Inflation, Output and Markup Dynamics with Purely Forward-Looking Wage and Price Setters^{*}

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Abstract

Medium-scale New Keynesian models are sometimes criticized for their use of backward-looking wage and price setting mechanisms, and for assuming several types of disturbances, including some that would be difficult to interpret economically. We propose a DSGE model with purely forward-looking wage and price setters and parsimoniously chosen disturbances. Our model emphasizes an interplay between production firms networking and working capital. Firms can use working capital to finance all (or a fraction) of their outlays on production factors, and use the output of other firms as an input in a what is often called a "roundabout" production structure. Our model generates a response of inflation which is mute on impact of a monetary policy shock, but highly persistent and very hump-shaped afterwards. It also yields a large contract multiplier for output, two times larger than the one implied by a model relying on indexation only. We also show that the response of the price markup can be positive on impact of an expansionary monetary policy shock, which differs from the standard countercyclical markup channel emphasized in conventional New Keynesian models. In contrast to models relying on indexation to past inflation, our model produces non-inertial responses of inflation to productivity and investment shocks, a finding which is broadly consistent with the existing VAR evidence.

JEL classification: E31, E32.

Keywords: New Keynesian Model; Firms Networking; Working Capital; Inflation Dynamics; Contract Multiplier for Output; Cyclical Markups

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1 Introduction

Medium-scale DSGE models (e.g. Christiano, Eichenbaum, and Evans 2005 or Smets and Wouters 2007) are commonly employed by academics and policymakers for counterfactual policy analysis and forecasting simulations. A useful test of these models is whether or not they can generate conditional responses to shocks which are broadly consistent with available empirical evidence. So as to match the inertial inflation response to identified monetary shocks from the VAR literature, these models often rely upon ad-hoc backward-looking price and wage-setting mechanisms such as the indexation of prices and wages to the previous period's rate of inflation. The use of indexation has been criticized by a number of researchers.¹ Backward indexation also generates inertial, hump-shaped inflation responses to non-policy shocks. This is inconsistent with the non-inertial responses of inflation to productivity and investment shocks typically found in the empirical literature.

This paper proposes a framework which does not rely upon backward-looking price and wagesetting at all but which can nevertheless generate persistent, inertial inflation responses to monetary shocks and non-inertial inflation responses to non-policy shocks. We also use our model to discuss some issues related to the cyclicality of markups and the measurement of markups in the data. The core of our model is quite similar to the standard medium-scale DSGE model. In particular, the model features nominal price and wage stickiness in the form of Calvo (1983) staggered contracts, variable capital utilization, habit formation in consumption, and investment adjustment costs. Monetary policy is characterized by a conventional Taylor type rule for the nominal interest rate.

Our model departs from conventionally specified medium-scale models along several dimensions. First, unlike for example Christiano, Eichenbaum, and Evans (2005), it does not feature backward price and wage indexation. Second, our model features production networking, or the use by firms of intermediate goods in an input-output production structure. This is a feature of U.S. production which is well documented empirically, with a typical firm selling 50 percent or more of its output to other firms (Basu, 1995; Huang, Liu, and Phaneuf, 2004). This is often referred to as a "roundabout production structure." Following Christiano (2015), we refer to this production structure as one of "firms networking." Firms networking introduces strategic complementarities and makes marginal cost less sensitive to input factor prices.

Third, in our model firms must borrow to finance some or all of their payments to all factors of production – intermediate inputs, capital services, and labor. In Christiano and Eichenbaum

¹For example, Woodford (2007) argues that "the model's implication that prices should continuously adjust to changes in prices elsewhere in the economy flies in the face of the survey evidence," while Cogley and Sbordone (2008) mention that backward wage and price setting mechanisms "lack a convincing microeconomic foundation."

(1992), Christiano, Eichenbaum, and Evans (1997, 2005), Ravenna and Walsh (2006), and Tillmann (2008), working capital serves only to finance wage payments before the proceeds of sale are received. There are a few exceptions to models where working capital finances only the wage bill. Assuming that working capital is used to purchase commodities and finance wage payments, Chowdhury, Hoffmann, and Schabert (2006) provide VAR evidence for the G7 countries supporting their specification. In Christiano, Trabandt, and Walentin (2011), working capital is used to finance payments to labor and materials input, with the intent of showing that intermediate inputs and working capital can possibly lead to indeterminacy even if the central bank complies with the Taylor principle. In our model, working capital can be used in an extended form to finance the cost of all inputs, a case to which we refer as "extended borrowing." It can also be used in a limited form, a case we call "limited borrowing," to finance only subsets of these three inputs. For our baseline specification, we focus on the extended borrowing case. As we later show, varying the extent to which working capital finances inputs has rich consequences for the short-run dynamics of inflation and output, as well as for the cyclical behavior of the price markup conditioned on a monetary policy shock.

A fourth dimension along which our model differs from many medium scale DSGE models is that we focus on a limited number of structural disturbances. This is in part motivated by the critique in Chari, Kehoe, and McGrattan (2009) that medium-scale New Keynesian models often rely on a number of "dubiously structural" shocks in way that renders them unsuitable for policy analysis. It is also driven by our desire to focus on understanding mechanisms in our model and how they relate to the conventionally-specified medium scale model without trying to force the model to match all dimensions of the data.

We use our model with forward-looking price and wage setting to address four main questions. A first question is: can it generate a highly persistent and hump-shaped response of inflation to a monetary policy shock without assuming backward-looking elements in price and wage setting? A second question is: does it deliver "large" contract multipliers for output in the terminology of Chari, Kehoe, and McGrattan (2000)? The third question is: what does our model imply about the cyclicality of the price markup conditioned on a monetary shock and on the measurement of markups more generally? Finally, the fourth question is: can our model generate impulse responses to non-policy shocks which are broadly consistent with available empirical evidence?

Our baseline model predicts a response of inflation which is nearly mute on impact of a monetary policy shock and very persistent and hump-shaped thereafter. Absent firms networking and working capital, the response of inflation is largest on impact and only weakly persistent thereafter. Our model is also able to address the "persistence problem" emphasized by Chari, Kehoe, and McGrattan (2000). Output responds significantly to a monetary policy shock and in a humpshaped and inertial fashion. In our model, the half-life of output conditional on a monetary shock is fourteen quarters. This is substantially larger than the half-life of output in a model without extended borrowing and firms networking, and is larger than a version of our model with backward price and wage indexation. Our model delivers all of these results without relying upon empirically implausible average waiting times between price and wage adjustments.

The key ingredient accounting for these findings is the interaction between firms networking and working capital. Firms networking introduces strategic complementarity into price setting, making inflation less sensitive to changes in real marginal cost by a factor of proportionality reflecting the share of intermediate inputs in production. Firms networking therefore makes the inflation response to a policy shock smaller on impact and more persistent, while at the same time making the output response to a policy shock larger. Working capital in its extended form contributes to making the inflation response to a policy shock very hump-shaped. Because of working capital, the nominal interest rate has a direct effect on marginal cost. This limits the initial increase in marginal cost associated with an expansionary policy shock. If firms borrow working capital to finance the costs of all of their inputs, the impact of the nominal interest on real marginal cost is the strongest. If borrowing is limited, the impact of the nominal interest rate is naturally smaller, but is stronger if working capital serves to finance the purchase of intermediate inputs rather than wage payments. Via the Phillips Curve, a smaller increase in marginal cost keeps inflation from initially rising by much. Since the cut in interest rates is only temporary, as the interest rate begins to rise after impact due to the expansionary effects of the policy shock, marginal cost also begins to rise, which puts upward pressure on inflation and results in hump-shaped inflation dynamics.

Some other substantive findings in our paper pertain to the cyclical behavior of markups. Conventional wisdom from the textbook New Keynesian model (e.g. Woodford, 2003, 2011) is that a countercyclical price markup is the key transmission mechanism of aggregate demand shocks. In contrast, in our model, the aggregate markup of price over marginal cost is procyclical conditional on a monetary shock. This is because, with extended borrowing, the nominal interest rate has a direct impact on marginal cost which works in the opposite direction of movements in other factor prices. The cyclicality of the price markup is therefore quite sensitive to assumptions concerning which factors must be financed via working capital. In contrast, the wage markup and labor wedge are countercyclical conditional on a policy shock and these responses are not very sensitive to assumptions about working capital.

Our analysis has relevance for a large and unsettled literature on the cyclical behavior of price markups. Galí, Gertler, and López-Salido (2007) report evidence of a price markup which is either weakly countercyclical or weakly procyclical depending on alternative methods and measures. The evidence in Nekarda and Ramey (2013) points to a mildly procyclical price markup. In contrast, Bils, Klenow, and Malin (2016) argue that the price markup is countercyclical. Common to this literature is the measurement of the price markup as the ratio of the real wage to the marginal product of labor. While this conforms to the theoretical definition of the price markup in a conventional New Keynesian model, it is misspecified with respect to our model, where the nominal interest rate has a direct effect on marginal cost. If working capital is an important feature of the economy, failing to account for movements in the interest rate can lead to spurious conclusions concerning the behavior of the price markup. In the context of our model, we show that a conventional definition of the price markup omitting the interest rate is countercylical conditional on a monetary shock, even though the true price markup in the model is procyclical.

While the bulk of the paper focuses on the behavior of the economy conditional on monetary policy shocks, we also study responses to non-policy shocks. Our model features three other shocks which feature prominently in the literature – a permanent shock to neutral productivity, a permanent shock to investment-specific technology, and a shock to the marginal efficiency of investment. Our model produces impulse responses of output and inflation to these shocks which are broadly consistent with available empirical evidence. Our model generates non-inertial responses of inflation to these shocks, even while producing hump-shaped and very persistent inflation responses to a monetary shock. This is an issue with which the existing literature has grappled and is an important success of our model.

