

Online Appendix to: “The Output and Welfare Effects of Government Spending Shocks over the Business Cycle

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This appendix contains additional supporting material to accompany “The Output and Welfare Effects of Government Spending Shocks over the Business Cycle” by Eric Sims and Jonathan Wolff.

A Equilibrium Conditions of the Medium Scale DSGE Model

This Appendix lists the full set of equilibrium conditions for the model of Section 3.

A.1 Household Optimality Conditions

The optimality conditions for the household problem described in Subsection 3.3.1 are:

$$(A.1) \quad (1 + \tau_t^C)\lambda_t = v_t \frac{1}{\widehat{C}_t} \phi_G(C_t - bC_{t-1})^{-\frac{1}{\nu}} - \beta b E_t v_{t+1} \frac{1}{\widehat{C}_{t+1}} \phi_G(C_{t+1} - bC_t)^{-\frac{1}{\nu}}$$

$$(A.2) \quad \widehat{C}_t = \phi_G(C_t - bC_{t-1})^{\frac{\nu-1}{\nu}} + (1 - \phi_G) G_{C,t}^{\frac{\nu-1}{\nu}}$$

$$(A.3) \quad (1 - \tau_t^K)\lambda_t R_t = \mu_t (\delta_1 + \delta_2(u_t - 1))$$

$$(A.4) \quad \lambda_t = \mu_t Z_t \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t \mu_{t+1} Z_{t+1} \kappa \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

$$(A.5) \quad \mu_t = \beta E_t \lambda_{t+1} (1 - \tau_{t+1}^K) R_{t+1} u_{t+1} + \beta E_t \mu_{t+1} \left(1 - \delta_0 - \delta_1 (u_{t+1} - 1) - \frac{\delta_2}{2} (u_{t+1} - 1)^2 \right)$$

$$(A.6) \quad \lambda_t = \beta (1 + i_t) E_t \lambda_{t+1} (1 + \pi_{t+1})^{-1}$$

$$(A.7) \quad w_t^\# = \frac{\epsilon_w}{\epsilon_w - 1} \frac{F_{1,t}}{F_{2,t}}$$

$$(A.8) \quad F_{1,t} = v_t \xi_t \left(\frac{w_t^\#}{w_t} \right)^{-\epsilon_w (1+\chi)} N_t^{1+\chi} + \beta \theta_w E_t \left(\frac{w_t^\#}{w_{t+1}^\#} \frac{(1 + \pi_t)^{\zeta_w}}{1 + \pi_{t+1}} \right)^{-\epsilon_w (1+\chi)} F_{1,t+1}$$

$$(A.9) \quad F_{2,t} = \lambda_t (1 - \tau_t^N) \left(\frac{w_t^\#}{w_t} \right)^{-\epsilon_w} N_t + \beta \theta_w E_t \left(\frac{w_t^\#}{w_{t+1}^\#} \right)^{-\epsilon_w} \left(\frac{(1 + \pi_t)^{\zeta_w}}{1 + \pi_{t+1}} \right)^{1-\epsilon_w} F_{2,t+1}$$

$$(A.10) \quad K_{t+1} = Z_t \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t + \left(1 - \delta_0 - \delta_1 (u_t - 1) - \frac{\delta_2}{2} (u_t - 1)^2 \right) K_t$$

λ_t is the Lagrange multiplier on the flow budget constraint, (4), and μ_t is the multiplier on the capital accumulation equation, (5). (A.1) defines λ_t in terms of the marginal utility of consumption. Composite consumption, \widehat{C}_t , is defined in (A.2). The first order condition for capital utilization is given by (A.3). (A.4) is the optimality condition for the choice of investment, and (A.5) is the optimality condition for the choice of next period's capital stock. The Euler equation for bonds is given by (A.6). (A.7)-(A.8) characterize optimal wage-setting for updating households. The optimal reset wage, $w_t^\#$, is common to all updating households. $F_{1,t}$ and $F_{2,t}$ are auxiliary variables. The accumulation equation for physical capital is given by (A.10).

