Problem Set 6

Graduate Macro II, Spring 2017
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due at my office by 5:00 pm on Friday April 14.

(1) Solving a New Keynesian Model: Consider the baseline New Keynesian model with no capital. Rather than explicitly modeling money in the utility function, I follow much of the literature and assume that the economy is "cashless." Rather than setting the money supply, the central bank instead sets the nominal interest rate according to a simple rule. Implicitly, there is a demand for money generated via money in the utility function; the central bank prints the amount of money necessary to make the money market clear at its target interest rate. Hence, we can avoid money altogether.

There is a representative household who solves the following problem, where P_t is the nominal price of goods, W_t is the nominal wage, i_{t-1} the nominal interest rate on bonds carried over from period $t-1$ into period t, Π_t^n is nominal profits distributed from firms (which the household takes as given), and T_t is a nominal lump sum tax/transfer from the government:

$$\max_{C_t, N_t, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi \frac{N_t^{1+\eta}}{1+\eta} \right)$$

s.t.

$$P_t C_t + B_{t+1} \leq W_t N_t + \Pi_t^n + P_t T_t + (1+i_{t-1}) B_t$$

(a) Find the first order conditions for an interior solution to the household problem.

Production is split up into two sectors. There is a competitive, representative final goods firm which produces final output, Y_t, which is a CES aggregate of a continuum of intermediate goods, $Y_t(j)$, $j \in (0,1)$.

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1$$

(b) Write down the profit maximization problem of the final good firm in nominal terms (nominal output is $P_t Y_t$) and derive the demand curve for each intermediate good.

(c) Define nominal output as $P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$. Use your answer from (b) to derive an expression for the aggregate price level.

Intermediate goods producers produce output using a linear production technology in labor input:

$$Y_t(j) = A_t N_t(j)$$

(d) Intermediate goods producers are price-takers in input markets, taking W_t as given. Set up the cost-minimizing problem of a generic intermediate producer, subject to the constraint that it produce as much as is demanded at a given price. Find the first order condition and provide an economic interpretation of the Lagrange multiplier.

Intermediate goods producers are not freely able to adjust their prices period-by-period. Each period they face a probability of $1-\phi$, $\phi \in (0,1)$, of being able to adjust their price. The probability of being able to adjust is independent of when they last updated their price. Firms discount future profit flows by the stochastic discount factor and the probability of being stuck with their currently chosen price.

(e) Write down the (dynamic) pricing problem of the firm, and derive the optimal "reset price" for a firm able to update its price in a given period.

Assume that the central bank sets the nominal interest rate according to a simple Taylor rule of the form:

$$i_t = (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\phi_\pi(\pi_t - \pi^*) + \varepsilon_{i,t}, \quad \phi_\pi > 1,\ 0 \leq \rho_i < 1$$

Here i^* is the steady state nominal interest rate and π^* is the target inflation rate, which will also be equal to the inflation rate in the non-stochastic steady state. ϕ_π is a parameter restricted to be greater than 1, and ρ_i is a smoothing parameter between zero and one. Assume also that A_t follows a mean-zero stationary AR(1) in the log:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}$$

(f) Write down the definition of an equilibrium for this economy. Assume that households do not begin time with any debt, and that firms and the government also do not participate in debt markets. Since we ignore money, and there is no government spending, lump sum taxes are always zero: $P_t T_t = 0$. Write down the market-clearing conditions.

(g) Use the properties of Calvo pricing (in particular, that firms are randomly selected to update their price and that $1-\phi$ is not only the probability of being able to adjust price but also the fraction of all firms who get to update their price) to derive an expression for the aggregate price level which depends on the optimal reset price and the lagged aggregate price level.

(h) Define inflation as $1+\pi_t = \frac{P_t}{P_{t-1}}$ and reset price inflation as $1+\pi_t^{\#} = \frac{P_t^{\#}}{P_{t-1}}$. Re-write your expression for the evolution of the aggregate price level above in terms of inflation and reset price inflation. Also, re-write the expression for the optimal reset price in terms of inflation rates instead of levels.

