gamma} \left(\left((1+\lambda)\widetilde{C}_{h,t+j} \times \exp\left(-\theta \frac{\widetilde{N}_{h,t+j}^{1+\phi}}{1+\phi} \right) \right)^{1-\gamma} - 1 \right) \right) \tag{B.14}$$

Simplifying:

$$V_l(K_t, Z_t) = (1+\lambda)^{1-\gamma} \left(V_h(K_t, Z_t) + \frac{1}{(1-\gamma)(1-\beta)} \right) - \frac{1}{(1-\gamma)(1-\beta)}$$

We can solve for λ as:

$$\lambda = \left(\frac{V_l(Z_t, K_t) + \frac{1}{(1-\gamma)(1-\beta)}}{V_h(K_t, Z_t) + \frac{1}{(1-\gamma)(1-\beta)}} \right)^{\frac{1}{1-\gamma}} - 1 \tag{B.15}$$

B.4 GHH (1988) Preferences

With GHH (1988) preferences, the value of being in a particular state under a given volatility regime is:

$$V_i(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{1}{1-\gamma} \left(\widetilde{C}_{i,t+j} - \psi \frac{\widetilde{N}_{i,t+j}^{1+\phi}}{1+\phi} \right)^{1-\gamma} \right) \tag{B.16}$$

The conditional compensating variation for the high- and low-volatility regimes is the solution to the following equality:

$$V_l(Z_t, K_t) = E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{1}{1-\gamma} \left((1+\lambda)\widetilde{C}_{h,t+j} - \psi \frac{\widetilde{N}_{h,t+j}^{1+\phi}}{1+\phi} \right)^{1-\gamma} \right) \tag{B.17}$$

There does not exist a closed form expression for the λ that makes this expression hold with equality. We therefore approximate the solution using numerical methods.

The unconditional compensating variation is solved in a similar way via numerical techniques:

$$E\left(V_l(Z_t, K_t)\right) = E\sum_{j=0}^{\infty}\beta^j\left(\frac{1}{1-\gamma}\left((1+\lambda^u)\widetilde{C}_{h,t+j} - \psi\frac{\widetilde{N}_{h,t+j}^{1+\phi}}{1+\phi}\right)^{1-\gamma}\right) \tag{B.18}$$

C Preference Shocks in the Model without Capital

As in the text, the model without capital can be reduced to a sequence of one period problems. Set the productivity shock equal to its unconditional mean, $Z_t = 1$. Also normalize the scaling parameter on the disutility from labor, ψ, to unity. The static planner's problem is:

$$\max_{C_t, N_t} \quad \nu_t\frac{C_t^{1-\gamma}-1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi}$$

$$\text{s.t.}$$

$$C_t \leq N_t$$

The first-order condition is:

$$N_t = \nu_t^{\frac{1}{\phi+\gamma}} \tag{C.1}$$

The indirect utility function is:

$$\widetilde{U}(\nu_t) = \nu_t^{\frac{1+\phi}{\phi+\gamma}}\left(\frac{\phi+\gamma}{(1-\gamma)(1+\phi)}\right) - \frac{\nu_t}{1-\gamma} \tag{C.2}$$

The first and second derivatives of the indirect utility function are:

$$\widetilde{U}'(\nu_t) = \frac{1}{1-\gamma}\left(\nu_t^{\frac{1-\gamma}{\phi+\gamma}} - 1\right) \tag{C.3}$$

$$\widetilde{U}''(\nu_t) = \frac{1}{\phi+\gamma}\nu_t^{\frac{1-\phi-2\gamma}{\phi+\gamma}} \geq 0 \tag{C.4}$$

The second derivative is always positive for finite values of ϕ and γ. This means that a mean-preserving spread on ν_t must increase welfare.