Money Demand
ECON 30020: Intermediate Macroeconomics

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Readings

- GLS Ch. 13

What is Money?

- ▶ Might seem like an obvious question but really not so clear
- ▶ Money is an *asset* – i.e. it's a stock that can be taken across time
- ▶ But it's a special kind of asset in that it can be used in exchange
- ▶ Whereas with most things we define them according to intrinsic characteristics (e.g. coffee is a dark liquid substance necessary for sustaining life), with money we instead give a *functional* definition
- ▶ Money is any asset which serves the following three functions:
  1. Medium of exchange
  2. Store of value
  3. Unit of account

Medium of Exchange

- ▶ The most important role played by money is its role as a medium of exchange
- ▶ This solves the "double coincidence of wants" problem associated with barter
- ▶ Bonds and capital can serve as stores of values (any asset does so, so money is not unique), and anything can serve as a unit of account
- ▶ But money is unique in its role as medium of exchange
- ▶ No exaggeration that money's role as a medium of exchange has been critical to the historical growth in economic activity
- ▶ *Fiat money* is the best medium of exchange, so long as people believe it has value

Including Money in the Neoclassical Model

- ▶ Is not so easy
- ▶ Why? Model only features one good (e.g. fruit). Makes medium of exchange role uninteresting
- ▶ We will include money essentially as a store of value and will use money as a nominal unit of account
- ▶ But money is a crummy store of value – bonds pay interest, money does not
- ▶ We will take a reduced form shortcut and assume that the household receives utility from holding money
- ▶ New variables:
    - ▶ M_t: stock of money (held between periods t and $t+1$ (i.e. store of value like S_t))
    - ▶ P_t: price of goods measured in units of money
    - ▶ i_t: nominal interest rate

Nominal Budget Constraints

- Period t:

$$P_t C_t + P_t S_t + M_t \leq P_t w_t N_t - P_t T_t + P_t D_t$$

- Period $t+1$:

$$P_{t+1} C_{t+1} + P_{t+1} S_{t+1} - P_t S_t + M_{t+1} - M_t \leq$$
$$P_{t+1} w_{t+1} N_{t+1} - P_{t+1} T_{t+1} + i_t P_t S_t + P_{t+1} D_{t+1} + P_{t+1} D_{t+1}^I$$

- Terminal conditions: $S_{t+1} = 0$ and $M_{t+1} = 0$. Writing constraints in real terms:

$$C_t + S_t + \frac{M_t}{P_t} = w_t N_t - T_t + D_t$$

$$C_{t+1} = w_{t+1} N_{t+1} - T_{t+1} + D_{t+1} + (1 + i_t) \frac{P_t}{P_{t+1}} S_t + D_{t+1}^I + \frac{M_t}{P_{t+1}}$$

- $\frac{M_t}{P_t}$: real money balances

Fisher Relationship

- The Fisher relationship is a relationship between the real and nominal interest rates

- It is given by:

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$$

- Define expected inflation between t and $t+1$ as:

$$1 + \pi_{t+1}^e = \frac{P_{t+1}}{P_t}$$

- Fisher relationship is then approximately:

$$r_t = i_t - \pi_{t+1}^e$$

- We will treat expected one period ahead inflation rate, π_{t+1}^e, as exogenous. Means movements and nominal and real rates are the same for a given rate of expected inflation

The Real Intertemporal Budget Constraint

- Can write $t+1$ constraint as:

$$C_{t+1} = w_{t+1}N_{t+1} - T_{t+1} + D_{t+1} + D_{t+1}^I + (1+r_t)S_t + \frac{1+r_t}{1+i_t}\frac{M_t}{P_t}$$

- Solve out for S_t, combining with period t constraint:

$$C_t + \frac{C_{t+1}}{1+r_t} + \frac{i_t}{1+i_t}\frac{M_t}{P_t} = w_t N_t - T_t + D_t + \frac{w_{t+1}N_{t+1} - T_{t+1} + D_{t+1} + D_{t+1}^I}{1+r_t}$$

- Exactly the same as before, just this additional "expenditure" category of $\frac{M_t}{P_t}$ – how many period t goods you choose to hold in money

Preferences

- Note that money is held *across* periods, not within a period (i.e. it is a stock variable, not a flow)
- Assume household receives a utility flow from its holding of *real balances* via the function $v(\cdot)$. Increasing and concave (e.g. log)
- This utility flow is received in period t
- Lifetime utility:

$$U = u(C_t, 1 - N_t) + v\left(\frac{M_t}{P_t}\right) + \beta u(C_{t+1}, 1 - N_{t+1})$$

