Problem Set 2

Graduate Macro II, Spring 2017
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday, February 14.

(1) Comparing Solution Techniques: Deterministic Growth Model: In this problem you will compare policy functions from a deterministic neoclassical growth model using (a) value function iteration and (b) log-linearization.

The equilibrium of the economy can be described as the solution to the following social planner's problem:

$$\max_{c_t, k_{t+1}} \quad \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

s.t.

$$\begin{aligned} k_{t+1} &= k_t^{\alpha} - c_t + (1-\delta) k_t \\ & k_0 \text{ given} \end{aligned}$$

(a) Use L'Hopital's rule to prove that, as $\sigma \to 1$, the within period utility function goes to $\ln c_t$.

(b) Set this problem up as a dynamic programming problem. What is the state variable? What is the control variable? Write down the Bellman equation. Find the first order condition necessary for an optimal solution.

(c) Find an expression for the steady state capital stock and the steady state value of consumption.

(d) Suppose that the parameter values are as follows: $\beta = 0.95$, $\alpha = 0.36$, $\sigma = 2$, and $\delta = 0.1$. What are the numerical values of the steady state capital stock and consumption for these parameters? *Write your own code* to numerically solve for the value and policy functions. To do so, create a grid of the capital stock, with the minimum value 0.25 of the steady state capital stock and the maximum value 1.75 times the steady state capital stock, with 300 grid points between. Use linear interpolation to evaluate points off the grid. Show a graph of both the final value function and the policy function.

(e) Now set up the problem using a Lagrangian. Write out the first order conditions, including the transversality condition. Provide a verbal explanation for the intuition behind the transversality condition.

(f) Log-linearize the first order conditions about the steady state. Form a VAR(1) of the form:

$$\mathbf{X}_{t+1} = \mathbf{M}\mathbf{X}_t$$

Where $\mathbf{X}_t$ contains the variables expressed as percentage deviations about the steady state. Write out an expression for $\mathbf{M}$.

(g) Solve for the linear policy function mapping the state variable into the jump variable. Write out the numerical policy function here.

(h) Show a plot of the linearized policy function and the policy function from the value function iteration procedure obtained above together. Be sure to transform the linearized policy function, which is expressed as a percentage deviation about the steady state, into actual levels so as to make the comparison appropriate. Comment on the quality of the linear approximation.

(i) Repeat the exercise in (h) for the following different values of σ: 5 and 10. How does the quality of the linear approximation vary with σ? Why does this make sense?

(2) Log-Linearizing a Stochastic Growth Model with Redundant Variables: Consider a stochastic neoclassical growth model. The first order optimality conditions of the planner's problem are:

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + (1-\delta) \right)$$
$$K_{t+1} = A_t K_t^{\alpha} - C_t + (1-\delta) K_t$$

Assume that productivity follows a stationary AR(1) process in the log:

$$\ln A_t = \rho \ln A_{t-1} + s_A \varepsilon_{A,t}$$

(a) Solve for the non-stochastic steady state.

(b) Log-linearize these equilibrium conditions about the non-stochastic steady state.

Suppose that you also want to study the behavior of output and investment, which are given by:

$$Y_t = A_t K_t^{\alpha}$$
$$Y_t = C_t + I_t$$

(c) Log-linearize these conditions about the non-stochastic steady state.

(d) What are the state variable(s)? What are the control/jump variable(s)? What are the redundant/static variable(s)?

(e) Express the log-linearized system (all three types of variables) in the form, where $\mathbf{X}_t$ is a vector of state, control, and redundant variables:

$$\mathbf{B} E_t \mathbf{X}_{t+1} = \mathbf{C} \mathbf{X}_t$$

(f) Show how to write this system in the form, where $\widehat{\mathbf{X}}_t$ is a vector of only control/jump and state variables:

$$E_t \widehat{\mathbf{X}}_{t+1} = \mathbf{M} \widehat{\mathbf{X}}_t$$

(g) Create a Matlab file to solve for the policy function mapping the state(s) into the control(s), and then use this to form the state space representation of the system. Use the following values for the parameters: $\beta = 0.99$, $\alpha = 1/3$, $\sigma = 1$, $\delta = 0.025$, $\rho = 0.95$, $s_A = 0.01$.

(h) Compute impulse responses of consumption, capital, productivity, output, and investment to a one standard deviation productivity shock. Plot these impulse responses over a 40 quarter horizon.

(i) Use your model to simulate 10,000 periods of data by drawing shocks from a Normal Distribution. Begin the simulation in the non-stochastic steady state. Compute the standard deviations of simulated values of (log-linearized) consumption, output, and investment. Compute the ratio of the standard deviations of consumption and investment to output. Comment on these relative volatilities.

(j) Repeat (i), but this time solve the model assuming $\rho = 0.99$. What happens to the relative volatilities of consumption and investment? Can you provide any intuition for your answer?