Altig, Christiano, Eichenbaum, and Linde (2011) build off the Christiano, Eichenbaum, and Evans (2005) model. While their model with firm-specific capital is capable of generating an inertial and hump-shaped inflation response to a policy shock, it is incapable of delivering a large impact decline in inflation after a neutral productivity shock. In our model, in contrast, the inflation decline after a positive productivity shock is largest on impact and inflation quickly reverts to its pre-shock value in a way consistent with the VAR evidence. Christiano, Trabandt, and Walentin (2010) are able to generate a persistent and hump-shaped response of inflation to a policy shock and a non-inertial response of inflation to a productivity shock. While their model abstracts from price indexation, importantly it does rely on backward wage indexation to lagged inflation. Christiano, Eichenbaum, and Trabandt (2015) and Christiano, Eichenbaum, and Trabandt (2016) dispense with wage rigidity altogether, combining Calvo price stickiness into a search and matching model of the labor market with no backward indexation. These models do deliver a persistent inflation response to a policy shock and a non-inertial inflation response conditional on a productivity shock, but they fall short in generating a significant hump-shape in the inflation response to a policy shock. In particular, inflation responds positively on impact of an expansionary monetary policy shock and its peak response is soon thereafter in most specifications of the models in these papers.

The remainder of the paper is organized as follows. Section 2 presents our model with firms networking and working capital. Section 3 studies the output and inflation effects of monetary policy shocks. Section 4 discusses issues related to the cyclicality of markups and measurement of markups in the data. Section 5 studies the dynamic effects of non-policy shocks in our model. Section 6 offers concluding thoughts.

2 A Medium-Scale DSGE Model with Firms Networking and Extended Borrowing

The core of our medium-scale DSGE model is similar to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). The model includes nominal rigidities in the form of Calvo wage and price contracts, habit formation in consumption, investment adjustment costs, variable capital utilization, and a Taylor rule. We augment the model to include firms networking and an extended working capital or cost channel. The subsections below lay out the decision problems of the relevant model actors. The full set of conditions characterizing the equilibrium are shown in Appendix A.

2.1 Good and Labor Composites

There is a continuum of firms, indexed by $j \in (0, 1)$, producing differentiated goods with the use of a composite labor input. The composite labor input is aggregated from differentiated labor skills supplied by a continuum of households, indexed by $h \in (0, 1)$. Differentiated goods are bundled into a gross output good, X_t . Some of this gross output good can be used as a factor of production by firms. Net output is then measured as gross output less intermediate inputs. Households can either consume or invest the final net output good. The composite gross output and labor input are:

$$X_t = \left(\int_0^1 X_t(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}},\tag{1}$$

$$L_t = \left(\int_0^1 L_t(h)^{\frac{\sigma-1}{\sigma}} dh\right)^{\frac{\sigma}{\sigma-1}}.$$
(2)

The parameters $\theta > 1$ and $\sigma > 1$ denote the elasticities of substitution between goods and labor, respectively. The demand schedules for goods of type j and labor of type i respectively are:

$$X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} X_t \quad \forall j,$$
(3)

$$L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\sigma} L_t \quad \forall h.$$
(4)

The aggregate price and wage indexes are:

$$P_t^{1-\theta} = \int_0^1 P_t(j)^{1-\theta} dj,$$
 (5)

$$W_t^{1-\sigma} = \int_0^1 W_t(h)^{1-\sigma} dh.$$
 (6)

2.2 Households

There is a continuum of households, indexed by $h \in (0, 1)$, who are monopoly suppliers of labor. They face a downward-sloping demand curve for their particular type of labor given in (4). Following Calvo (1983), each period, there is a fixed probability, $(1 - \xi_w)$, that households can adjust their nominal wage, with $0 \leq \xi_w < 1$. Although we abstract from backward indexation in our baseline model, we write the model in such a way as to permit indexation. Non-updated wages may be indexed to lagged inflation via the parameter $\zeta_w \in [0, 1]$. As in Erceg, Henderson, and Levin (2000), we assume that utility is separable in consumption and labor. State-contingent securities insure households against idiosyncratic wage risk arising from staggered wage setting. With this setup, households are identical along all dimensions other than labor supply and nominal wages. We therefore suppress dependence on h except for choice variables related to the labor market.

The problem of a particular household is to optimize the present discounted value of flow utility, (7), subject to a flow budget constraint, (8), a law of motion for physical capital, (9), the demand curve for labor, (10), and a constraint describing the Calvo wage setting process, (11). Preferences are given by:

$$\max_{C_t, L_t(h), K_{t+1}, B_{t+1}, I_t, Z_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln \left(C_t - b C_{t-1} \right) - \eta \frac{L_t(h)^{1+\chi}}{1+\chi} \right).$$
(7)

The flow budget constraint and physical capital accumulation process are, respectively:

$$P_t\left(C_t + I_t + \frac{a(Z_t)K_t}{\varepsilon_t^I}\right) + \frac{B_{t+1}}{1 + i_t} \le W_t(h)L_t(h) + R_t^k Z_t K_t + \Pi_t + B_t + T_t,$$
(8)

$$K_{t+1} = \varepsilon_t^I \vartheta_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) \right) I_t + (1-\delta)K_t.$$
(9)

Here, P_t is the nominal price of goods, C_t is consumption, I_t is investment measured in units of consumption, K_t is the physical capital stock, and Z_t is the level of capital utilization. $W_t(h)$ is the nominal wage paid to labor of type h, and R_t^k is the common rental price on capital services (the product of utilization and physical capital). Π_t and T_t are, respectively, distributed dividends from firms and lump sum taxes from the government, both of which households take as given. B_t is a stock of nominal bonds with which the household enters the period. $a(Z_t)$ is a resource cost of utilization, satisfying a(1) = 0, a'(1) = 0, and a''(1) > 0. This resource cost is measured in units of physical capital. $S\left(\frac{I_t}{I_{t-1}}\right)$ is an investment adjustment cost, satisfying $S(g_I) = 0$, $S'(g_I) = 0$, and $S''(g_I) > 0$, where $g_I \ge 1$ is the steady state (gross) growth rate of investment. i_t is the nominal interest rate. $0 < \beta < 1$ is a discount factor, $0 < \delta < 1$ is a depreciation rate, and $0 \le b < 1$ is a parameter for internal habit formation. χ is the inverse Frisch labor supply elasticity and η is a scaling parameter on the disutility from labor.

 ε_t^I , which enters the capital accumulation equation by multiplying investment and the budget constraint in terms of the resource cost of capital utilization, measures the level of investment specific technology (IST). The exogenous variable ϑ_t , which enters the capital accumulation equation in the same way as the IST term, is a stochastic MEI shock.² Justiniano, Primiceri, and Tambalotti (2011) draw the distinction between these two types of investment shocks, showing that IST shocks map one-to-one into the relative price of investment goods, while MEI shocks do not impact the relative price of investment.³

The demand curve for labor and the constraint describing Calvo wage setting are:

$$L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\sigma} L_t,$$
(10)

$$W_{t}(h) = \begin{cases} W_{t}^{*}(h) & w/ \ prob \ 1 - \xi_{w} \\ (1 + \pi_{t-1})^{\zeta_{w}} W_{t-1}(h) & otherwise \end{cases}$$
(11)

It is straightforward to show that all households given the opportunity to change their wage will adjust to a common reset wage, W_t^* .

²Note that Justiniano, Primiceri, and Tambalotti (2011) argue that smoothed MEI shocks obtained from Bayesian estimation of their model closely correlate with observed credit spread dynamics. As an extension, we run a version of our model with an exogenous credit spread shock and compute impulse responses to it. The credit spread shock generates impulse responses in our model which are qualitatively similar to those empirically identified in a VAR by Gilchrist and Zakrajsek (2012). These results are available from the authors upon request.

³In the model, the relative price of investment goods is easily seen to be $\frac{1}{\varepsilon_t^I}$. The division by ε_t^I in the resource cost of utilization is therefore necessary so that capital is priced in terms of consumption goods.

2.3 Firms

The production function for a typical producer j is:

$$X_t(j) = \max\left\{A_t\Gamma_t(j)^{\phi} \left(\widehat{K}_t(j)^{\alpha} L_t(j)^{1-\alpha}\right)^{1-\phi} - \Upsilon_t F, 0\right\},\tag{12}$$

where A_t is neutral productivity, F is a fixed cost, and production is required to be non-negative. Υ_t is a growth factor. Given Υ_t , F is chosen to keep profits zero along a balanced growth path, so that entry and exit of firms can be ignored. $\Gamma_t(j)$ is the amount of intermediate input, and $\phi \in (0, 1)$ is the intermediate input share. Intermediate inputs come from aggregate gross output, X_t . $\hat{K}_t(j)$ is capital services, while $L_t(j)$ is labor input. This production function differs from the standard in the New Keynesian DSGE literature in its addition of intermediate goods, $\Gamma_t(j)$.

A firm gets to choose its price, $P_t(j)$, as well as quantities of the intermediate input, capital services, and labor input. It is subject to Calvo pricing, where each period there is a $(1 - \xi_p)$ probability that a firm can re-optimize its price, with $0 \le \xi_p < 1$. Even though we abstract from indexation in our baseline model, we write a firm's problem in such a way as to permit indexation. Non-updated prices may be indexed to lagged inflation at $\zeta_p \in [0, 1]$. In other words, a firm's price satisfies:

$$P_t(j) = \begin{cases} P_t^*(j) & w/ \ prob \ 1 - \xi_p \\ (1 + \pi_{t-1})^{\zeta_p} P_{t-1}(j) & otherwise \end{cases}$$
(13)

An updating firm will choose its price to maximize the present discounted value of flow profit, where discounting is by the stochastic discount factor of households as well as the probability that a price chosen today will still be in effect in the future. It is straightforward to show that all firms given the ability to change their price will adjust to a common reset price, P_t^* .

Regardless of whether a firm can re-optimize its price, it will always choose inputs so as to minimize cost, subject to the constraint of meeting demand at its price. A key assumption is that firms must finance some or all of their variable inputs through intra-period loans from a financial intermediary. The financial intermediary returns the interest earned on these loans to the household lump sum. The cost-minimization problem of a typical firm is:

$$\min_{\Gamma_t(j),\hat{K}_t(j),L_t(j)} (1 - \psi_{\Gamma} + \psi_{\Gamma}(1+i_t))P_t\Gamma_t(j) + (1 - \psi_K + \psi_K(1+i_t))R_t^k\hat{K}_t(j) + (1 - \psi_L + \psi_L(1+i_t))W_tL_t(j) \quad (14)$$

$$A_t \Gamma_t(j)^{\phi} \left(\widehat{K}_t(j)^{\alpha} L_t(j)^{1-\alpha} \right)^{1-\phi} - \Upsilon_t F \ge \left(\frac{P_t(j)}{P_t} \right)^{-\theta} X_t.$$

s.t.