Optimality Conditions

- FOC for consumption and labor *exactly* the same as before:

$$u_C(C_t, 1 - N_t) = \beta(1 + r_t)u_C(C_{t+1}, 1 - N_t)$$
$$u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$$

- New FOC for money:

$$v'\left(\frac{M_t}{P_t}\right) = \frac{i_t}{1 + i_t}u_C(C_t, 1 - N_t)$$

- Interpretation ... marginal benefit equals marginal cost!
- If no utility benefit from holding money ($v'(\cdot) = 0$), then could only hold if $i_t = 0$: money dominated as a store of value by bonds if $i_t > 0$

Optimal Decision Rules

- ► Can go from FOC to optimal decision rules as before
- ► Presence of money does not impact optimal decision rules for consumption or labor supply:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$N_t = N^s(w_t, \theta_t)$$

- ► Cutting a few corners (i.e. treating C_t and Y_t as interchangebale), optimal decision rule for money is:

$$M_t = P_t M^d(\underset{-}{i_t}, \underset{+}{Y_t})$$

- ► Or, using the Fisher relationship:

$$M_t = P_t M^d(\underset{-}{r_t + \pi^e_{t+1}}, \underset{+}{Y_t})$$

- ► This is our money demand function – demand for real balances is decreasing in the nominal rate and increasing in total expenditure

Government

- ▶ Firm and financial intermediary problems can be written either in real or nominal terms. Firms do not hold money across periods, so optimal decision rules are identical
- ▶ Government "prints" money, and we take this to be exogenous. Period t budget constraint:

$$P_t G_t \leq P_t T_t + P_t B_t + M_t$$

- ▶ Government can use money as an additional "revenue" source (way to finance spending). Period $t+1$ constraint:

$$P_{t+1} G_{t+1} + i_t P_t B_t + M_t \leq P_{t+1} T_{t+1} + P_{t+1} B_{t+1} - P_t B_t$$

- ▶ Government essentially has to "buy back" in period $t+1$ the money it issues in period t. Terminal condition: $B_{t+1} = 0$, implying:

$$P_{t+1} G_{t+1} + (1 + i_t) P_t B_t + M_t \leq P_{t+1} T_{t+1}$$

Government's IBC

▶ In real terms, the two flow budget constraints for the government are:

$$G_t = T_t + B_t + \frac{M_t}{P_t}$$

$$G_{t+1} + (1 + i_t)\frac{P_t}{P_{t+1}}B_t + \frac{M_t}{P_{t+1}} = T_{t+1}$$

▶ Combining the two and using the Fisher relationship, we get:

$$G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t}\frac{M_t}{P_t}$$

▶ Similar to before, but additional "revenue" category related to money (what we call seignorage). Analogous to household IBC which features the same term but as an expenditure category. When combining firm and household IBCs, these terms cancel (money irrelevant for consumption decision)

Equilibrium Conditions

▶ These are:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$N_t = N^s(w_t, \theta_t)$$
$$N_t = N^d(w_t, A_t, K_t)$$
$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$
$$Y_t = A_t F(K_t, N_t)$$
$$Y_t = C_t + I_t + G_t$$
$$M_t = P_t M^d(i_t, Y_t)$$
$$r_t = i_t - \pi_{t+1}^e$$

▶ First six are *identical* to what we had before and have no reference to any nominal variable
▶ Eight endogenous variables: Y_t, C_t, I_t, N_t, w_t, r_t, P_t, and i_t
▶ New exogenous variables: M_t and π_{t+1}^e (treat expected inflation as exogenous)

Classical Dichotomy

- First six equations feature six real endogenous variables and no nominal variables
- Means that the real endogenous variables are determined independently of nominal variables
- This is known as the *classical dichotomy*
- Do not need to know nominal variables to determine real variables
- Converse not true: nominal variables will be affected by real variables

Graphing the Equilibrium

- Can use the same five part graph as before to determine equilibrium of the real side of the economy

- The real interest rate, r_t, and output, Y_t, are relevant for money demand

- Once we know r_t and Y_t, along with the exogenous quantity of money supplied, can determine P_t

- Given an exogenous π_{t+1}^e, given r_t can determine i_t (i_t and r_t always move in same direction absent a change in π_{t+1}^e)