A.2 Firm Optimality Conditions

The optimality conditions for the firm problem described in Subsection 3.1.2 are:

$$(A.11) \quad w_t = mc_t(1 - \alpha)A_t K_{G,t}^\varphi \left(\frac{\widehat{K}_t}{N_t} \right)^\alpha$$

$$(A.12) \quad R_t = mc_t \alpha A_t K_{G,t}^\varphi \left(\frac{\widehat{K}_t}{N_t} \right)^{\alpha-1}$$

$$(A.13) \quad 1 + \pi_t^\# = \frac{\epsilon_p}{\epsilon_p - 1} (1 + \pi_t) \frac{X_{1,t}}{X_{2,t}}$$

$$(A.14) \quad X_{1,t} = \lambda_t mc_t Y_t + \theta_p \beta E_t (1 + \pi_t)^{-\zeta_p \epsilon_p} (1 + \pi_{t+1})^{\epsilon_p} X_{1,t+1}$$

$$(A.15) \quad X_{2,t} = \lambda_t Y_t + \theta_p \beta E_t (1 + \pi_t)^{\zeta_p (1 - \epsilon_p)} (1 + \pi_{t+1})^{\epsilon_p - 1} X_{2,t+1}$$

Real marginal cost is denoted by mc_t . It is common across all firms, as is the ratio of capital services to labor. (A.11) implies defines a demand curve for labor and (A.12) implicitly defines a demand curve for capital services. Optimal pricing for updating firms is described in (A.13)-(A.15). $1 + \pi_t^\# = \frac{P_t^\#}{P_{t-1}^\#}$ is reset price inflation. $X_{1,t}$ and $X_{2,t}$ are auxiliary variables.

A.3 Government

The equations below describe the behavior of both the fiscal and monetary authorities in the model:

$$(A.16) \quad G_{C,t} + G_{I,t} + i_{t-1}(1 + \pi_t)^{-1} b_{g,t} \leq \tau_t^C C_t + \tau_t^N w_t N_t + \tau_t^K R_t \widehat{K}_t + T_t + b_{g,t+1} - b_{g,t}(1 + \pi_t)^{-1}$$

$$(A.17) \quad K_{G,t+1} = G_{I,t} + (1 - \delta_G)K_{G,t}$$

$$(A.18) \quad \ln G_{C,t} = (1 - \rho_{G_C}) \ln G_C + \rho_{G_C} \ln G_{C,t-1} + s_{G_C} \varepsilon_{G_C,t}$$

$$(A.19) \quad \ln G_{I,t} = (1 - \rho_{G_I}) \ln G_I + \rho_{G_I} \ln G_{I,t-1} + s_{G_I} \varepsilon_{G_I,t}$$

$$(A.20) \quad \tau_t^C = (1 - \rho_C) \tau^C + \rho_C \tau_{t-1}^C + (1 - \rho_C) \gamma_C \left(\frac{B_{G,t}}{Y_t} - \frac{B_G}{Y} \right)$$

$$(A.21) \quad \tau_t^N = (1 - \rho_N) \tau^N + \rho_N \tau_{t-1}^N + (1 - \rho_N) \gamma_N \left(\frac{B_{G,t}}{Y_t} - \frac{B_G}{Y} \right)$$

$$(A.22) \quad \tau_t^K = (1 - \rho_K) \tau^K + \rho_K \tau_{t-1}^K + (1 - \rho_K) \gamma_K \left(\frac{B_{G,t}}{Y_t} - \frac{B_G}{Y} \right)$$

$$(A.23) \quad T_t = (1 - \rho_T) T + \rho_T T_{t-1} + (1 - \rho_T) \gamma_T \left(\frac{B_{G,t}}{Y_t} - \frac{B_G}{Y} \right)$$

$$(A.24) \quad i_t = (1 - \rho_i) i + \rho_i i_{t-1} + (1 - \rho_i) [\phi_\pi \pi_t + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_i \varepsilon_{i,t}$$

(A.16) is the government's flow budget constraint. Government capital accumulates according to (A.17). (A.18)-(A.19) describe the exogenous stochastic processes for government consumption and investment. (A.20)-(A.23) are processes for the different tax instruments. Monetary policy is characterized by (A.24).

