# Problem Set 1

Graduate Macro II, Spring 2017
The University of Notre Dame
Professor Sims

**Instructions:** You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday, January 24.

**(1) ARMA Processes and Impulse Responses:** Suppose that you have a random variable, $x_t$, which obeys a MA(4) process given below:

$$x_t = \epsilon_t + 0.9\epsilon_{t-1} + 0.7\epsilon_{t-2} + 0.4\epsilon_{t-3} - 0.1\epsilon_{t-4} \qquad \epsilon_t \sim N(0,1)$$

The $\epsilon$ are white noise drawn from a standard normal distribution.

(a) What is the mean, of $x_t$, i.e. $E(x_t)$? What is the variance of $x_t$, i.e. $\text{var}(x_t)$?

Draw $\epsilon$ in Matlab using the "randn" command. Construct a data set with $T = 300$ observations using the MA(4) process given above. Assume as initial conditions that the realizations of all $\epsilon$ prior to the start of the sample are 0. Repeat this process $N = 1000$ times – this should leave you with 1000 samples with 300 observations each.

(b) For each of the $N$ samples you created, calculate the time series mean and variance. In doing this, drop the first 100 periods so as to limit the influence of assumed initial conditions. In other words, calculate the mean and variance of the last 200 observations of each of the 1000 artificial samples. Then, calculate the mean of the means across the $N = 1000$ artificial samples and the mean of the variances across the $N = 1000$ artificial samples. Report those here, and comment on how close they are to what you reported in part (a).

(c) On the last 200 observations of each of the $N = 1000$ artificial samples you created, estimate an AR(p) process for the following different lag lengths: $p = 1$, $p = 2$, $p = 4$, and $p = 8$. You should always include a constant in any estimated regression, even if you don't think it needs to be there (here, it doesn't). Use the estimated AR(p) model to construct the impulse response function (to a one unit shock) up to horizon $H = 10$ (the easiest way to do this is re-write the AR(p) as a VAR(1)). So, for the four different values of $p$, you should produce an impulse response function for each of the $N = 1000$ artificial samples. Average these impulse responses across the $N$ different samples. Plot the average impulse response function for each different value of $p$, and comment on how closely the average estimated impulse response function corresponds to the (known) parameters of the MA process given above.

**(2) The Hodrick-Prescott Filter:** In this problem you will derive the HP filter and create your own code for implementing it. Suppose you have a sequence of data, $y_t$ with $t = 1, ..., T$ (i.e. there are $T$ total observations). Our objective is to find a trend, $\{\tau_t\}_{t=1}^{T}$ to minimize the following objective function:

$$\min_{\tau_t} \quad \sum_{t=1}^{T}(y_t-\tau_t)^2+\lambda\sum_{t=2}^{T-1}\left[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})\right]^2$$

The parameter $\lambda \geq 0$.

(a) Provide a verbal interpretation of this objective function and what exactly one is trying to get at when choosing a trend, $\{\tau_t\}_{t=1}^{T}$.

(b) Prove that, if $\lambda = 0$, then the solution is for $\tau_t = y_t \ \forall \ t$. In other words, the trend and the actual series would be identical.

(c) Prove that, as $\lambda \to \infty$, then the solution is for for the trend to be a linear time trend; i.e. for $\tau_t = \alpha t$ for some $\alpha$.

(d) For the more general case in which $0 < \lambda < \infty$, derive analytical conditions which implicitly define the trend. To do this, take the derivative with respect to $\tau_t$ for each $t = 1, ..., T$ and set them equal to zero, yielding $T$ first order conditions.

(e) Write a Matlab code to find the trend taking the actual data series, $y_t$, and parameter, $\lambda$, as inputs. To do this, express the first order conditions in (d) in matrix form as follows:

$$\mathbf{\Lambda T} = \mathbf{Y}$$

$$\mathbf{T} = \mathbf{\Lambda}^{-1}\mathbf{Y}$$

Where $\mathbf{\Lambda}$ is a $T \times T$ matrix whose elements are functions of $\lambda$; $\mathbf{T}$ is a $T \times 1$ vector equal to $[\tau_1 \ \ \tau_2 \ \ . \ . \ . \ \ \tau_T]'$ and $\mathbf{Y}$ is a $T \times 1$ vector equal to $[y_1 \ \ y_2 \ \ . \ . \ . \ \ y_T]'$.

(f) Download quarterly, seasonally adjusted data on US real GDP from 1947 quarter 1 to 2014 quarter 3 (this should be available, for example, on the St. Louis Fed FRED data website). Take natural logs of the data, and then use your code to compute the HP trend using $\lambda = 1600$. Show a plot of the trend. Get the "cycle" series by subtracting the trend from the actual series. Show a plot of the cyclical component. What is the standard deviation of the HP detrended GDP?

(g) What is the peak decline relative to trend in GDP during the Great Recession era (i.e. the period since the third quarter of 2007)?

(h) Repeat part (f), but this time use $\lambda = 10,000,000$. What is the peak decline in real GDP relative to the trend during the Great Recession era? How does this compare to what you found in (g)? Can you speculate intelligently about the intuition for any differences in your answers?