Money Market Equilibrium
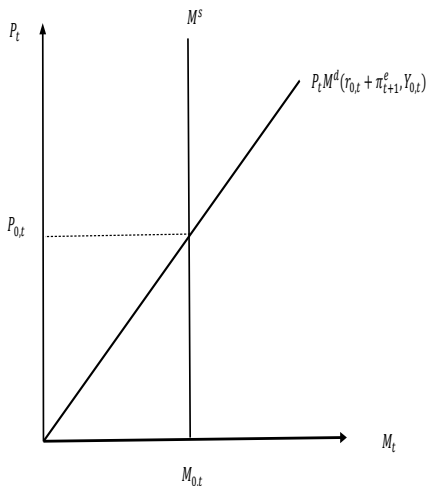
- Looks funny to have "demand" upward-sloping, but P_t is price of goods in terms of money, so $\frac{1}{P_t}$ is price of money in terms of goods. Demand *decreasing* in $\frac{1}{P_t}$
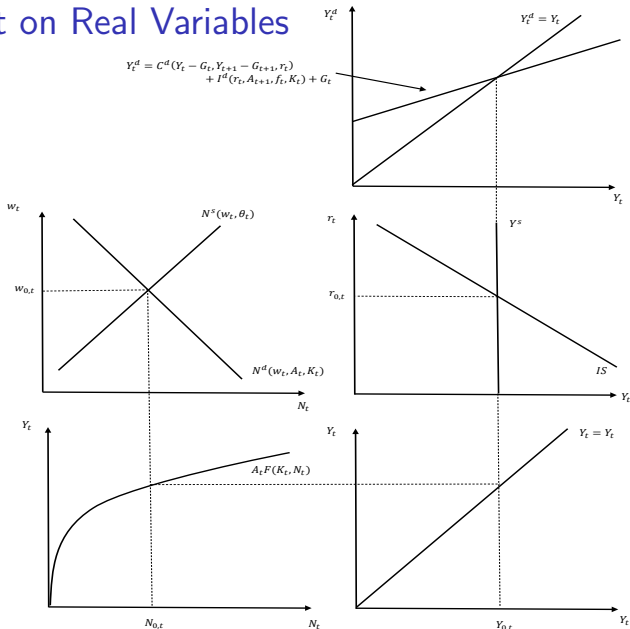
Monetary Neutrality

- Increase in M_t does not affect first six equations – no effect of change in M_t on any real endogenous variable
- We say that money is *neutral*
- Useful medium run benchmark, but in the short run nominal rigidities may break monetary neutrality
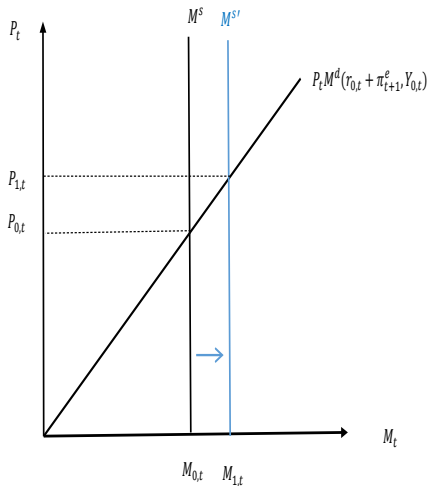- Only effect of an increase in M_t is an increase in P_t

Increase in M_t
No Effect on Real Variables

$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + I^d(r_t, A_{t+1}, f_t, K_t) + G_t$$