(i) Derive an expression for the aggregate production function in terms of A_t, N_t, and a term related to price dispersion. Write the term related to price dispersion recursively in terms of inflation and reset price inflation in a way that does not depend on j.

(j) Assume that inflation is zero in steady state, so $\pi^* = 0$. Solve for the non-stochastic steady state values of $\pi_t^{\#}$, i_t, Y_t, N_t, w_t (the real wage), and mc_t (real marginal cost).

(k) Argue that the steady state of this economy is distorted relative to what a planner would choose. Suppose that a government had access to a (fixed) tax rate on labor income paid by households (e.g. in the budget constraint labor income would appear as $(1-\tau)W_t N_t$). Tax revenues/losses from this tax (note that the tax rate can be negative) are remitted to households lump sum. What value of τ would eliminate the steady state distortion in the economy? Provide some intuition.

(l) Create a Dynare file to quantitatively solve for the policy functions of this model and compute impulse responses to the productivity and monetary policy shocks. In doing so you may ignore the hypothetical tax you solved for in (k). Use the following parameter values: $\beta = 0.99$, $\phi = 0.75$, $\phi_\pi = 1.5$, $\rho_i = 0.8$,

$\eta = 1$, $\epsilon = 10$, $\rho_a = 0.95$, $\psi = 1$, $s_i = 0.0025$, and $s_a = 0.01$ (the latter two are the standard deviations of the monetary policy and productivity shocks, respectively). Produce impulse responses of output, employment, the nominal and real interest rates, and inflation to each of the two shocks.

(m) Re-do these exercises, but consider two different values of ϕ, $\phi = 0.90$ and $\phi = 0.50$. Comment on how the responses to both shocks differ from the baseline responses in (l).

(n) Go back to the original value of $\phi = 0.75$. Now consider two different values of the policy rule coefficient on inflation, $\phi_\pi = 2.5$ and $\phi_\pi = 1.1$. Comment on how the impulse responses differ relative to the baseline.

(2) Solving a Linearized NK Model by Hand: Suppose that you have the baseline linearized New Keynesian model as developed in class. For simplicity, assume that the flexible price level of output is held fixed at its non-stochastic steady state. In terms of log deviations, this means that $\widetilde{Y}_t^f = 0$. The two main non-policy equations of the model can be written:

$$\widetilde{Y}_t = E_t\widetilde{Y}_{t+1} - (\widetilde{i}_t - E_t\pi_{t+1})$$

$$\pi_t = \lambda\widetilde{Y}_t + \beta E_t\pi_{t+1}$$

Here I have normalized the elasticity of intertemporal substitution to be 1. The coefficient in the Phillips Curve is $\lambda = \frac{(1-\phi)(1-\phi\beta)}{\phi}(1+\eta)$, where η is the inverse Frisch labor supply elasticity, and 1 is the intertemporal elasticity of substitution. Note that since I am assuming $\widetilde{Y}_t^f = 0$, it is omitted from the Phillips Curve, but inflation would ordinarily depend on the output gap, not just output. Note also that since the model is linearized about a zero inflation steady state, actual inflation is the same as the deviation of inflation from steady state – i.e. $\widetilde{\pi}_t = \pi_t$.

Instead of obeying a money supply targeting rule or a Taylor rule, suppose instead that the central bank obeys a strict inflation targeting rule. In particular, let π_t^* be an exogenous inflation target. The central bank will adjust i_t so that $\pi_t = \pi_t^*$ is consistent with these equations holding. Assume that the inflation target follows an exogenous AR(1) process:

$$\pi_t^* = \rho\pi_{t-1}^* + \epsilon_{\pi,t}, \quad 0 < \rho < 1$$

(a) Derive an analytic expression for $\widetilde{i}_t$ as a function of π_t^*.

(b) Suppose that ρ is 0. In which direction must the central bank adjust $\widetilde{i}_t$ in order to achieve an increase in π_t^*?

(c) If ρ is sufficiently close to 1, is it possible that $\widetilde{i}_t$ must increase in order to implement an increase in π_t? How does this answer depend on the value of λ?

(d) Try to provide some intuition for your answers on (b) and (c).