Problem Set 6

ECON 30020: Intermediate Macroeconomics
Professor Sims
University of Notre Dame, Spring 2018

Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member's name on the first page of your solutions. This problem set is due in class on Thursday March 8.

1. **Ricardian Equivalence:** Critically evaluate the following statement. "Ricardian Equivalence means that changes in G_t have the same effect on the equilibrium of an endowment economy as do changes in G_{t+1}."

2. **A Static One Period Model of the Macroeconomy:** Consider a static, one period model of the macroeconomy. There is a representative household. The household can choose how much to consume and how much to work. There is no saving since the model is static. The household problem is:

$$\max_{C_t, N_t} \quad U = \ln\left[C_t - \frac{\theta_t}{2} N_t^2\right]$$

s.t.

$$C_t = w_t N_t + D_t$$

Here N_t is labor, C_t consumption, D_t a dividend received from ownership of the firm, and w_t is the real wage. θ_t is an exogenous parameter governing the disutility from work. A firm produces output according to the production technology:

$$Y_t = A_t N_t$$

The firm's dividend is:

$$D_t = Y_t - w_t N_t$$

The firm's objective is to pick N_t to maximize D_t:

$$\max_{N_t} \quad D_t = A_t N_t - w_t N_t$$

(a) Use calculus to derive a first order condition characterizing optimal behavior by the household.

(b) Use calculus to derive a first order condition characterizing optimal behavior by the firm.

(c) Given your previous answers, what will be true about D_t in equilibrium?

(d) Given previous answers, derive the aggregate resource constraint.

(e) Use previous answers to derive an expression for equilibrium Y_t as a function of exogenous variables, A_t and θ_t. Verify that Y_t is increasing in A_t and decreasing in θ_t.

(f) Re-do previous parts, but with the more general utility specification:

$$U = \ln\left[C_t - \frac{\theta_t}{1+\chi} N_t^{1+\chi}\right], \quad \chi \geq 0$$

The original problem is a special case of this with $\chi = 1$. For the more general case, re-derive an expression for Y_t as a function of A_t and θ_t. Assuming that $A_t = \theta_t$ for simplicity, is the sensitivity of Y_t to A_t higher, lower, or unaffected by the value of χ (i.e. is the partial derivative of Y_t with respect to A_t bigger or smaller as χ is bigger)? Try to use demand and supply curves for labor to provide some intuition for your answer.

3. **A Firm's Investment Problem:** Suppose that a firm produces output according to the following production function:

$$Y_t = A_t K_t^{\alpha}, \quad 0 < \alpha < 1$$

The production function looks the same in period $t+1$:

$$Y_{t+1} = A_{t+1} K_{t+1}^{\alpha}$$

A_t and A_{t+1} are exogenous and known by the firm. Capital accumulates according to a standard law of motion, except that there is full depreciation (so $\delta = 1$). This means that tomorrow's capital is today's investment:

$$K_{t+1} = I_t$$

The period t capital stock, K_t, is taken as given by the firm. The firm does not use labor. The firm must borrow to finance investment from a financial intermediary at interest rate $r_t + f_t$, where f_t is an exogenous credit spread variable. Period t and $t+1$ dividends are therefore:

$$D_t = Y_t$$

$$D_{t+1} = Y_{t+1} - (1 + r_t + f_t) I_t$$

The firm's objective is pick I_t to maximize its value (present discounted value of dividends) subject to the capital accumulation equation and production function:

$$\max_{I_t} \quad V_t = D_t + \frac{D_{t+1}}{1 + r_t}$$

s.t.

$$K_{t+1} = I_t$$

$$D_t = Y_t$$

$$D_{t+1} = Y_{t+1} - (1 + r_t + f_t) I_t$$

Use calculus to derive an optimal demand function for investment. Show that investment is decreasing in both r_t and f_t, and increasing in A_{t+1} (but not a function of A_t).