Problem Set 5

Graduate Macro II, Spring 2017
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Thursday, March 30, 2017.

(1) Preference Shocks and Variable Capital Utilization: Suppose we have a model with variable capital utilization and a time-varying disutility of labor. A representative household owns the capital stock, K_t, and makes capital accumulation decisions. This agent also gets to pick how intensively to utilize capital, u_t, leasing capital services (the product of utilization and physical capital), $\widehat{K}_t \equiv u_t K_t$, to a representative firm on a period-by-period basis. The cost of more intensive utilization of capital is faster depreciation.

The problem of the household can be written as follows:

$$\max_{C_t, K_{t+1}, B_{t+1}, N_t, u_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \nu_t \theta \frac{N_t^{1+\chi}}{1+\chi} \right\}$$

s.t.

$$C_t + K_{t+1} - (1-\delta(u_t))K_t + B_{t+1} \leq w_t N_t + R_t u_t K_t + \Pi_t + (1+r_{t-1})B_t$$

$$\delta(u_t) = \delta_0 + \phi_1(u_t - 1) + \frac{\phi_2}{2}(u_t - 1)^2$$

ν_t is a time-varying shock to preferences over labor, obeying a stationary AR(1) process that is mean zero in the log:

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \varepsilon_{\nu,t}$$

(a) Derive the first order conditions for the household problem.

The firm produces output using labor and capital services, $\widehat{K}_t \equiv u_t K_t$, and takes factor prices, w_t and R_t, as given. Its problem is:

$$\max_{N_t, \widehat{K}_t} \quad \Pi_t = A_t \widehat{K}_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t \widehat{K}_t$$

(b) Derive the first order conditions for the firm problem.

Productivity obeys an AR(1) process in the log:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}$$

(c) Derive the aggregate resource constraint using the the definition of profit and the bond market-clearing condition.

(d) Provide a condition on the parameter ϕ_1 to ensure that steady state capital utilization is 1.

(e) Derive a condition giving the value of θ necessary to target steady state labor hours of $N^* = 1/3$.

(f) Write a Dynare code to solve the model and compute impulse responses to both productivity and preference shocks using a first order log-linear approximation. Use the following parameter values: $\beta = 0.99$, $\delta = 0.02$, $\alpha = 1/3$, $\chi = 1$, $\rho_a = 0.97$, $\rho_\nu = 0.95$, $s_a = 0.01$ (standard deviation of productivity shock), and $s_\nu = 0.02$ (standard deviation of the preference shock). Use the value of θ consistent with steady state hours of 1/3 found in (e) above. Consider three different values of ϕ_2: 100, 0.1, 0.01. Graphically show impulse responses of output, hours, consumption, investment, utilization, the real wage, and the real interest rate to both shocks. Comment on how the impulse responses to both shocks vary with the parameter ϕ_2, and provide some intuition for your results.

(g) Suppose that you measure total factor productivity not taking into account variable utilization. That is, let $\widehat{A}_t = \ln Y_t - \alpha \ln K_t - (1-\alpha)\ln N_t$. Show impulse responses of measured TFP to both shocks for the three different values of ϕ_2 from above. How does ϕ_2 affect the response of measured TFP to both shocks?

(h) How does the inclusion of the preference shock affect the *relative* volatility of HP filtered hours (relative to GDP) in the model? To see this, compute the relative volatility of hours to output with both shocks "turned on" and again with the preference shock "turned off" (e.g. set the standard deviation of that shock to zero). Use a value of $\phi_2 = 0.1$ in doing this part.

(2) GHH vs. Traditional Preferences and the Effects of Government Spending and Productivity Shocks: Suppose you have a RBC model with two stochastic shocks: a shock to productivity and a shock to government spending. We will consider two different preference specifications: standard separable preferences and GHH preferences.

The household problem can be written:

$$\max_{C_t, K_{t+1}, N_t, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

s.t.

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} \leq w_t N_t + R_t K_t + \Pi_t - T_t + (1 + r_{t-1})B_t$$

(a) For the arbitrary specification of household preferences, $U(C_t, N_t)$, find the first order conditions for a solution to the problem.

The firm problem is the same in both setups:

$$\max_{N_t, K_t} \quad \Pi_t = A_t K_t^\alpha N_t^{1-\alpha} - w_t N_t - R_t K_t$$

(b) Find the first order conditions necessary for a solution to the firm problem.

(c) The government chooses its spending exogenously and balances its budget each period, $G_t = T_t$. What then must be true about bond-holding by households in equilibrium? Write down the aggregate resource constraint.

Suppose that preferences are given by the standard separable form:

$$U(C_t, N_t) = \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi}$$

(d) Suppose that the non-stochastic steady state value of A_t is $A = 1$, while the steady state value of government spending is $G = \omega Y$, where Y is the steady state value of output and $0 < \omega < 1$. Using these preferences, derive an expression for the value of θ consistent with steady state hours of $N = 1/3$, and provide expressions for the steady state values of Y, C, I, K, w, and R as a function of parameters.

Suppose that G_t and A_t follows stationary AR(1) processes in the log:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{A,t}$$

$$\ln G_t = (1 - \rho_G) \ln G + \rho_G \ln G_{t-1} + \varepsilon_{G,t}$$

(e) Solve the model using a first order log-linear approximation in Dynare using the following parameter values: $\beta = 0.99$, $\delta = 0.02$, $\alpha = 1/3$, $\chi = 1$, $\rho_A = 0.97$, $\rho_G = 0.95$, $s_A = 0.01$ (standard deviation of productivity shock), $s_G = 0.01$ (standard deviation of government spending shock), $\omega = 0.20$, and the value of θ consistent with $N = 1/3$. Produce impulse response graphs of Y_t, C_t, I_t, N_t, w_t, and r_t to each shock over a 20 period horizon. Calculate the "government spending multiplier," defined as the ratio of the impact response of the level of output to the impact response of the level of government spending following a government spending shock.