Here ψ_l , $l = \Gamma$, K, L, denotes the fraction of payments to a factor that must be financed at the gross nominal interest rate, $1 + i_t$. With $\psi_l = 0$ for all l, firms do not have to borrow to pay any of their factors. In contrast, when $\psi_l = 1$ for all l, firms must borrow the entirety of their factor payments each period. With this setup, the factor prices relevant for firms are the product of the gross nominal interest rate and the factor price. We refer to this case as extended borrowing (EB).

The use of working capital may be limited to a subset of factors. When used to finance only wage payments as in Christiano, Eichenbaum, and Evans (1997), Christiano, Eichenbaum, and Evans (2005), or Ravenna and Walsh (2006), we set $\psi_{\Gamma} = \psi_{K} = 0$ and $\psi_{L} = 1$, a case to which we refer as LBW. When used to finance only the purchase of intermediate goods, a case we refer to as LBI, we set $\psi_{L} = \psi_{K} = 0$ and $\psi_{\Gamma} = 1$. To economize on notation, we define $\Psi_{l,t} = (1 - \psi_{l} + \psi_{l}(1 + i_{t}))$ for $l = \Gamma, K, L$.

Applying some algebraic manipulations to the first order conditions for the cost-minimization problem yields an expression for real marginal cost, v_t , which is common across all firms:

$$v_{t} = \overline{\phi} \Psi_{\Gamma,t}^{\phi} \left(\Psi_{K,t} r_{t}^{k} \right)^{\alpha(1-\phi)} \left(\Psi_{L,t} w_{t} \right)^{(1-\alpha)(1-\phi)} A_{t}^{-1},$$
(15)

with $\overline{\phi} \equiv \frac{1}{\phi} \left(\frac{\phi}{1-\phi}\right)^{1-\phi} \left(\frac{1}{\alpha}\right)^{1-\phi} \left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)(1-\phi)}$. The variables r_t^k and w_t are the real rental rate on capital services and the real wage for labor, respectively. Much of the intuition for the results which follow can be gleaned from (15), so we pause to discuss some special cases.

In a model where both firms networking ($\phi = 0$) and financial intermediation ($\Psi_{l,t} = 0$ for all l) are excluded, the expression for real marginal cost reduces to:

$$v_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} (r_t^k)^{\alpha} (w_t)^{1-\alpha} A_t^{-1}.$$
(16)

(16) is the standard expression for real marginal cost in the literature, where marginal cost depends only on factor prices and neutral productivity. In contrast, when firms networking and extended borrowing are both turned on, where $\phi > 0$ and $\Psi_{l,t} = 1 + i_t$ for all l, the expression for real marginal cost becomes:

$$v_t = (1+i_t)\overline{\phi} \left(r_t^k\right)^{\alpha(1-\phi)} w_t^{(1-\alpha)(1-\phi)} A_t^{-1}.$$
 (17)

It is instructive to compare the differences between (17) and (16). Firms networking, as captured by the parameter ϕ , reduces the sensitivity of real marginal cost to factor prices. This will have the effect of "flattening" the New Keynesian Phillips Curve and resulting in larger monetary nonneutralities. Extended borrowing results in the nominal interest rate directly affecting real marginal cost. This feature will play an important role in generating hump-shaped inflation dynamics in response to monetary policy shocks.

2.4 Monetary Policy

Monetary policy follows a Taylor rule:

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\rho_i} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}g_Y^{-1}\right)^{\alpha_y} \right]^{1-\rho_i} \varepsilon_t^r.$$
(18)

The nominal interest rate responds to deviations of inflation from an exogenous steady-state target, π , and to deviations of output growth from its trend level, g_Y . ε_t^r is an exogenous shock to the policy rule.⁴ The parameter ρ_i governs the smoothing-effect on nominal interest rates while α_{π} and α_y are control parameters. We restrict attention to parameter configurations resulting in a determinate rational expectations equilibrium.⁵

2.5 Aggregation

Given properties of Calvo (1983) price and wage setting, aggregate inflation and the real wage evolve according to:

$$1 = \xi_p \pi_t^{\theta - 1} \pi_{t-1}^{\zeta_p (1-\theta)} + (1 - \xi_p) \left(p_t^* \right)^{1-\theta},$$
(19)

$$w_t^{1-\sigma} = \xi_w \left(\frac{w_{t-1}\pi_{t-1}^{\zeta_w}}{\pi_t}\right)^{1-\sigma} + (1-\xi_w) \left(w_t^*\right)^{1-\sigma}.$$
 (20)

⁴Note that in our model the monetary policy shock is observed by agents in period t and can therefore affect endogenous variables immediately. This differs from the standard recursive timing structure in the VAR literature, as well as from the model in Christiano, Eichenbaum, and Evans (2005) where economic variables react to policy shocks only with a lag. We also solved a version of our model where agents make decisions based on the period t - 1expectation of the policy shock, which renders the model consistent with common VAR timing assumptions. The results which follow are similar under this alternative assumption and are available from the authors upon by request.

⁵Our policy rule is written where the nominal interest rate reacts to output growth relative to trend growth rather than the level of output relative to potential. Policy rules written this way have empirical support, particularly over the last thirty years – see, e.g., Coibion and Gorodnichenko (2011). In a model with borrowing required to finance some or all input costs, the relatively simple Taylor principle is no longer guaranteed to support a determinate equilibrium, a point which is noted by Christiano, Trabandt, and Walentin (2011). In our quantitative exercises we have found that responding too strongly to the output gap can result in indeterminacy, which, in addition to the empirical support from Coibion and Gorodnichenko (2011), motivates our specification of the policy rule in terms of output growth.

The notation here is that $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is aggregate gross inflation, $p_t^* \equiv \frac{P_t}{P_t}$ is the relative reset price, $w_t \equiv \frac{W_t}{P_t}$ is the real wage, and $w_t^* \equiv \frac{W_t^*}{P_t}$ is the real reset wage. Market-clearing for capital services, labor, and intermediate inputs requires that $\int_0^1 \hat{K}_t(j)dj = \hat{K}_t$, $\int_0^1 L_t(j)dj = L_t$, and $\int_0^1 \Gamma_t(j)dj = \Gamma_t$. This means that aggregate gross output can be written:

$$s_t X_t = A_t \Gamma_t^{\phi} \left(\widehat{K}_t^{\alpha} L_t^{1-\alpha} \right)^{1-\phi} - \Upsilon_t F,$$
(21)

where s_t is a price dispersion variable that can be written recursively:

$$s_t = (1 - \xi_p) p_t^{*-\theta} + \xi_p \pi_{t-1}^{-\zeta_p \theta} \pi_t^{\theta} s_{t-1}.$$
 (22)

Using the market-clearing conditions, the aggregate factor demands can be written:

$$\Gamma_t = \phi v_t \Psi_{\Gamma,t}^{-1} \left(s_t X_t + \Upsilon_t F \right), \tag{23}$$

$$\widehat{K}_t = \alpha (1 - \phi) \frac{v_t}{\Psi_{K,t} r_t^k} \left(s_t X_t + \Upsilon_t F \right), \qquad (24)$$

$$L_t = (1 - \alpha)(1 - \phi) \frac{v_t}{\Psi_{L,t} w_t} \left(s_t X_t + \Upsilon_t F \right).$$

$$(25)$$

Note that factor demands depend both on factor prices as well as potentially on the nominal interest rate.⁶ Aggregate net output, Y_t , is gross output minus intermediate input:

$$Y_t = X_t - \Gamma_t \tag{26}$$

Integrating over household budget constraints yields the aggregate resource constraint:

$$Y_t = C_t + I_t + a(Z_t)K_t \tag{27}$$

2.6 Shock Processes

Our model features several exogenous variables – neutral productivity, A_t ; investment-specific technology, ε_t^I ; marginal efficiency of investment, ϑ_t ; and the monetary policy disturbance, ε_t^r .

⁶Note that since output is the numeraire the factor price of intermediates is normalized to unity.

We assume that neutral productivity follows a random walk with drift in the log, where g_A is the gross trend growth rate and u_t^A is a shock drawn from mean zero normal distribution with known standard deviation of s^A :

$$A_t = g_A A_{t-1} \exp\left(s_A u_t^A\right). \tag{28}$$

The IST term follows a random walk with drift in the natural log, where g_{ε^I} is the gross growth rate and $u_t^{\varepsilon^I}$ is a shock drawn from a normal distribution with standard error s_{ε^I} :

$$\varepsilon_t^I = g_{\varepsilon^I} \varepsilon_{t-1}^I \exp\left(s_{\varepsilon^I} u_t^{\varepsilon^I}\right) \tag{29}$$

The MEI shock follows a stationary AR(1) process, with innovation drawn from a mean zero normal distribution with standard deviation s_I :

$$\vartheta_t = (\vartheta_{t-1})^{\rho_I} \exp(s_I u_t^I), \quad 0 \le \rho_I < 1 \tag{30}$$

The only remaining shock in the model is the monetary policy shock, ε_t^r . We assume that it is drawn from a mean zero normal distribution with known standard deviation s_r .

2.7 Growth

Most variables inherit trend growth from the drift terms in neutral and investment-specific productivity. Let this trend factor be Υ_t . Output, consumption, investment, intermediate inputs, and the real wage all grow at the rate of this trend factor on a balanced growth path: $g_Y = g_I = g_{\Gamma} =$ $g_w = g_{\Upsilon} = \frac{\Upsilon_t}{\Upsilon_{t-1}}$. The capital stock grows faster due to growth in investment-specific technology, with $\widetilde{K}_t \equiv \frac{K_t}{\Upsilon_t \varepsilon_t^I}$ being stationary. Given our specification of preferences, labor hours are stationary. The full set of equilibrium conditions re-written in stationary terms can be found in Appendix A.