A.4 Exogenous Processes

Other exogenous processes in the model are given by:

$$(A.25) \quad \ln A_t = (1 - \rho_A) \ln A + \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t}$$

$$(A.26) \quad \ln Z_t = \rho_Z \ln Z_{t-1} + s_Z \varepsilon_{Z,t}$$

$$(A.27) \quad \ln v_t = \rho_v \ln v_{t-1} + s_v \varepsilon_{v,t}$$

$$(A.28) \quad \ln \xi_t = (1 - \rho_\xi) \ln \xi + \rho_\xi \ln \xi_{t-1} + s_\xi \varepsilon_{\xi,t}$$

A.5 Aggregate Conditions

$$(A.29) \quad Y_t = C_t + I_t + G_{C,t} + G_{I,t}$$

$$(A.30) \quad v_t^p Y_t = A_t K_{G,t}^\varphi \widehat{K}_t^\alpha N_t^{1-\alpha} - F$$

$$(A.31) \quad v_t^p = (1 + \pi_t)^{\varepsilon_p} \left[(1 - \theta_p)(1 + \pi_t^\#)^{-\varepsilon_p} + \theta_p(1 + \pi_{t-1})^{-\zeta_p \varepsilon_p} v_{t-1}^p \right]$$

$$(A.32) \quad \widehat{K}_t = u_t K_t$$

$$(A.33) \quad (1 + \pi_t)^{1-\varepsilon_p} = (1 - \theta_p)(1 + \pi_t^\#)^{1-\varepsilon_p} + \theta_p(1 + \pi_{t-1})^{\zeta_p(1-\varepsilon_p)}$$

$$(A.34) \quad w_t^{1-\varepsilon_w} = (1 - \theta_w) w_t^{\#, 1-\varepsilon_w} + \theta_w \left(\frac{(1 + \pi_{t-1})^{\zeta_w}}{1 + \pi_t} w_{t-1} \right)^{1-\varepsilon_w}$$

A.6 Equilibrium

Expressions (A.1)-(A.34) comprise thirty-four equations in thirty-four variables: $\{C_t, I_t, Y_t, G_{C,t}, G_{I,t}, K_{G,t}, K_t, u_t, \widehat{K}_t, N_t, B_{G,t}, \tau_t^C, \tau_t^N, \tau_t^K, T_t, \widehat{C}_t, \lambda_t, \mu_t, i_t, \pi_t, \pi_t^\#, R_t, w_t, w_t^\#, mc_t, X_{1,t}, X_{2,t}, F_{1,t}, F_{2,t}, A_t, Z_t, v_t, \xi_t\}$.

The model features six stochastic shocks – $\{\varepsilon_{G_C,t}, \varepsilon_{G_I,t}, \varepsilon_{A,t}, \varepsilon_{Z,t}, \varepsilon_{v,t}, \varepsilon_{\xi,t}\}$.

B Measuring Welfare in the Medium Scale DSGE Model

We define aggregate welfare in the model of Section 3 as the equally weighted sum of welfare across households. Let $V_t(h)$ be the welfare of household h . Welfare is the presented discounted value of flow utility, which can be written recursively:

$$(B.1) \quad V_t(h) = v_t \left\{ \frac{\nu}{\nu-1} \ln \widehat{C}_t - \xi_t \frac{N_t(h)^{1+\chi}}{1+\chi} \right\} + \beta E_t V_{t+1}(h)$$

Aggregate welfare, \mathbb{W}_t , is defined as:

$$(B.2) \quad \mathbb{W}_t = \int_0^1 V_t(h) dh$$

Since households are identical along all non-labor market margins, combining (B.1) with (B.2) yields:

$$(B.3) \quad \mathbb{W}_t = v_t \frac{\nu}{\nu-1} \ln \widehat{C}_t - v_t \xi_t \int_0^1 \frac{N_t(h)^{1+\chi}}{1+\chi} dh + \beta E_t \mathbb{W}_{t+1}$$