$Y_t^d = Y_t$

$N^s(w_t, \theta_t)$

$w_{0,t}$

$N^d(w_t, A_t, K_t)$

$r_{0,t}$

Y^s

IS

$A_t F(K_t, N_t)$

$Y_t = Y_t$

$N_{0,t}$

$Y_{0,t}$
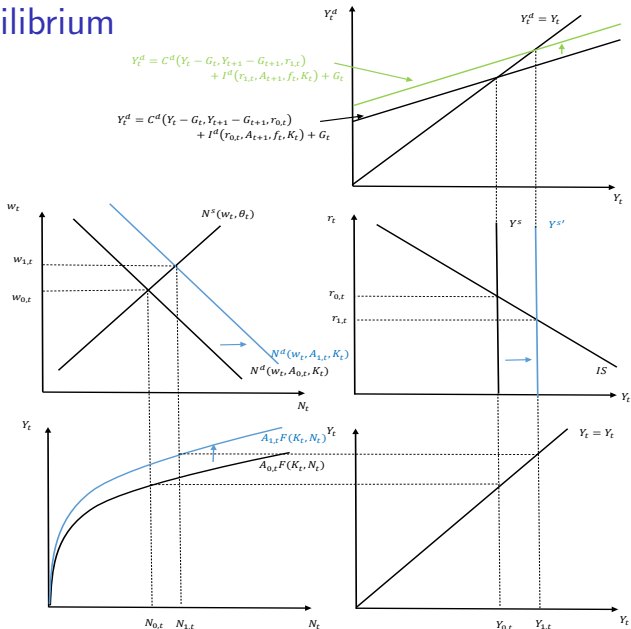
Increase in M_t
Raises P_t

Real Shocks Affect Nominal Variables

- Increase in A_t: lowers r_t and raises Y_t, both of which pivot money demand to the right, and hence lower P_t
- Increase in θ_t: raises r_t and lowers Y_t, both of which pivot money demand to the left, and hence raise P_t
- Positive "demand" shocks (increases in A_{t+1} or G_t, or decreases in f_t or G_{t+1}): raise r_t, no effect on Y_t. Hence, money demand shifts left, and price level rises
- Increase in π_{t+1}^e: i_t rises by same amount. Money demand pivots in, so price level increases. "Self-fulfilling" inflation

Supply Shock: $\uparrow A_t$
New Equilibrium

$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{1,t})$
$\qquad + I^d(r_{1,t}, A_{t+1}, f_t, K_t) + G_t$

$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{0,t})$
$\qquad + I^d(r_{0,t}, A_{t+1}, f_t, K_t) + G_t$

$Y_t^d = Y_t$

$N^s(w_t, \theta_t)$

$N^d(w_t, A_{1,t}, K_t)$

$N^d(w_t, A_{0,t}, K_t)$

Y^s $Y^{s\prime}$

IS

$A_{1,t}F(K_t, N_t)$

$A_{0,t}F(K_t, N_t)$

$Y_t = Y_t$

Increase in A_t
Lowers P_t
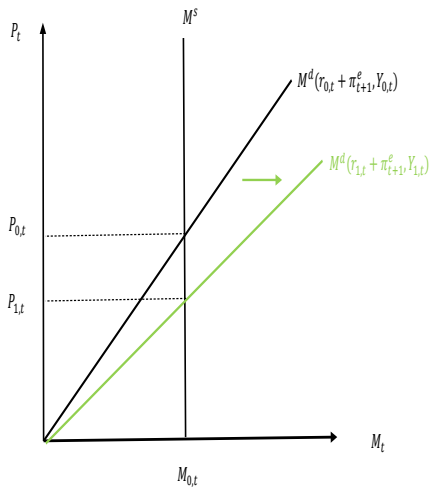
Demand Shock: $\uparrow f_t$
New Equilibrium

$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{0,t}) + I^d(r_{0,t}, A_{t+1}, f_{0,t}, K_t) + G_t$

$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{1,t}) + I^d(r_{1,t}, A_{t+1}, f_{1,t}, K_t) + G_t$

$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{0,t}) + I^d(r_{0,t}, A_{t+1}, f_{1,t}, K_t) + G_t$

$Y_t^d = Y_t$

w_t

$N^s(w_t, \theta_t)$

$w_{0,t}$

$N^d(w_t, A_t, K_t)$

N_t

r_t

Y^s

$r_{0,t}$

$r_{1,t}$

IS

IS'

Y_t

Y_t

$A_t F(K_t, N_t)$

$N_{0,t}$

N_t

Y_t

$Y_t = Y_t$

$Y_{0,t}$

Y_t

Increase in f_t
Lowers P_t

Qualitative Effects

				Exogenous Shock				
Variable	$\uparrow A_t$	$\uparrow \theta_t$	$\uparrow f_t$	$\uparrow A_{t+1}$	$\uparrow G_t$	$\uparrow G_{t+1}$	$\uparrow M_t$	$\uparrow \pi^e_{t+1}$
Y_t	+	-	0	0	0	0	0	0
C_t	+	-	-	?	-	-	0	0
I_t	+	-	+	?	-	+	0	0
N_t	+	-	0	0	0	0	0	0
w_t	+	+	0	0	0	0	0	0
r_t	-	+	-	+	+	-	0	0
i_t	-	+	-	+	+	-	0	+
P_t	-	+	-	+	+	-	+	+