One can show that the trend factor that induces stationarity among transformed variables is:

$$\Upsilon_t = (A_t)^{\frac{1}{(1-\phi)(1-\alpha)}} \left(\varepsilon_t^I\right)^{\frac{\alpha}{1-\alpha}}.$$
(31)

This reverts to the conventional trend growth factor in a model with growth in neutral and investment-specific productivity when $\phi = 0$. Under this restriction, intermediates are irrelevant for production, and the model reduces to the standard New Keynesian model. Interestingly, from (31), it is evident that a higher value of ϕ amplifies the effects of trend growth in neutral productivity on output and its components. For a given level of trend growth in neutral productivity, the economy grows faster the larger is the share of intermediates in production.

2.8 Calibration

We now turn to a discussion of the numerical values assigned to the parameters in our model. Many parameters are calibrated to conventional long run targets in the data. Others are chosen based on the previous literature. We choose to calibrate, rather than estimate, the parameters in our model for two reasons. First, given our focus on a limited number of structural shocks (motivated by the criticism in Chari, Kehoe, and McGrattan 2009), only a limited number of observable variables could be used in estimation, which would make identification of many parameters difficult. Second, we wish to compare different versions of our model to standard parameterizations of medium scale New Keynesian models, holding fixed all but the relevant parameter. This exercise allows us to cleanly shed light on the different mechanisms at work in our model.

The calibration for most parameters is summarized in Table 1. The unit of time is taken to be a quarter. Some parameter values, like β , b, η , χ , δ and α are standard in the literature. Others require some explanation.

We assume the following functional forms for the resource cost of capital utilization and the investment adjustment cost:

$$a(Z_t) = \gamma_1(Z_t - 1) + \frac{\gamma_2}{2}(Z_t - 1)^2,$$
(32)

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - g_I\right)^2.$$
(33)

The parameter governing the size of investment adjustment costs κ is 3. This is somewhat higher than the estimate in Christiano, Eichenbaum, and Evans (2005) (2.48), but somewhat lower than that in Justiniano, Primiceri, and Tambalotti (2011) (3.14). The parameter on the squared term in the utilization adjustment cost is set to $\gamma_2 = 0.05$. This is broadly consistent with the evidence in Basu and Kimball (1997) and Dotsey and King (2006), and represents a middle range between Justiniano, Primiceri, and Tambalotti (2010, 2011) who estimate this parameter to be about 0.15, and Christiano, Eichenbaum, and Evans (2005), who fix this parameter at 0.00035.⁷ The parameter θ is the elasticity of substitution between differentiated goods and is set at 6. This implies a steadystate price markup of 20 percent, which is consistent with Rotemberg and Woodford (1997). The

⁷CEE set $\frac{\gamma_2}{\gamma_1} = 0.01$; given the parameterization of γ_1 to be consistent with steady state utilization of unity, this implies $\gamma_2 = 0.000457$.

parameter σ is the elasticity between differentiated labor skills and is also set at 6 (e.g. Huang and Liu, 2002; Griffin, 1992).

The Calvo probabilities for wage and price non-reoptimization both take a value of 0.66. This implies an average waiting time between price and wage changes of 9 months. These are fairly standard values in the literature. The parameters of the Taylor rule include the smoothing parameter set at 0.8, the coefficient on inflation at 1.5, and the coefficient on output growth at 0.2. These are also fairly standard. The steady state gross inflation target is $\pi = 1$.

Our model explicitly allows for positive trend growth. Mapping the model to the data, the trend growth rate of the IST term, g_{ε^I} , equals the negative of the growth rate of the relative price of investment goods. To measure this in the data, we define investment as expenditures on new durables plus private fixed investment, and consumption as consumer expenditures of nondurables and services. These series are from the BEA and cover the period 1960:I-2007:III, to leave out the financial crisis.⁸ The relative price of investment is the ratio of the implied price index for investment goods to the price index for consumption goods. The average growth rate of the relative price from the period 1960:I-2007:III is -0.00472, so that $g_{\varepsilon^I} = 1.00472$. Real per capita GDP is computed by subtracting the log civilian non-institutionalized population from the log-level of real GDP. The average growth rate of the resulting output per capita series over the period is 0.005712, so that $g_Y = 1.005712$ or 2.28 percent a year. Given the calibrated growth of IST, we then use (31) to set $g_A^{1-\phi}$ to generate the appropriate average growth rate of output. This implies $g_A^{1-\phi} = 1.0022$ or a measured growth rate of TFP of about 1 percent per year.⁹

Our model differs from standard New Keynesian models with the joint addition of firms networking and borrowing to finance factor payments. The parameter ϕ measures the share of payments to intermediate inputs in total production. In the literature, this parameter is typically found to be in the range of 0.5 to 0.8. Using the fact that the weighted average revenue share of intermediate inputs in the U.S. private sector was about 50 percent in 2002, and knowing that the cost share of intermediate inputs equals the revenue share times the markup, Nakamura and Steinsson (2010) set $\phi = 0.7$. However, their calibration of θ implies a steady state price markup of 1.33, while ours corresponds to a steady state price markup of 1.2. We therefore set $\phi = 0.6$ as our benchmark. We consider the extended borrowing (EB) version of our model as a benchmark case, meaning we set $\psi_L = \psi_K = \psi_{\Gamma} = 1$. We assume that there is no backward indexation of prices or wages to lagged inflation, i.e. $\zeta_p = \zeta_w = 0$.

⁸A detailed explanation of how these data are constructed can be found in Ascari, Phaneuf, and Sims (2015).

⁹Note that this is a lower average growth rate of TFP than would obtain under traditional growth accounting exercises. This is due to the fact that our model includes firms networking, which would mean that a traditional growth accounting exercise ought to overstate the growth rate of true TFP.

3 Inflation and Output Dynamics to a Monetary Policy Shock

This section studies the transmission of monetary policy shocks in our baseline model and some alternative models. Woodford (2009) argues that studying conditional responses to monetary shocks is a particularly useful way of discriminating among alternative versions of the model; it should not be interpreted as claiming that such disturbances are a primary source of aggregate volatility. We focus on impulse responses to a monetary policy shock, and assess the roles that different model features play in generating the results. The model is solved via a first order linear approximation about the non-stochastic steady state.

3.1 Output and Inflation Dynamics

Figure 1 plots the model impulse responses of output and inflation to a twenty-five basis point expansionary monetary policy shock (i.e. a negative shock to the Taylor rule). The solid lines show the responses in our baseline model. For point of comparison, we also present impulse responses under three alternative specifications. The first two gauge the relative contributions of EB and FN in generating our main findings. The dotted lines show responses in which there is no extended borrowing (nor limited borrowing, so that none of the factors of production must be financed through working capital, i.e. $\psi_{\Gamma} = \psi_K = \psi_L = 0$). The dashed lines show the responses in which there is no firms networking (i.e $\phi = 0$). The third specification is one where there is no extended borrowing and no firms networking, but in which prices and wages are fully indexed to lagged inflation (i.e. $\zeta_p = \zeta_w = 1$); these responses are represented by dashed lines with "+" markers.

In our baseline model output rises by about 0.33 percent on impact of the shock. This jump is roughly one-half the magnitude of the peak output response, which is a little more than 0.6 percent and occurs about four quarters subsequent to the shock. The response of output is highly persistent, being positive more than five years after the shock; it also displays a pronounced humpshaped pattern. The response of inflation is nearly mute on impact of the shock, and reaches a peak after about four quarters. Like the output response, the inflation impulse response to the policy shock is very persistent. These responses are broadly consistent with results in the empirical literature on monetary policy shocks.

When there is no working capital at all (dotted lines), the impulse response of inflation is largest on impact, and exhibits no hump-shape. Thus, working capital is needed to generate a hump-shaped inflation response to a policy shock in our model. The response of output is also somewhat smaller compared to our baseline model. If instead there is no firms networking but all factors are financed via working capital (dashed lines), the response of inflation is positive on impact and hump-shaped, the peak response occurring roughly three quarters subsequent to the shock. The short-run response of inflation exceeds that in our baseline model and is also less persistent. The absence of firms networking implies that the slope of the New Keynesian Phillips Curve is steeper relative to our baseline model, making inflation more responsive to the monetary policy shock and lowering inflation persistence. As a result, the response of output is also significantly smaller and less persistent than in our baseline model.

The dashed lines with "+" markers take the "standard" New Keynesian model without EB and FN and modify it so that both prices and wages are fully indexed to one period lagged inflation (i.e. $\zeta_p = \zeta_w = 1$). Indexation has been advanced in the literature as a way to generate more inertia in the response of inflation to a policy shock. This specification does result in a hump-shaped response of inflation. But with indexation, the inflation response also reverts to zero from its peak more quickly than in our baseline model. In addition, the inclusion of backward indexation makes the output response to a policy shock significantly smaller and less persistent relative to our baseline model.

Table 2 presents some statistics summarizing the dynamics of inflation and output conditioned on monetary policy shocks. In our baseline model, the first order autocorrelation of inflation is 0.946. Inflation is highly persistent, with an autocorrelation coefficient at a one year lag of more than 0.5. The autocorrelation coefficients of inflation are higher (by 0.1 or more) at all lags in the base model relative to the version of the model with no working capital and no firms networking. The model without working capital and firms networking, but augmented with full backward indexation, produces a first order autocorrelation of inflation of 0.947, which is essentially the same as in the benchmark model, but the autocorrelations at lags of 2-5 quarters are higher in our baseline model than in the backward indexation model.

To measure the strength of internal propagation in models with nominal contracts, Chari, Kehoe, and McGrattan (2000) focus on the half-life of output, representing the number of quarters it takes for the response of output to equal one-half its impact response (rounded to the nearest integer). They provide evidence of a relatively small contract multiplier for output in a variety of DSGE models with intertemporal links.

In our baseline model, the half-life of output is 14 quarters, or three and a half years. Although price and wage setting are purely forward-looking, our model is therefore not prone to the Chari, Kehoe, and McGrattan (2000) criticism that models with nominal rigidities cannot generate a large contract multiplier for output. In the model with no working capital and no firms networking, the half-life of output is still substantial but half of a year shorter than in our baseline model at 12 quarters. Perhaps surprisingly, the half-life of output is significantly lower in the model with full backward indexation of prices and wages, with a half-life of only 7 quarters, half of what this multiplier is in our baseline model.