(B.3) can be written as:

$$(B.4) \quad \mathbb{W}_t = v_t \frac{\nu}{\nu-1} \ln \widehat{C}_t - v_t \xi_t \frac{N_t^{1+\chi}}{1+\chi} \int_0^1 \left(\frac{w_t(h)}{w_t} \right)^{-\epsilon_w(1+\chi)} dh + \beta E_t \mathbb{W}_{t+1}$$

Define $v_t^w = \int_0^1 \left(\frac{w_t(h)}{w_t} \right)^{-\epsilon_w(1+\chi)} dh$. Using properties of Calvo (1983) wage-setting, this can be

written without reference to h as:

$$(B.5) \quad v_t^w = (1 - \theta_w) \left(\frac{w_t^\#}{w_t} \right)^{-\epsilon_w(1+\chi)} + \theta_w \left(\frac{w_t}{w_{t-1}} \frac{1 + \pi_t}{(1 + \pi_{t-1})^{\zeta_w}} \right)^{\epsilon_w(1+\chi)} v_{t-1}^w$$

Hence, aggregate welfare can be written:

$$(B.6) \quad \mathbb{W}_t = v_t \frac{\nu}{\nu - 1} \ln \widehat{C}_t - v_t \xi_t v_t^w \frac{N_t^{1+\chi}}{1 + \chi} + \beta E_t \mathbb{W}_{t+1}$$

For the construction of the welfare multiplier, we simply include (B.5) and (B.6) as equilibrium conditions in the model.

C Separately Identifying ϕ_G and ν

We experimented with several different specifications in which we sought to jointly estimate the parameters ϕ_G and ν . We also considered several different fixed values of ϕ_G , and re-estimated the model (including ν). Our analysis suggests that these parameters cannot be jointly identified. Accordingly, as a baseline we set $\phi_G = 0.8$ as in [Bouakez and Rebei \(2007\)](#). These authors also report that they cannot jointly identify ϕ_G and ν .

In what follows, we provide some intuition for the non-identification of these parameters jointly. For simplicity, assume that there is no internal habit formation (i.e. $b = 0$). In log deviations, the Lagrange multiplier on the flow budget constraint facing a household can be written:

$$(C.1) \quad \widetilde{\lambda}_t = -\widehat{c}_t - \frac{1}{\nu} c_t$$

Here $\widetilde{\lambda}_t$ is the log-deviation of λ_t from steady state, \widehat{c}_t is the log-deviation of \widehat{C}_t from steady state, and c_t is the log-deviation of C_t from steady state. Defining $\bar{C}_t = \widehat{C}_t^{\frac{\nu}{\nu-1}}$, \widehat{c}_t can be written:

$$(C.2) \quad \widehat{c}_t = \frac{\nu-1}{\nu} \phi_G \left(\frac{C}{\bar{C}} \right)^{\frac{\nu-1}{\nu}} c_t + \frac{\nu-1}{\nu} (1-\phi_G) \left(\frac{G_C}{\bar{C}} \right)^{\frac{\nu-1}{\nu}} g_{C,t}$$

Here $g_{C,t}$ denotes the log-deviation of $G_{C,t}$ from its steady state, and variables without a time subscript are steady state values. Combining (C.2) with (C.1) yields:

$$(C.3) \quad \widetilde{\lambda}_t = -\phi_G \left(\frac{C}{\bar{C}} \right)^{\frac{\nu-1}{\nu}} c_t - \frac{\nu-1}{\nu} (1-\phi_G) \left(\frac{G_C}{\bar{C}} \right)^{\frac{\nu-1}{\nu}} g_{C,t}$$

