Practice Problem Set 7.1

ECON 30020: Intermediate Macroeconomics
Professor Sims
University of Notre Dame, Spring 2018

Instructions: This problem set covers material covered in class the class meetings of March 8, March 20, and March 22. It is only for practice in preparation for the upcoming midterm. You need not turn anything in. Solutions will be made available by the end of the business day on Friday March 23.

1. **The Fisher Relationship:** The Fisher relationship relates the real interest rate, r_t, to the nominal interest rate, i_t, as well as the current and future price of goods measured in terms of money, P_t and P_{t+1}. It is given by:

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$$

(a) Conceptually define how r_t and i_t differ from one another. Given this, provide some intuition for why the Fisher relationship as written above must hold.

(b) Define the (gross) expected inflation rate as $1 + \pi^e_{t+1} = \frac{P_{t+1}}{P_t}$. Use this to show that the Fisher relationship approximately implies:

$$r_t \approx i_t - \pi^e_{t+1}$$

(c) Suppose that, in the medium to long run, the real interest rate satisfies:

$$1 + r_t = \frac{1}{\beta}\frac{Y_{t+1}}{Y_t}$$

Define $\beta = \frac{1}{1+\rho}$, where $\rho \geq 0$. Take logs to derive an approximate relationship between the real interest rate, ρ, and the expected growth rate of output. Argue that the real interest rate is increasing in ρ and increasing in the expected growth rate of output. Provide some intuition (based on competitive equilibrium in an endowment economy) for your answers.

(d) If, as immediately above, the real interest rate in the medium to long run is independent of the inflation rate, and if expected inflation equals actual inflation in the medium to long run, what is the primary determinant of the level of the nominal interest rate in the medium to long run? Is your answer consistent with what we observe in the data?

2. **Money in the Utility Function in an Endowment Economy:** Suppose that you have an endowment economy with a representative household and a government. There is no

production. The household takes its income flow, Y_t and Y_{t+1}, as given. The household and government both live for two periods, t and $t+1$. Lifetime utility is:

$$U = \ln C_t + \ln\left(\frac{M_t}{P_t}\right) + \beta \ln C_{t+1}$$

The household faces a sequence of two flow budget constraints. Imposing terminal conditions and writing in nominal terms, these are:

$$P_t C_t + P_t S_t + M_t = P_t Y_t - P_t T_t$$

$$P_{t+1} C_{t+1} = P_{t+1} Y_{t+1} - P_{t+1} T_{t+1} + (1 + i_t) P_t S_t + M_t$$

The government issues no debt and does no spending. It issues M_t exogenously in period t; this is a source of revenue. It then "buys back" the money it issued in period t in period $t+1$. This is a cost to the government. The government's two flow budget constraints are:

$$P_t T_t = -M_t$$

$$P_{t+1} T_{t+1} = M_t$$

(a) Eliminate S_t and use the Fisher relationship (given above) to derive the real intertemporal budget constraint for the household.

(b) The household's objective is to pick a consumption plan (C_t and C_{t+1}) and money holdings between t and $t+1$, M_t, to maximize U subject to the intertemporal budget constraint you derived above. Find the first order conditions characterizing optimal behavior by the household.

(c) For markets to clear, what must be true about S_t in equilibrium given our assumption that the government does no borrowing? Use this, along with the government's budget constraints, to derive the aggregate resource constraint.

(d) Combine the government's two flow budget constraints with the household's real intertemporal budget constraint and make use of the Fisher relationship. Show that M_t, P_t, T_t, and T_{t+1} drop out.

(e) Combine the household's consumption Euler equation with its real intertemporal budget constraint derived immediately above (which makes use of knowledge of the government's budget balancing in an intertemporal sense) to derive the consumption function.

(f) Use the period t market-clearing condition along with your derived first order condition for money holdings to derive the money demand function. Argue that it is a special case of the more general demand specification given in class (i.e. $M_t = P_t M^d(i_t, Y_t)$) and show that money demand is decreasing in the nominal interest rate, i_t, and increasing in output, Y_t.

3. **Real and Nominal Shocks:** Consider the neoclassical model as augmented to include money as presented in class. The equations characterizing the equilibrium of the model are:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_t, r_t)$$

$$N_t = N^s(w_t, \theta_t)$$

$$N_t = N^d(w_t, A_t, K_t)$$

$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$

$$Y_t = A_t F(K_t, N_t)$$

$$Y_t = C_t + I_t + G_t$$

$$M_t = P_t M^d(i_t, Y_t)$$

$$r_t = i_t - \pi^e_{t+1}$$

   (a) What are the endogenous variables? What are the exogenous variables?

   (b) Explain what is meant by the *classical dichotomy* and what it implies for thinking about the effects of exogenous shocks in the model.

   (c) Suppose that there is an exogenous increase in A_t. Graphically show how this will impact the equilibrium of the model, including how the price level and nominal interest rate are affected.

   (d) Suppose that agents in the model exogenously expect more inflation in the future (i.e. π^e_{t+1} increases). Document what happens to each endogenous variable.