3.2 Intuition

Our model produces a large and persistent response of output to a policy shock and a hump-shaped, inertial response of inflation. The reasons why a model with backward indexation can produce a hump-shaped response of inflation to a monetary policy shock are well understood in the literature. But how does our model without wage and price indexation succeed in generating hump-shaped inflation dynamics? The two key model ingredients giving rise to this pattern are the combination of firms networking and a working capital channel.

To gain some intuition, note that our model generates a linearized price Phillips Curve expression that is identical to the textbook New Keynesian model:

$$\widehat{\pi}_t = \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p}\widehat{v}_t + \beta E_t\widehat{\pi}_{t+1},\tag{34}$$

where "hats" atop variables denote log deviations from steady state. Where our model differs relative to the textbook model is in the behavior of real marginal cost. Linearizing the expression for real marginal cost in our baseline model, (17), yields:

$$\widehat{v}_t = \widehat{i}_t + \alpha (1 - \phi) \widehat{r}_t^k + (1 - \alpha) (1 - \phi) \widehat{w}_t - \widehat{A}_t.$$
(35)

In contrast, in the more standard model without firms networking and extended borrowing, the expression for real marginal cost would be:

$$\widehat{v}_t = \alpha \widehat{r}_t^k + (1 - \alpha)\widehat{w}_t - \widehat{A}_t.$$
(36)

An expansionary policy shock requires higher factor prices to support higher output. Higher factor prices exert upward pressure on marginal cost and hence on inflation. This effect is weaker with firms networking (i.e. bigger ϕ), which has the effect of reducing the upward pressure on inflation from higher factor prices. Without an extended working capital channel, however, this would only serve to mute the inflation response and would not generate a hump-shape. The extended working capital channel affects inflation dynamics because the nominal interest rate becomes a direct argument in the expression for marginal cost. In the very short run, the exogenous cut in the interest rate works in the opposite direction from the upward pressure on factor prices. This limits upward pressure on real marginal cost and hence allows the inflation response to be very close to zero on impact. Because the cut in the interest rate is only temporary, as the interest rate begins to rise after impact, marginal cost begins to rise, which puts upward pressure on inflation and can result in a hump-shaped response pattern.

What happens if working capital serves to finance the costs of fewer inputs than in our baseline model? Figure 2 compares the responses of output, inflation, and real marginal cost (and its components – the real wage, the real rental price on capital, and the nominal interest rate) when working capital finances the costs of all inputs (solid lines), the cost of intermediate inputs only (dotted lines), and the cost of labor only (dashed lines). All versions of the models include firms networking. In our baseline model, real marginal cost actually falls on impact before rising. This occurs in spite of the fact that factor prices rise on impact. This decline in real marginal cost is driven by the influence of the nominal interest rate on marginal cost. When working capital applies to fewer factors of production, the impact decline in real marginal cost is much smaller. Correspondingly, the response of inflation is less hump-shaped.

When working capital is required to finance only payments for labor (LBW), the linearized expression for real marginal cost is:

$$\widehat{v}_t = (1 - \alpha)(1 - \phi)\widehat{i}_t + \alpha(1 - \phi)\widehat{r}_t^k + (1 - \alpha)(1 - \phi)\widehat{w}_t - \widehat{A}_t.$$
(37)

In the version of the model where working capital is used to finance intermediates only (LBI), the linearized expression for real marginal cost is:

$$\widehat{v}_t = \phi \widehat{i}_t + \alpha (1 - \phi) \widehat{r}_t^k + (1 - \alpha) (1 - \phi) \widehat{w}_t - \widehat{A}_t.$$
(38)

In our baseline parameterization, $\phi = 0.6$ and $\alpha = 1/3$. This means that the coefficient on \tilde{i}_t in the expression for marginal cost is 60 percent of the extended borrowing case when working capital is only needed to pay for intermediates and only about 25 percent of the extended borrowing case when working capital only applies to the wage bill. This is consistent with the patterns we observe in Figure 2, where the LBI model produces responses much closer to our baseline model than does the LBW model where working capital is only needed to finance payments to labor.

A final point we wish to mention in this section concerns the sensitivity of inflation to real marginal cost in our model. Given our calibration of parameters, the magnitude of the slope coefficient on real marginal cost in (34) is about 0.18, which is significantly higher than most empirical estimates (see, e.g., Galí and Gertler 1999 or Altig, Christiano, Eichenbaum, and Linde 2011, who report a sensitivity of inflation, or its quasi-difference, to labor's share of income of about 0.01-0.02). Empirically most papers use labor's share of income as a measure of real marginal

cost. In our model, labor's share does not correspond to real marginal cost. This is for three reasons: (i) a fixed cost of production, (ii) firms networking, as captured by the parameter ϕ , and (iii) extended borrowing, which makes the nominal interest rate a component of marginal cost. We conduct an exercise in which we project the quasi-difference of inflation on labor's share of income from a simulation of our model.¹⁰ We estimate a coefficient in such a regression that is about 0.01, which is in-line with the empirical estimates from Galí and Gertler (1999) and Altig, Christiano, Eichenbaum, and Linde (2011). Our model is therefore simultaneously consistent with a low sensitivity of inflation to labor's share of income, but a relatively high frequency of price re-adjustment.

3.3 Robustness

Our baseline model assumes that the entirety of factor payments must be financed via borrowing and that there is an important roundabout production structure. While we consider cases where these model features are turned off above, in this section we consider some robustness related to the values of key parameters in our model. The responses of output and inflation to a monetary policy shock for different cases are depicted in Figure 3.

As discussed above, firms networking reduces the sensitivity of real marginal cost to factor prices. In this way, one can think of firms networking as "flattening" the price Phillips Curve. Is the role of firms networking in the model isomorphic to having stickier prices? The answer turns out to be no. In the upper panel of Figure 3, we plot two sets of responses of output and inflation to a policy shock. The solid line considers our base model. The dashed line considers a version of the model in which firms networking is turned off (so that $\phi = 0$), while the Calvo price adjustment parameter is increased so that the sensitivity of inflation from the linearized Phillips Curve to the real wage is the same as in our benchmark model.¹¹ With a longer average duration between price adjustments and no firms networking, the output response to the policy shock is qualitatively similar, though smaller, compared to our baseline model. More substantive differences emerge when looking at the inflation response. In particular, with stickier prices and no firms networking, inflation rises by significantly more on impact and the hump-shaped response is much less apparent compared to our baseline model.

¹⁰We are grateful to an anonymous referee for asking us to consider the implications of our model for the sensitivity of inflation to labor's share of income.

¹¹In particular, in our baseline model, one can combine (34) with (35). The resulting coefficient on the real wage is $\frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p}(1-\phi)(1-\alpha)$. For our baseline parmaterization with $\xi_p = 0.66$ and $\phi = 0.6$, this coefficient is 0.0476. When we set $\phi = 0$, we need $\xi_p = 0.77$ to generate the same value of the coefficient.

Our baseline model assumes that the entirety of factor payments must be financed via borrowing. This is an admittedly strong assumption. The second panel of Figure 3 considers the case where there is only partial borrowing required for all inputs. In particular, we set $\psi_l = 0.5$ for $l = L, K, \Gamma$, so that one-half of the payments to each factor of production must be financed via borrowing. There is little difference in the output response relative to our baseline model. The inflation response remains hump-shaped and inertial, although inflation increases by more on impact when the extent of borrowing is smaller.

The third row of Figure 3 assumes a weaker degree of firms networking by setting $\phi = 0.3$ instead of its baseline value of 0.6. This change does not have much effect on the qualitative shapes of the output and inflation responses to a policy shock, though the output response is a bit smaller, and the inflation response a bit larger, at all horizons. This is entirely consistent with the intuition above that firms networking works to flatten the price Phillips curve.

The final row of Figure 3 considers robustness to our specification of the Taylor rule. Our Taylor rule assumes a moderate response to the growth rate of output. When we set this coefficient to zero, so that policy only reacts to deviations of inflation from target, the impulse responses are qualitatively the same as in our baseline model. The output response is (naturally) somewhat larger at most horizons, while the inflation response is somewhat smaller. There is no effect on the shapes of the responses. Interestingly, the impact response of inflation is slightly negative with no response to the output growth rate in the Taylor rule. In other words, our model generates a mild "price puzzle." We do not wish to take a strong stand on the empirical validity of the "price puzzle," but do wish to note that our model is capable of delivering one with reasonable parameter values. We obtain similar inflation responses when the Calvo price adjustment parameter is made smaller (so that price adjustment is more frequent) compared to our baseline assumption of $\xi_p = 0.66$.

4 Markup Dynamics

Having discussed the output and inflation dynamics in response to a monetary policy shock in our model, we now turn to a discussion of markup dynamics. The basic transmission mechanism by which positive demand shocks raise output in the textbook New Keynesian model is via a counter-cyclical price markup over marginal cost (e.g. Woodford, 2003, 2011). But the empirical evidence about the cyclicality of the price markup conditioned on demand shocks is mixed. Galí, Gertler, and López-Salido (2007) provide evidence of a rise in the price markup following a contractionary monetary policy shock, while Nekarda and Ramey (2013) offer evidence of a fall in that markup.

The evidence about the unconditional cyclicality of the price markup is also mixed. Galí, Gertler, and López-Salido (2007) present evidence of a price markup which is either weakly countercyclical or weakly procyclical unconditionally depending on alternative model specifications and measures. Nekarda and Ramey (2013) challenge these findings based on some evidence that points to a price markup which is moderately procyclical unconditionally. Bils, Klenow, and Malin (2016) report evidence of a countercyclical price markup. Therefore, whether the price markup is countercyclical or procyclical conditionally or unconditionally depends very much on the specific theory and methodology used by the authors. In contrast, the wage markup and the labor wedge are almost always found to be countercyclical.

We do not take a firm stand on the issue of the observed cyclicality of the price markup, but analyze instead the implications of different versions of our model for the conditional and unconditional cyclicality of markups and the labor wedge. We also use our model to point out some conceptual issues in the measurement of markups.