In the conventional case of additively separability (i.e. $\nu = 1$), $\widetilde{\lambda}_t$ depends only on c_t . In the more general case, $\widetilde{\lambda}_t$ depends on both c_t and $g_{C,t}$. Holding G_C and \bar{C} fixed, the elasticity of the Lagrange multiplier on the budget constraint with respect to government spending is given by $-\frac{\nu-1}{\nu}(1-\phi_G)$. What is relevant for the equilibrium dynamics of variables like consumption and output is this elasticity, not the individual parameters ν and ϕ_G . Values of $\nu < 1$ imply that increases in government spending raise the marginal utility of wealth. This complementarity is key for private and government consumption to be positively correlated. Once $\nu < 1$, the model can generate a given elasticity of the marginal utility of wealth with respect to government spending with a relatively low value of ν and a relatively high value of ϕ_G , or a relatively large value of ν and a smaller value of ϕ_G . In our different estimations, we find exactly this pattern – fixing ϕ_G at a relatively lower value results in a higher estimated value of ν and vice-versa, but has virtually no effect on unconditional moments or model fit. Given a fixed value of ϕ_G , the parameter ν does seem to be well-identified.

While ϕ_G and ν do not seem to be well-identified (at least in the region where $\nu < 1$), different values of ϕ_G are relevant for the size and magnitude of the welfare multiplier. We discuss this in the text in Section 4.4. In particular, the higher is ϕ_G , the smaller (or more negative) is the welfare multiplier for government consumption. This is intuitive – the larger is ϕ_G , the lower the utility weight households place on government consumption.

D Additional Parameter Robustness Exercises

This Appendix considers some additional robustness exercises to other parameters in our model. For these exercises, all but the relevant parameter(s) are set to their baseline values. We then generate the distributions of output and welfare multipliers. Results are summarized in Table D1.

We first consider the case in which the elasticities of substitution for both goods and labor are significantly higher than in our baseline by setting $\epsilon_w = \epsilon_p = 21$. Doing so makes very little difference for the properties of the output multipliers for both government consumption and investment. The distributions of the welfare multipliers for both spending categories are noticeably different. First, the average welfare multipliers are smaller (more negative in the case of government consumption, and less positive for government investment). This makes sense in light of the intuition developed above. When ϵ_p and ϵ_w are larger, the economy is less distorted on average. This tends to lower the welfare benefit of government spending.

We also consider the case in which prices and wages are perfectly flexible, i.e. $\theta_w = \theta_p = 0$. The lack of nominal rigidity results in smaller average output multipliers for both types of government spending, though the effect is more pronounced for the government investment shock than for government consumption. The average welfare multiplier for government investment is close to the same as in our baseline. The average welfare multiplier for government consumption, while still negative, is actually larger than in our baseline. The lower output multipliers for each type of government expenditure result in welfare multipliers for both types of government spending becoming more positively correlated with output.

We next consider a case in which there is no variable capital utilization. We implement this by setting $\delta_2 = 1,000$, which effectively results in capital utilization being fixed. This results in smaller average output multipliers for both types of government spending. It also results in smaller average welfare multipliers. For both types of government spending, a lack of capital utilization results in the welfare multipliers being more strongly positively correlated with output.

A final robustness exercise we consider is to lower the autoregressive parameters for government consumption and investment, setting each of these to 0.75 instead of their baseline estimated values. Less persistent shocks result in higher average output multipliers for both types of spending. For government consumption, this results in a larger (less negative) average welfare multiplier, and

also leads to the welfare multiplier being less positively correlated with output. For government investment, the average welfare multiplier is actually smaller than in our baseline, in spite of the fact that the output multiplier is larger on average. This arises because the benefits of government investment are felt most in the future, and with a less persistent shock these future benefits are smaller.