Now instead suppose that preferences are given by the GHH variety:

$$U(C_t, N_t) = \ln \left(C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} \right)$$

(f) Using these preferences, derive an expression for the value of θ consistent with steady state hours of $N = 1/3$, and provide expressions for the steady state values of Y, C, I, K, w, and R as a function of parameters.

(g) Solve the model using a first order log-linear approximation using the same parameter values as above. Produce impulse response graphs of Y_t, C_t, I_t, N_t, w_t, and r_t to each shock over a 20 period horizon. Calculate the "government spending multiplier," defined as the ratio of the impact response of the level of output to the impact response of the level of government spending following a government spending shock.

(h) Compare and contrast your impulse responses to the two shocks with the two different preference specifications. Provide some intuition for your findings.

(3) News Shocks: This problem asks you to work out a business cycle model where there are anticipated shocks to productivity, which the literature has called "news shocks." In particular, suppose that the process for exogenous productivity, A_t, obeys the following stochastic process:

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_{t-4}$$

This is the standard AR(1) in the log, but the shock is observed 4 periods in advance of when productivity changes. In this way, a positive (or negative) realization of ε_t provides "news" about the level of A_t in 4 periods.

The model that we will consider is a variant of Jaimovich and Rebelo (2009, *American Economic Review*): "Can News About the Future Drive the Business Cycle?" It is a real model (nothing nominal) with three main twists relative to a standard RBC model: investment adjustment costs, variable capital utilization, and GHH type preferences. For now, let's assume a generic preference specification, where flow utility is is $U = U(C_t, N_t)$ and is increasing in consumption and decreasing in labor hours. A representative household owns the capital stock, makes investment decisions, and also makes utilization decisions. It leases capital services (the product of utilization and the physical capital stock) to a representative firm. The household problem can be written:

$$\max_{C_t, K_{t+1}, I_t, N_t, u_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

s.t.

$$C_t + I_t \leq w_t N_t + R_t u_t K_t + \Pi_t$$

$$K_{t+1} = \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1 - \delta(u_t))K_t$$

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$$

(a) Derive the first order conditions for the household problem. It is best to write a Lagrangian with two constraints; e.g. let λ_t be the multiplier on the flow budget constraint and μ_t the multiplier on the capital accumulation equation.

The firm problem is standard, where it picks capital services, $\widehat{K}_t = u_t K_t$, not capital or utilization individually:

$$\max_{N_t, K_t} \quad \Pi_t = A_t \widehat{K}_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t \widehat{K}_t$$

(b) Derive the first order conditions for the firm problem.

(c) Write down the definition of a competitive equilibrium and derive the aggregate resource constraint.

(d) Derive a restriction on the parameter δ_2 necessary to ensure that steady state utilization equals 1 (you do not need to know the specific form of preferences to do this).

Suppose that preferences are given by the standard additively separable form:

$$U(C_t, N_t) = \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi}$$

(e) Write a Dynare file to solve this model using a first order log-linear approximation. Use parameter values $\alpha = 1/3$, $\beta = 0.99$, $\delta_0 = 0.025$, $\chi = 1$, the value of δ_1 you derived above, and a value of θ consistent with steady state labor hours of $1/3$. Use values of $\rho = 0.95$ and the standard deviation of the shock of 0.01. It is straightforward to include the news shock in Dynare – just write "e(-4)" where you would normally write "e" in the shock process. For now, assume that $\delta_2 = 1000$ (effectively, no variable utilization) and $\kappa = 0$ (no investment adjustment costs). Produce and print out impulse responses of output, consumption, hours, and investment to the news shock. What happens to these variables in the period between the news hits (on "impact") and when the shock translates into higher productivity four periods later? Can you provide some intuition for this?

(f) Re-do the exercise in (e), but this time set $\delta_2 = 0.01$, effectively "turning on" variable utilization (but keep $\kappa = 0$). Comment on how the impulse responses are different, and try to provide some intuition.

(g) Repeat the exercise in (e), but instead "turn" on investment adjustment costs, setting $\kappa = 3$ (but set $\delta_2 = 1000$). Comment on how the impulse responses differ from the baseline, and try to provide some intuition.

(h) Now turn both of these features on simultaneously, setting $\delta_2 = 0.01$ and $\kappa = 3$. Produce the impulse responses to the news shock. Knowing what you know about co-movements among aggregate variables in the actual time series data, can news shocks be an important driving force of the business cycle in this model?

Now instead consider the GHH preference specification, where:

$$U(C_t, N_t) = \ln\left(C_t - \theta \frac{N_t^{1+\chi}}{1+\chi}\right)$$

(i) Use the same parameter values you did in part (e) (e.g. "turn off' variable utilization and the investment adjustment cost), compute impulse responses to the news shock when the household has these preferences. Use a value of θ consistent with steady state labor hours of 1/3 (note, this value of θ will be different than what you found above). Comment on how the impulse responses look different from what you found in part (e), and try to provide some intuition.

(j) Continue to use GHH preferences, but now "turn on" both variable utilization and the investment adjustment cost ($\delta_2 = 0.01$ and $\kappa = 3$). Produce impulse response to the news shock, and compare them what you found for the base RBC model with separable preferences, no utilization, and no adjustment costs. Comment on the differences, and try to provide some intuition for how each of the three different changes relative to the baseline (preferences, utilization, and adjustment costs) help account for the differences.