Figure 4 plots impulse responses of the price markup, wage markup, and labor wedge in response to an expansionary monetary policy shock in our model. The price markup is defined as the inverse of real marginal cost in our model. The wage markup is defined as the log difference between the real wage and the marginal rate of substitution between consumption and labor. The labor wedge is as defined in the literature as the log difference between the marginal product of labor and the marginal rate of substitution. In our baseline model, the price markup increases significantly on impact of a monetary policy shock and remains positive for several quarters thereafter. Therefore, going against the conventional wisdom concerning the cyclicality of markups in the New Keynesian model, our model generates expansionary effects of an exogenous cut in the interest rate with a conditionally procyclical price markup.

The response of the price markup to a monetary policy shock varies significantly across specifications of our model. In specifications without firms networking and borrowing to finance input costs, the markup is conditionally countercyclical. With firms networking and borrowing limited to either labor payments or intermediate payments, the price markup still increases on impact, albeit not as much as in our baseline model, before turning negative. In contrast, the responses of the wage markup and the labor wedge to the monetary policy shock are not very sensitive to model specifications. In all specifications, the wage markup and the labor wage both decrease significantly after an expansionary monetary policy shock.

As noted at the outset of this section, the empirical evidence on the cyclicality of the price markup is mixed. We wish to highlight an important measurement issue that arises in our model. Most of the empirical literature (see, e.g. Nekarda and Ramey 2013 and the references therein) measures the price markup as the log difference between a measure of the marginal product of labor and a measure of the aggregate real wage. From the perspective of our model, this is *not* the appropriate definition of the price markup. In our model, the price markup equals the ratio of the marginal product of labor to the product of the gross nominal interest rate and the real wage:

$$\mu_t^p = \frac{MPL_t}{(1+i_t)w_t} \tag{39}$$

Figure 5 plots the impulse response of the price markup in our baseline model to a monetary policy shock in a solid line. In the dashed line, we plot the impulse response of what one would measure as the price markup using a conventional definition of that markup (i.e. in omitting the $1 + i_t$ term in the denominator in (39)). Even though the true price markup in our model is conditionally procyclical, the conventional definition would yield a conditionally countercyclical price markup in response to an expansionary monetary policy shock.

Because the markup concept in our model does not coincide with traditional empirical counterparts, we cannot directly speak to this literature. We simply wish to note two things. First, contrary to the conventional wisdom, our model with extended borrowing does not rely upon a countercylical price markup as the transmission mechanism for a monetary policy shock. Second, to the extent to which borrowing to finance factor payments is an important feature of reality, empirical measures of price markups may be mis-specified. The notion that empirical measurement of markups is difficult is not new per se. For example, Basu and House (2016) stress that if empirical measures of remitted real wages do not correspond to allocative wages, then one may mis-measure the cyclicality of the price markup. We are raising a related point that the true cost of additional labor may include both a wage term but also the interest rate.

5 Effects of Other Shocks

We have thus far focused on the dynamic effects of monetary policy shocks in our model. Our model with extended borrowing and firms networking produces a large contract multiplier for output and an inertial and persistent inflation response. It does so in spite of the fact that we assume only moderate amounts of nominal rigidity and price and wage-setting that is purely forward-looking.

It would be disappointing if our model helped to capture the dynamic effects of monetary policy shocks but produced empirically dubious responses to other exogenous shocks. In this section we therefore study the responses to other shocks in our model and compare those responses both to different specifications of our model and to reduced form empirical evidence from the literature. In addition to the monetary policy shock, our model allows for three other exogenous shocks – a neutral productivity shock, a marginal efficiency of investment shock, and a shock to investmentspecific technology. We assume that the neutral and investment specific shocks follow random walks with drift in the log. We calibrate the autoregressive parameter in the MEI process to $\rho_I = 0.8$. This is based on the estimate in Justiniano, Primiceri, and Tambalotti (2011). Because the model is solved via a first order approximation, the magnitudes of shocks do not affect the shape of impulse responses. For that reason, we consider positive one percentage point shocks to each of these additional exogenous variables.¹²

Impulse responses are presented in Figure 6. The left panel plots output responses, while the right panel considers inflation. We consider the same four cases that we do in Figure 1, with responses in our base model shown in solid black lines. Focus first on a permanent shock to neutral productivity, displayed in the upper panel of the figure. Output increases on impact, but by less than measured TFP does. Output then grows for several quarters. There is virtually no difference between the impulse response in our baseline model compared to a version of the model with no extended borrowing or a version of the model with no firms networking. The version of the model without these two features but with complete price and wage indexation produces a qualitatively similar response, though there is a noticeable hump-shape. In our baseline model inflation declines immediately but returns to trend quickly. The responses of inflation in other variants of the model are similar, with the exception being the version with complete price and wage indexation, in which case the inflation response is noticeably hump-shaped and slower to return to trend.

The non-inertial response of inflation to a productivity shock is a noticeable success of our model. Altig, Christiano, Eichenbaum, and Linde (2011) estimate a VAR model and identify a permanent neutral productivity shock. The impulse responses of output and inflation in our model are very similar to the responses estimated in their VAR. They engage in an estimation exercise wherein parameters are chosen to minimize the distance between model impulse responses and those obtained from the VAR. In their VAR, inflation decreases immediately on impact and returns to trend very quickly. Their model is unable to match this, instead generating essentially no response of inflation to the permanent productivity shock at any horizon. Their DSGE model, like ours, is successful in generating an inertial response of inflation to a policy shock. But their model fails to generate a non-inertial inflation response to a productivity shock. Our model, in contrast, succeeds on both counts.

¹²A subtle point worth mentioning concerns the neutral productivity shock. The size of this shock is set to $0.01 \times (1-\phi)$, which, given the roundabout production structure, means that measured TFP, which is $A_t \Gamma_t^{\phi}$, increases by one percent on impact in response to a shock.

The second row of Figure 6 plots responses of output and inflation to a MEI shock. Our model produces a positive and hump-shaped response of output to this shock. Indeed, all versions of the model presented produce a qualitatively similar response, though the response is largest in the version of the model with no extended borrowing. Inflation increases on impact of the MEI shock in our model. Indeed, the impact response of inflation is larger than the response at any other horizon. In the version of the model with backward-indexation, in contrast, the inflation response is hump-shaped and persistent. We are aware of no VAR-based identifications of MEI shocks, which, differently than IST shocks, are not associated with the relative price of investment goods. Nevertheless, the inflation response to a MEI shock in our model is similar to the response in the estimated DSGE model of Justiniano, Primiceri, and Tambalotti (2011).

Finally, consider the response to a permanent investment-specific technology shock (IST). This is shown in the last row of Table 6. Output increases in response to the IST shock in our baseline model, though this increase is the smallest and most protracted in our model compared to the other variations of the model. This is potentially consistent with the estimation results in Justiniano, Primiceri, and Tambalotti (2011), who argue that IST shocks are not an important driver of output fluctuations in an unconditional sense, even though variations in the relative price of investment are large. Our model generates a small increase in inflation in response to the IST shock. The sign, magnitude, and shape of the response are consistent with the empirical inflation response to an IST shock identified in Altig, Christiano, Eichenbaum, and Linde (2011). In particular, their empirically identified inflation response to an IST shock is only very mildly hump-shaped, with the peak inflation response occurring one period subsequent to impact. The inflation response in our model is consistent with their empirical work. Other versions of our model (versions without extended borrowing or firms networking) produce more pronounced hump-shapes in the response of inflation to an IST shock in a way inconsistent with the empirical evidence. The version of our model with complete backward-indexation produces a very large hump-shape and has the wrong sign on impact relative to the empirical evidence.

In summary, our model produces output and inflation impulse responses to other exogenous shocks which are broadly consistent with available empirical and theoretical evidence. For the most part, the output responses to other shocks are not strongly affected by the key ingredients in our model relative to a more standard New Keynesian DSGE model. Importantly, our model produces non-inertial impulse responses of inflation to these other shocks. This is in spite of the fact that the inflation response to a monetary policy shock is very hump-shaped and quite persistent. Other versions of our model, or other models in the literature (e.g. Altig, Christiano, Eichenbaum, and Linde 2011) generally fail to simultaneously generate an inertial inflation response to a policy shock but non-inertial responses to non-policy shocks.

What is the intuition for these results about the inflation responses to shocks in our model? To understand the intuition, refer back to (35), which shows the linearized expression for real marginal cost in our model. In our model with extended borrowing the nominal interest rate is an explicit argument in the expression for real marginal cost. Conditional on an expansionary policy shock, the nominal interest rate declines while movements in factor prices put upward pressure on marginal cost. These effects roughly cancel out in the very short run, so that the marginal cost response (and hence also the inflation response) after a monetary policy shock is noticeably hump-shaped. Conditional on other shocks, in contrast, the nominal interest rate tends to move in the same direction as pressures on marginal cost. This is because the primary determinant of movements in the nominal rate in response to non-policy shocks is inflation. Shocks which would be deflationary (e.g. productivity shocks) push the nominal interest rate down, which only works to make the shock even more deflationary. Similarly, shocks which are inflationary (e.g. MEI or IST shocks) push the nominal interest rate up, which just reinforces the upward pressure on marginal cost and inflation from the shock.

6 Conclusion

This paper has built a medium-scale DSGE model allowing for firms networking in the form of a roundabout production structure and a working capital channel wherein firms must borrow to finance the full cost of factor payments. Conditional on a monetary policy shock, the model delivers a large contract multiplier for output and a persistent, hump-shaped, and inertial response of inflation. In our model, marginal cost depends explicitly on the nominal interest rate, and the transmission of the monetary policy shock does not rely upon a countercyclical price markup, which runs counter to the conventional wisdom from a textbook New Keynesian model.

Many papers include backward-indexation or rule of thumb price-setters into New Keynesian models to help account for the sluggish behavior of inflation. These features are theoretically unattractive and counterfactually imply that prices and wages adjust every quarter. Our analysis suggests that these features are not necessary to understand the inertial behavior of inflation conditional on a monetary policy shock. Our model with purely forward-looking price and wage setting does at least as well as a model with backward-indexation in accounting for inertial inflation dynamics, and does better at producing a large contract multiplier for output. We also study the behavior of key variables conditional on non-policy shocks. Our model generates impulse responses of output and inflation to productivity and investment shocks which are broadly consistent with evidence from the VAR literature. Importantly, our model generates non-inertial responses of inflation to non-policy shocks. A model relying on backward-indexation fails along this dimension, and other modeling attempts eschewing backward-indexation generally fail to simultaneously generate hump-shaped and inertial inflation responses to policy shocks but non-inertial responses to non-policy shocks.

