Problem Set 4

Graduate Macro II, Spring 2017
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday, March 7.

(1) Debt, Taxes, and Government Spending: A representative household gets utility from consumption and government spending and disutility from labor. It gets to choose how much labor to supply and how much to consume; it takes government spending as given. It can save by accumulating bonds or capital. It leases capital to a representative firm on a period-by-period basis. It owns the firm and receives profit from the firm via a lump sum payment. It faces a distortionary tax rate on its labor income. Its problem can be written:

$$\max_{C_t, N_t, K_{t+1}, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} + \psi \ln G_t \right\}$$

s.t.

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} \leq (1-\tau_t) w_t N_t + R_t K_t + \Pi_t + (1 + r_{t-1}) B_t$$

(a) Derive the first order conditions for the household problem.

(b) A representative firm produces output according to $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. Each period, it solves the static problem of picking K_t and N_t to maximize static profit. Derive the first order condition for the firm.

The government's budget constraint is given by:

$$G_t + r_{t-1} D_t = \tau_t w_t N_t + D_{t+1} - D_t$$

D_t is government debt (so that positive values of D_t mean that the government faces an interest expense). Government spending follows an exogenous AR(1) process with non-stochastic mean of G:

$$\ln G_t = (1-\rho_G) \ln G + \rho_G \ln G_{t-1} + \epsilon_{G,t}$$

The tax rate follows an AR(1) process with non-stochastic mean of τ, but reacts to deviation of the debt-gdp ratio from an exogenous steady state target, $\frac{D}{Y}$:

$$\tau_t = (1-\rho_\tau)\tau + \rho_\tau \tau_{t-1} + (1-\rho_\tau)\gamma_\tau \left(\frac{D_t}{Y_t} - \frac{D}{Y} \right)$$

The parameter γ_τ is such that the there is a determinate, non-explosive equilibrium. Exogenous productivity follows an AR(1) in the log with non-stochastic mean of unity:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{A,t}$$

(c) Assume that $B_t = D_t$ initially. What must be true about household bond-holdings and government debt going forward? Write down the definition of a competitive equilibrium. Derive an expression for the aggregate resource constraint (i.e. the expenditure identity expression for output).

(d) Assume that in steady state the government consumes an exogenous fraction of output, i.e. $\frac{G}{Y} = \omega > 0$. Assume further that the steady state debt-gdp ratio is exogenous, $\frac{D}{Y}$. Solve for the steady state value of τ consistent with the government's budget constraint holding in steady state.

(e) Given exogenous values of ω and $\frac{D}{Y}$, as well as the steady state value of τ you found above, solve for expressions for the steady state values of all other endogenous variables in terms of these parameters and the other deep parameters of the model.

(f) Assume the following values of the parameters: $\beta = 0.99$, $\delta = 0.025$, $\alpha = 1/3$, $\theta = 5$, $\chi = 1$, $\psi = 0.3$, $\frac{D}{Y} = 1$, $\rho_A = 0.95$, $\rho_G = 0.9$, $\rho_\tau = 0.8$, $\gamma_\tau = 0.1$, $s_A = 0.01$ (standard deviation of productivity shock), and $s_G = 0.01$ (standard deviation of the government spending shock). Numerically solve for the values of ω and τ which maximize steady state flow utility for the household with these parameters. Also report the value of flow utility in this optimal steady state.

(g) Repeat (f), but this time for a steady state debt-gdp target of $\frac{D}{Y} = 0$. Report the optimal values of ω and τ, as well as the value of flow utility in this steady state. Is flow utility higher with a lower value of $\frac{D}{Y}$?

(h) To quantify how much better off the household would be in the low debt-gdp regime, compute a consumption equivalent welfare metric. Define $\varDelta$ as the the fraction of consumption that you would take away from the household with $\frac{D}{Y} = 0$ to give that household the same steady state utility as when $\frac{D}{Y} = 1$. What is $\varDelta$ here?

(i) Would the welfare metric you found in (h) likely over- or understate the potential benefit of eliminating debt from 100 percent of GDP to to 0 percent of GDP? Explain.

(j) Go back to assuming that $\frac{D}{Y} = 1$, and use the other parameter values given to you in part (f) above. Create a Dynare file. Solve the model via a first order approximation. Plot impulse responses (over a 40 quarter period) to a positive shock to government spending for output, consumption, investment, labor hours, the real interest rate, the real wage, the labor tax rate, and the level of government debt. Calculate the government spending multiplier as the ratio of the impact response of Y_t to the impact response of G_t to the shock to government spending. If you write your code where Dynare interprets the variables as logs, what the ratio of the impulse responses on impact will give you is an elasticity. To put this in multiplier form, divide the ratio by the steady state government spending share of output.

(k) Repeat (j), but instead assume that $\rho_G = 0.99$. What is the multiplier? Is it bigger or smaller than what you found in (j)? What is the intuition for this?

(l) Continue to assume that $\rho_G = 0.99$. But now assume that $\gamma_\tau = 0.3$. Compute impulse responses to an increase in government spending and report the multiplier. What is the multiplier? Is it bigger or smaller than what you found in (k)? Are you surprised by this? What is the intuition for what is going on?