Problem Set 7

Graduate Macro II, Spring 2017
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday, April 25.

(1) Interest Rate Rules in the Linearized NK Model: Consider the following NK model linearized about a zero inflation steady state:

$$\widetilde{Y}_t = E_t\widetilde{Y}_{t+1} - \left(\widetilde{i}_t - E_t\pi_{t+1}\right) \tag{1}$$

$$\pi_t = \gamma\widetilde{X}_t + \beta E_t\pi_{t+1} \tag{2}$$

$$\widetilde{i}_t = \phi_\pi\pi_t + \phi_x\widetilde{X}_t \tag{3}$$

$$\widetilde{X}_t = \widetilde{Y}_t - \widetilde{Y}_t^f \tag{4}$$

$$\widetilde{Y}_t^f = \rho\widetilde{Y}_{t-1}^f + \varepsilon_t \tag{5}$$

In these expressions we think about the flexible price level of output, $\widetilde{Y}_t^f$, as exogenous, and following an AR(1) process.

(a) Define the "natural rate of interest." Denote this by $\widetilde{r}_t^f$. Derive an expression for $\widetilde{r}_t^f$.

(b) Re-write the IS equation in terms of the output gap and the interest rate gap, the gap between the real interest rate and the natural rate.

(c) Show that you can eliminate the exogenous process for $\widetilde{Y}_t^f$, instead writing an exogenous AR(1) process for $\widetilde{r}_t^f$. Derive this expression.

(d) From here on out, use the process for $\widetilde{r}_t^f$ you derived in (c) and the output gap representation of the IS curve derived in (b). Use the Taylor rule to eliminate $\widetilde{i}_t$ from the IS equation. This should leave you three equations – the modified IS equation, the Phillips Curve, and the process for $\widetilde{r}_t^f$.

(e) Guess that the policy functions are given by $\pi_t = \theta_1\widetilde{r}_t^f$ and $\widetilde{X}_t = \theta_2\widetilde{r}_t^f$. Solve for expressions for θ_1 and θ_2.

(f) Instead suppose that the monetary policy rule is a "stochastic intercept rule" in which $\widetilde{i}_t = \widetilde{r}_t^f + \phi_\pi\pi_t + \phi_x\widetilde{X}_t$. That is, the central bank adjusts the nominal interest rate one-for-one with the natural rate of interest, as well as to deviations of inflation and the output gap. Re-do part (e) using this monetary policy rule.

(g) Under what value of ρ will the equilibrium under the standard Taylor rule (part (e)) correspond to the equilibrium with the stochastic intercept Taylor rule (part (f))? Explain.

(2) The Government Spending Multiplier in the NK Model: Consider a linearized version of the New Keynesian model with government spending. The only exogenous shock is the shock to government spending. I will spare you the details of the model derivation and instead only present the equilibrium conditions linearized about a zero inflation steady state:

$$\widetilde{C}_t = E_t\widetilde{C}_{t+1} - \left(\widetilde{i}_t - E_t\pi_{t+1}\right) \tag{6}$$

$$\pi_t = \gamma\widetilde{X}_t + \beta E_t\pi_{t+1} \tag{7}$$

$$\widetilde{X}_t = \widetilde{Y}_t - \widetilde{Y}_t^f \tag{8}$$

$$\widetilde{Y}_t = (1-g)\widetilde{C}_t + g\widetilde{G}_t \tag{9}$$

$$\widetilde{G}_t = \rho\widetilde{G}_{t-1} + \varepsilon_t \tag{10}$$

$$\widetilde{i}_t = \phi_\pi\pi_t \tag{11}$$

Here $\widetilde{G}_t$ is the log-deviation of government spending from its steady state, assumed to obey a stationary AR(1) process. g is the steady state share of government spending in output (i.e. $g = \frac{G}{Y}$, so that $1-g = \frac{C}{Y}$).

In a hypothetical equilibrium in which prices are flexible, the following equilibrium conditions must hold:

$$\theta\left(N_t^f\right)^\chi = \frac{1}{C_t^f}\frac{\epsilon-1}{\epsilon} \tag{12}$$

$$Y_t^f = N_t^f \tag{13}$$

$$Y_t^f = C_t^f + G_t \tag{14}$$

(a) Analytically solve for a *log-linearized* expression for the flexible price level of output, $\widetilde{Y}_t^f$. In doing this, assume that $\frac{G}{Y^f} = g$, the same as the government spending share of output in the actual economy.

(b) Use this expression to solve for the flexible price equilibrium government spending multiplier, $\frac{dY_t^f}{dG_t}$. Since the above are log deviations from steady state, you can compute this as $\frac{\widetilde{Y}_t^f}{\widetilde{G}_t}\frac{Y^f}{G} = \frac{\widetilde{Y}_t^f}{\widetilde{G}_t}g^{-1}$. Argue that this multiplier is bound between 0 and 1. Will the multiplier be bigger or smaller the bigger is χ (the inverse Frisch elasticity)? Will the multiplier be bigger or smaller the bigger is g (the steady state government spending share of output)?

(c) Solve for an analytic expression for $\widetilde{r}_t^f$, the hypothetical real interest rate which would emerge if prices were flexible.

(d) Given your answers on (a)-(d), write the Euler equation in terms of the output gap and the deviation of the real interest rate, $\widetilde{i}_t - E_t\pi_{t+1}$, from the natural rate of interest, $\widetilde{r}_t^f$.

(e) Show that you can model the exogenous process for $\widetilde{G}_t$ instead as an exogenous process for $\widetilde{r}_t^f$, in a way similar to what you did on the previous problem. Show your expression here.

(f) You should now have the model down to the Euler equation written in terms of the output gap, the Phillips Curve, the Taylor rule, and the exogenous process for $\widetilde{r}_t^f$. Substitute out the nominal interest rate using the Taylor rule. Guess that the policy functions are $\pi_t = \theta_1\widetilde{r}_t^f$ and $\widetilde{X}_t = \theta_2\widetilde{r}_t^f$. Solve for θ_1 and θ_2.

(g) Use your answer from (f) to derive an analytic expression showing $\widetilde{X}_t$ as a function of $\widetilde{G}_t$ alone. Does the output gap go up or down after an increase in $\widetilde{G}_t$? Use this to derive an expression for the government spending multiplier, $\frac{dY_t}{dG_t}$. Is the multiplier bigger or smaller than with flexible prices? How does the multiplier depend on the value of γ?