References

- Altig, David, Lawrence Christiano, Martin Eichenbaum, and Jesper Linde. 2011. "Firm-Specific Capital, Nominal Rigidities, and the Business Cycle." *Review of Economic Dynamics* 14(2):225–247.
- Ascari, Guido, Louis Phaneuf, and Eric Sims. 2015. "On the Welfare and Cyclical Implications of Moderate Trend Inflation." NBER Working Paper No. 21392.
- Basu, Susanto. 1995. "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare." American Economic Review 85(3):512–531.
- Basu, Susanto and Christopher L. House. 2016. "Allocated and Remitted Wages: New Facts and Challenges for New Keynesian Models." In *Handbook of Macroeconomics Volume 2*, edited by John B. Taylor and Harald Uhlig. Amsterdam, North-Holland, 297–354.
- Basu, Susanto and Miles Kimball. 1997. "Cyclical Productivity with Unobserved Input Variation." NBER Working Paper 5915.
- Bils, Mark, Peter J Klenow, and Benjamin A Malin. 2016. "Resurrecting the Role of the Product Market Wedge in Recessions." Tech. rep.
- Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximising Framework." Journal of Monetary Economics 12:383–398.
- Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2000. "Sticky Prices Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica* 68 (5):1151–1180.
- ———. 2009. "New Keynesian Models: Not Yet Useful for Policy Analysis." *American Economic Journal: Macroeconomics* 1(1):242–266.
- Chowdhury, Ibrahim, Mathias Hoffmann, and Andreas Schabert. 2006. "Inflation Dynamics and the Cost Channel of Monetary Transmission." *European Economic Review* 50:995–1016.
- Christiano, Lawrence, Martin Eichenbaum, and Mathias Trabandt. 2015. "Understanding the Great Recession." American Economic Journal: Macroeconomics 7(1):110–167.
- ———. 2016. "Unemployment and Business Cycles." *Econometrica* 84(4):1523–1569.
- Christiano, Lawrence, Mathias Trabandt, and Karl Walentin. 2010. "DSGE Models for Monetary Policy Analysis." In *Handbook of Monetary Economics*, edited by Benjamin Friedman and Michael Woodford. Elsevier, 285–367.
- Christiano, Lawrence J. 2015. "Comment on" Networks and the Macroeconomy: An Empirical Exploration"." In *NBER Macroeconomics Annual 2015, Volume 30.* University of Chicago Press.
- Christiano, Lawrence J. and Martin Eichenbaum. 1992. "Liquidity Effects and the Monetary Transmission Mechanism." *American Economic Review* 82:346–353.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 1997. "Sticky Price and Limited Participation Models of Money: A Comparison." *European Economic Review* 41:1201– 1249.

——. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal* of *Political Economy* 113 (1):1–45.

- Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin. 2011. "DSGE Models for Monetary Policy Analysis." In *Handbook of Monetary Economics*, edited by Benjamin M. Friedman and Michael Woodford, chap. 7, Volume 3. San Diego CA: Elsevier, 285–367.
- Cogley, Timothy and Argia M. Sbordone. 2008. "Trend Inflation, Indexation and Inflation Persistence in the New Keynesian Phillips Curve." *American Economic Review* 98 (5):2101–2126.
- Coibion, Olivier and Yuriy Gorodnichenko. 2011. "Monetary Policy, Trend Inflation and the Great Moderation: An Alternative Interpretation." American Economic Review 101 (1):341–370.
- Dotsey, Michael and Robert G. King. 2006. "Pricing, Production, and Persistence." Journal of the European Economic Association 4(5):893–928.
- Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics* 46:281–313.
- Galí, Jordi and Mark Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." Journal of Monetary Economics 44:195–222.
- Galí, Jordi, Mark Gertler, and David López-Salido. 2007. "Markups, Gaps, and the Welfare Costs of Business Fluctuations." *Review of Economics and Statistics* 89:44–59.
- Gilchrist, Simon and Egon Zakrajsek. 2012. "Credit Spreads and Business Cycle Fluctuations." American Economic Review 102:1692–1720.
- Griffin, Peter. 1992. "The Impact of Affirmative Action on Labor: a Test of Some Implications of the Le Chatelier Principle." *Review of Economics and Statistics* 74:251–260.
- Huang, Kevin X.D. and Zheng Liu. 2002. "Staggered Contracts and Business Cycle Persistence." Journal of Monetary Economics 49:405–433.
- Huang, Kevin X.D., Zheng Liu, and Louis Phaneuf. 2004. "Why Does the Cyclical Behavior of Real Wages Change Over Time?" American Economic Review 94(4):836–856.
- Justiniano, Alejandro, Giorgio Primiceri, and Andrea Tambalotti. 2010. "Investment Shocks and Business Cycles." Journal of Monetary Economics 57(2):132–145.
- ——. 2011. "Investment Shocks and the Relative Price of Investment." *Review of Economic Dynamics* 14(1):101–121.
- Nakamura, Emi and Jon Steinsson. 2010. "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model." Quarterly Journal of Economics 125 (3):961–1013.
- Nekarda, Christopher and Valerie Ramey. 2013. "The Cyclical Behavior of the Price-Cost Markup." NBER Working Paper No. 19099.
- Ravenna, Federico and Carl Walsh. 2006. "Optimal Monetary Policy with the Cost Channel." Journal of Monetary Economics 53:199–216.
- Rotemberg, Julio J. and Michael Woodford. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." In NBER Macroeconomics Annual 1997, edited by Julio J. Rotemberg and B. S. Bernanke. Cambridge, MA, The MIT Press, 297–346.

- Smets, Frank and Raf Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." American Economic Review 97 (3):586–606.
- Tillmann, Peter. 2008. "Do Interest Rates Drive Inflation Dynamics? An Analysis of the Cost Channel of Monetary Transmission." Journal of Economic Dynamics and Control 32:2723–2744.
- Woodford, Michael. 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton: Princeton University Press.
- ——. 2007. "Interpreting Inflation Persistence: Comments on the Conference 'Quantitative Evidence on Price Determination'." Journal of Money, Credit, and Banking 39(2):203–210.
- ———. 2009. "Convergence in Macroeconomics: Elements of the New Synthesis." *American Economic Journal: Macroeconomics* 1(1):267–279.
- ——. 2011. "Simple Analytics of the Government Expenditure Multiplier." American Economic Journal: Macroeconomics 3(1):1–35.

Parameter	Value	Description			
β	0.99	Discount factor			
b	0.8	Internal habit formation			
η	6	Labor disutility			
χ	1	Frisch elasticity			
κ	3	Investment adjustment cost			
δ	0.025	Depreciation rate			
γ_1	$Z^{*} = 1$	Utilization adjustment cost linear term			
γ_2	0.05	Utilization adjustment cost squared term			
ξ_p	0.66	Calvo price			
ξ_w	0.66	Calvo wage			
ζ_p	0	Price indexation			
ζ_w	0	Wage indexation			
θ	6	Elasticity of substitution: goods			
σ	6	Elasticity of substitution: labor			
ϕ	0.6	Intermediate share			
α	1/3	Capital share			
ψ_L	1	Fraction of labor financed			
ψ_K	1	Fraction of capital financed			
ψ_{Γ}	1	Fraction of intermediates financed			
ρ_i	0.8	Taylor rule smoothing			
α_{π}	1.5	Taylor rule inflation			
α_y	0.2	Taylor rule output growth			
π	1	Steady State Gross Inflation			
g_{ε^I}	1.0047	Gross Growth of IST			
g_A	$1.0022^{1-\phi}$	Gross Growth of Neutral Productivity			

Table 1: Parameter Values

Note: This table shows the values of the parameters used in quantitative analysis of the model. A description of each parameter is provided in the right column. The parameter on the linear term in the utilization adjustment cost function, γ_1 , is chosen to be consistent with a steady state normalization of utilization to 1. Given other parameters this implies a value $\gamma_1 = 0.0457$. The fixed cost of production, F, is chosen so that profits equal zero in the non-stochastic steady state. Given other parameters, this implies a value of F = 0.0183.

		Inflation autocorrelation				
		Lag				
	Output half-life	1	2	3	4	5
Base model	14	0.9457	0.8269	0.6799	0.5287	0.3874
No EB, no FN	12	0.8220	0.6437	0.4839	0.3499	0.2426
No EB, no FN, full indexation	7	0.9473	0.8187	0.6476	0.4641	0.2914

=

Table 2: Output and Inflation Dynamics

Note: This table shows some statistics from different versions of the model. The column labeled "Output half-life" shows the half-life of output in response to a monetary policy shock, which we define as the number of quarters (rounded to the nearest integer) after which the impulse response of output is one-half its impact response. The remaining columns show autocorrelations of inflation at different lags. The row labeled "No EB, no FN" refers to a version of the model with no extended borrowing and no firms networking. The remaining row augments this case to consider full indexation of prices and wages to lagged inflation.





Note: This figure plots the impulse responses of output and inflation to a monetary policy shock. The solid lines show the responses in the baseline calibrated model. The dashed lines show responses when there is no firms networking ("No FN"). The dotted lines show responses when there is no extended borrowing ("No EB"). The dashed lines with "+" show responses when there is no firms networking and no extended borrowing, but prices and wages are fully indexed to the lagged inflation rate ("No FN, No EB, Full Backward Indexation").



Figure 2: Output, Inflation, and Real Marginal Cost Responses to a Monetary Policy Shock

Note: This figure plots the impulse responses of output, inflation, and real marginal cost (and its components) to a monetary policy shock. The solid lines show the responses in the baseline calibrated model. The dotted lines show responses when there is firms networking and limited borrowing with working capital financing only intermediate goods ("FN, LBI"). The dashed lines show responses when there is firms networking and limited borrowing with working capital financing only wages ("FN, LBW").



Note: This figure plots impulse responses of output and inflation to a monetary policy shock under different assumptions about parameter values. These assumptions are described in the left axes and in the text. Solid lines plot responses in our baseline calibrated model, while dashed lines show responses for an alternative parameterization of the model.



