Problem Set 8

Graduate Macro II, Spring 2017
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday, May 2.

(1) The Government Spending Multiplier in a New Keynesian Model: Suppose that you have a New Keynesian model with government spending (financed by lump sum taxes). The equilibrium conditions of the model linearized about a zero inflation steady state are (note all variables are either percent deviations or absolute deviations from steady state, and hence I omit "tilde" terms):

$$C_t = E_t C_{t+1} - (i_t - E_t \pi_{t+1}) \tag{1}$$

$$\chi N_t = -C_t + w_t \tag{2}$$

$$mc_t = w_t \tag{3}$$

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi} mc_t + \beta E_t \pi_{t+1} \tag{4}$$

$$Y_t = N_t \tag{5}$$

$$Y_t = (1-g)C_t + gG_t \tag{6}$$

$$r_t = i_t - E_t \pi_{t+1} \tag{7}$$

$$G_t = \rho_G G_{t-1} + s_G \varepsilon_{G,t} \tag{8}$$

In writing down the model, I have assumed that productivity is constant, i.e. $A_t = 0$. $g \in [0,1]$ is the steady state government spending share of output in the economy. I have also intentionally left off a description of monetary policy for the time being.

(a) Briefly explain what each equation represents and where it comes from.

(b) What would be true about mc_t if $\phi = 0$ (i.e. if prices were flexible). Use this fact, plus the given equations of the model, to find an expression for Y_t^f, the flexible price level of output, in terms of the exogenous variable G_t.

(c) Use your answer from the previous part that the government spending multiplier must be between 0 and 1 if prices are flexible. The government spending multiplier can be written as $\frac{dY_t^f}{dG_t}\frac{1}{g}$. Division by g at the end is necessary because the multiplier is about level changes in output for a level change in government spending, but the variables in the model are logs.

(d) Given the Fisher relationship between the real and nominal interest rates, derive an expression for r_t^f, the so-called natural rate of interest, in terms of G_t.

(e) Use your results from the previous part to show that you can write the Phillips Curve as:

$$\pi_t = \gamma X_t + \beta E_t \pi_{t+1} \tag{9}$$

Where $X_t = Y_t - Y_t^f$. Solve for an expression for γ.

(f) Use your results form previous parts to derive a NK IS curve of the sort:

$$X_t = E_t X_{t+1} - \theta(i_t - E_t \pi_{t+1} - r_t^f) \tag{10}$$

Where X_t again is the output gap. What is the expression for θ?

(g) Show that we can re-cast the exogenous process for G_t in terms of an exogenous process for r_t^f, where:

$$r_t^f = \rho_G r_{t-1}^f + \omega s_G \varepsilon_{G,t} \tag{11}$$

Solve for an expression for ω.

(h) The model can be summarized by equations (9)-(11), plus a monetary policy rule. Suppose that the monetary policy rule is a Taylor rule of the form:

$$i_t = a\pi_t \tag{12}$$

Assume that $a > 1$. Guess that the policy functions are:

$$\pi_t = \psi_1 r_t^f \tag{13}$$

$$X_t = \psi_2 r_t^f \tag{14}$$

Use the method of undetermined coefficients to solve for expressions for ψ_1 and ψ_2.

(i) Use your answer from the previous part to solve for an expression for the government spending multiplier, $\frac{dY_t}{dG_t}$. Compare your answer to the flexible price multiplier you found in part (c). Argue that the multiplier is weakly bigger than what you found for the flexible price case in (c), and that the gap between the multiplier and the flexible price multiplier is increasing in the amount of price stickiness.

(j) Suppose that the nominal interest rate were pegged at a fixed value (say, the steady state, so $i_t = 0$) in period t. In the next period, it will exit the peg with probability $1-p$, after which time policy returns to being characterized by the Taylor rule. Once policy returns to the Taylor rule, it is expected to remain forever in that regime. With probability p, the interest rate remains pegged. *Without actually doing it*, describe how you would solve for an analytic expression for the multiplier.