Figure 4: The Price Markup, the Wage Markup, and the Labor Wedge Responses to a Monetary Policy Shock

Note: This figure plots the impulse responses of the price and wage markups and the labor wedge to a monetary policy shock. The solid lines show the responses in the baseline calibrated model. The dotted lines show responses when there is firms networking and limited borrowing with working capital financing only intermediate goods ("FN, LBI"). The dashed lines show responses when there is firms networking and limited borrowing with "+" markers shows responses in a version of the model with no firms networking, no extended borrowing, and full price and wage indexation. The dashed-dotted lines represent responses in the same version of the model but without indexation.

Figure 5: The Price Markup and Conventionally Measured Price Markup Response to a Monetary Policy Shock



Note: This figure plots the price markup response to a monetary policy shock in our baseline model (solid line). The dashed line shows the response when the markup is measured incorrectly as the log difference between the marginal product of labor and the real wage.



Figure 6: Output and Inflation Responses to Other Shocks

Note: This figure plots the impulse responses of output and inflation to three different exogenous shocks – permanent neutral productivity, marginal efficiency of investment, and permanent investment-specific technology shocks. Rows correspond to different shocks. The solid lines show the responses in the baseline calibrated model. The dashed lines show responses when there is no firms networking ("No FN"). The dotted lines show responses when there is no firms networking and no extended borrowing ("No EB"). The dashed lines with "+" show responses when there is no firms networking and no extended borrowing, but prices and wages are fully indexed to the lagged inflation rate ("No FN, No EB, Full Backward Indexation").

A Full Set of Equilibrium Conditions

This appendix lists the full set of equilibrium conditions. These conditions are expressed in stationary transformations of variables, e.g. $\tilde{X}_t = \frac{X_t}{\Upsilon_t}$ for most variables.

$$\widetilde{\lambda}_{t}^{r} = \frac{1}{\widetilde{C}_{t} - bg_{\Upsilon,t}^{-1}\widetilde{C}_{t-1}} - E_{t}\frac{\beta b}{g_{\Upsilon,t}\widetilde{C}_{t+1} - b\widetilde{C}_{t}}$$
(A1)

$$\widetilde{r}_t^k = \gamma_1 + \gamma_2(Z_t - 1) \tag{A2}$$

$$\widetilde{\lambda}_{t}^{r} = \widetilde{\mu}_{t}\vartheta_{t} \left(1 - \frac{k}{2} \left(\frac{\widetilde{I}_{t}}{\widetilde{I}_{t-1}}g_{\Upsilon,t} - g_{\Upsilon}\right)^{2} - \kappa \left(\frac{\widetilde{I}_{t}}{\widetilde{I}_{t-1}}g_{\Upsilon,t} - g_{\Upsilon}\right)\frac{\widetilde{I}_{t}}{\widetilde{I}_{t-1}}g_{\Upsilon}\right) + \dots \\ \beta E_{t}g_{\Upsilon}^{-1}\widetilde{\mu}_{t+1}\vartheta_{t+1}\kappa \left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_{t}}g_{\Upsilon,t} - g_{\Upsilon}\right)\left(\frac{\widetilde{I}_{t+1}}{\widetilde{I}_{t}}g_{\Upsilon,t}\right)^{2}$$
(A3)

$$g_{\varepsilon^{I},t}g_{\Upsilon,t}\widetilde{\mu}_{t} = \beta E_{t}\widetilde{\lambda}_{t+1}^{r} \left(\widetilde{r}_{t+1}^{k}Z_{t+1} - \left(\gamma_{1}(Z_{t+1}-1) + \frac{\gamma_{2}}{2}(Z_{t+1}-1)^{2}\right)\right) + \beta(1-\delta)E_{t}\widetilde{\mu}_{t+1}$$
(A4)

$$\widetilde{\lambda}_t^r = \beta g_{\Upsilon,t}^{-1} E_t (1+i_t) \pi_{t+1}^{-1} \widetilde{\lambda}_{t+1}^r$$
(A5)

$$\widetilde{w}_t^* = \frac{\sigma}{\sigma - 1} \frac{f_{1,t}}{\widetilde{f}_{2,t}} \tag{A6}$$

$$\widetilde{f}_{1,t} = \eta \left(\frac{\widetilde{w}_t}{\widetilde{w}_t^*}\right)^{\sigma(1+\chi)} L_t^{1+\chi} + \beta \xi_w E_t(\pi_{t+1})^{\sigma(1+\chi)} \left(\frac{\widetilde{w}_{t+1}^*}{\widetilde{w}_t^*}\right)^{\sigma(1+\chi)} g_{\Upsilon,t}^{\sigma(1+\chi)} \widetilde{f}_{1,t+1}$$
(A7)

$$\widetilde{f}_{2,t} = \widetilde{\lambda}_t^r \left(\frac{\widetilde{w}_t}{\widetilde{w}_t^*}\right)^\sigma L_t + \beta \xi_w E_t (\pi_{t+1})^{\sigma-1} \left(\frac{\widetilde{w}_{t+1}^*}{\widetilde{w}_t^*}\right)^\sigma g_{\Upsilon,t}^{\sigma-1} \widetilde{f}_{2,t+1}$$
(A8)

$$\widetilde{\widehat{K}}_{t} = g_{\varepsilon^{I},t} g_{\Upsilon,t} \alpha (1-\phi) \frac{mc_{t}}{\Psi_{K,t} \widetilde{r}_{t}^{k}} \left(s_{t} \widetilde{X}_{t} + F \right)$$
(A9)

$$L_t = (1 - \alpha)(1 - \phi)\frac{mc_t}{\Psi_L \widetilde{w}_t} \left(s_t \widetilde{X}_t + F\right)$$
(A10)

$$\widetilde{\Gamma}_t = \phi m c_t \Psi_{\Gamma,t}^{-1} \left(s_t \widetilde{X}_t + F \right)$$
(A11)

$$p_t^* = \frac{\theta}{\theta - 1} \frac{x_t^1}{x_t^2} \tag{A12}$$

$$x_t^1 = \tilde{\lambda}_t^r m c_t \tilde{X}_t + \xi_p \beta \left(\frac{1}{\pi_{t+1}}\right)^{-\theta} x_{t+1}^1 \tag{A13}$$

$$x_t^2 = \tilde{\lambda}_t^r \tilde{X}_t + \xi_p \beta \left(\frac{1}{\pi_{t+1}}\right)^{1-\theta} x_{t+1}^2$$
(A14)

$$1 = \xi_p \left(\frac{1}{\pi_t}\right)^{1-\theta} + (1-\xi_p) p_t^{*1-\theta}$$
 (A15)

$$\widetilde{w}_t^{1-\sigma} = \xi_w g_{\Upsilon,t}^{\sigma-1} \left(\frac{\widetilde{w}_{t-1}}{\pi_t}\right)^{1-\sigma} + (1-\xi_w) \widetilde{w}_t^{*1-\sigma} \tag{A16}$$

$$\widetilde{Y}_t = \widetilde{X}_t - \widetilde{\Gamma}_t \tag{A17}$$

$$s_t \widetilde{X}_t = \widetilde{\Gamma}_t^{\phi} \widetilde{\widehat{K}}_t^{\alpha(1-\phi)} L_t^{(1-\alpha)(1-\phi)} g_{\Upsilon,t}^{\alpha(\phi-1)} g_{\varepsilon^I,t}^{\alpha(\phi-1)} - F$$
(A18)

$$\widetilde{Y}_t = \widetilde{C}_t + \widetilde{I}_t + g_{\Upsilon,t}^{-1} g_{\varepsilon^I,t}^{-1} \left(\gamma_1 (Z_t - 1) + \frac{\gamma_2}{2} (Z_t - 1)^2 \right) \widetilde{K}_t$$
(A19)

$$\widetilde{K}_{t+1} = \vartheta_t \left(1 - \frac{\kappa}{2} \left(\frac{\widetilde{I}_t}{\widetilde{I}_{t-1}} g_{\Upsilon,t} - g_{\Upsilon} \right)^2 \right) \widetilde{I}_t + (1 - \delta) g_{\Upsilon,t}^{-1} g_{\varepsilon^I,t}^{-1} \widetilde{K}_t$$
(A20)

$$\frac{1+i_t}{1+i} = \left(\left(\frac{\pi_t}{\pi}\right)^{\alpha_{\pi}} \left(\frac{\widetilde{Y}_t}{\widetilde{Y}_{t-1}} g_{\Upsilon,t}\right)^{\alpha_y} \right)^{1-\rho_i} \left(\frac{1+i_{t-1}}{1+i}\right)^{\rho_i} \exp\left(\varepsilon_t^r\right)$$
(A21)

$$\widehat{K}_t = Z_t \widetilde{K}_t \tag{A22}$$

$$s_t = (1 - \xi_p) p_t^{*-\theta} + \xi_p \left(\frac{1}{\pi_t}\right)^{-\theta} s_{t-1}$$
(A23)

$$\vartheta_t = \left(\vartheta_{t-1}\right)^{\rho_\vartheta} \exp\left(s_\vartheta u_t^\vartheta\right) \tag{A24}$$

$$g_{A,t} = g_A \exp\left(s_A u_t^A\right) \tag{A25}$$

$$g_{\varepsilon^{I},t} = g_{\varepsilon^{I}} \exp\left(s_{\varepsilon^{I}} u_{t}^{\varepsilon^{I}}\right) \tag{A26}$$

Equation (A1) defines the real multiplier on the flow budget constraint. (A2) is the optimality condition for capital utilization. (A3) and (A4) are the optimality conditions for the household choice of investment and next period's stock of capital, respectively. The Euler equation for bonds is given by (A5). (A6)-(A8) describe optimal wage setting for households given the opportunity to adjust their wages. Optimal factor demands are given by equations (A9)-(A11). Optimal price setting for firms given the opportunity to change their price is described by equations (A12)-(A14). The evolutions of aggregate inflation and the aggregate real wage index are given by (A15) and (A16), respectively. Net output is gross output minus intermediates, as given by (A17). The aggregate production function for gross output is (A18). The aggregate resource constraint is (A19), and the law of motion for physical capital is given by (A20). The Taylor rule for monetary policy is (A21). Capital services are defined as the product of utilization and physical capital, as in (A22). The law of motion for price dispersion is (A23). (A24)-(A26) give the assumed laws of motion for other exogenous variables.