Table D1: Other Parameter Robustness

	<i>Consumption</i>			<i>Investment</i>		
	Mean	SD	Corr w/ Output	Mean	SD	Corr w/ Output
$\epsilon_w = \epsilon_p = 21$						
Output	1.0609	0.0161	0.2571	0.9067	0.0055	-0.1631
Welfare	-5.1631	1.5554	0.4645	0.6308	0.8354	-0.1531
Cons Eq	-0.2589	0.0400	0.4472	0.1343	0.1246	-0.1696
$\theta_w = \theta_p = 0$						
Output	1.0420	0.0213	0.3636	0.8229	0.0089	-0.4431
Welfare	-1.3362	1.6606	0.5769	3.1440	0.5307	0.3528
Cons Eq	-0.0742	0.1377	0.5232	0.3234	0.0186	0.2536
$\delta_2 = 1000$						
Output	1.0152	0.0160	0.4049	0.8673	0.0048	-0.1707
Welfare	-3.3958	1.7335	0.6350	2.5579	0.7342	0.2028
Cons Eq	-0.1858	0.0739	0.5697	0.2954	0.0411	0.1280
$\rho_{G_C} = \rho_{G_I} = 0.75$						
Output	1.1815	0.0162	0.4687	0.9696	0.0021	0.1992
Welfare	-0.3649	0.6337	0.2687	1.1271	0.3653	-0.0946
Cons Eq	-0.0251	0.0793	0.2790	0.2045	0.0478	-0.1067

Note: this table is structured similarly to Table 4, but fixes the listed parameter values at different values than those used in our baseline simulations. All other parameter values other than the ones listed in the relevant rows are set to their baseline values.

E Automatic Stabilizer Components to Government Spending

This appendix considers adding an automatic stabilizer component to both types of government spending. We continue to assume that the exogenous components of government consumption and investment are governed by (12) and (13), respectively. Actual government consumption and investment, $G_{C,t}^*$ and $G_{I,t}^*$, are given by:

$$(E.1) \quad \ln G_{C,t}^* = \ln G_{C,t} + \gamma_{G_C}^Y (\ln Y_t - \ln Y)$$

$$(E.2) \quad \ln G_{I,t}^* = \ln G_{I,t} + \gamma_{G_I}^Y (\ln Y_t - \ln Y)$$

In (E.1)-(E.2), the parameters $\gamma_{G_C}^Y$ and $\gamma_{G_I}^Y$ measure the responses of actual government spending

to the deviation of output from steady state. In the accumulation equation for government capital, (11), and the aggregate resource constraint, (21), actual government consumption and investment replace the exogenous components of spending.

We revert to assuming that all fiscal finance comes from lump sum taxes. Other parameters are set at the posterior mode from the baseline estimation. We consider five different scenarios concerning different values of $\gamma_{G_C}^Y$ and $\gamma_{G_I}^Y$ and conduct the same quantitative experiments as described earlier. Results are summarized in [E1](#). Regime 1 features a positive response of government consumption to output relative to steady state and no response of government investment, while Regime 2 features no response of government consumption to output and a positive response of government investment. Regimes 3 and 4 are similar but with negative responses to the deviation of output from steady state. In Regime 5, both components of government spending respond negatively to the deviation of output from its steady state. Overall, our main conclusions are similar when accounting for endogenous spending responses.

Table E1: Endogenous Spending Response

	<i>Consumption</i>			<i>Investment</i>		
	Mean	SD	Corr w/ Output	Mean	SD	Corr w/ Output
<i>Regime 1</i>						
Output	1.0673	0.0165	0.2393	0.9125	0.0041	-0.3182
Welfare	-2.3095	1.5457	0.4763	3.1020	0.6290	-0.0075
Cons Eq	-0.1511	0.0990	0.4303	0.3238	0.0252	-0.0815
<i>Regime 2</i>						
Output	1.0693	0.0168	0.2760	0.9046	0.0040	-0.2634
Welfare	-2.3229	1.5834	0.4993	3.1317	0.6232	-0.0303
Cons Eq	-0.1414	0.0952	0.4500	0.3221	0.0240	-0.1126
<i>Regime 3</i>						
Output	1.0671	0.0171	0.3165	0.8967	0.0040	-0.2295
Welfare	-2.3446	1.6186	0.5204	3.1365	0.6219	-0.0193
Cons Eq	-0.1407	0.0970	0.4685	0.3222	0.0244	-0.1033
<i>Regime 4</i>						
Output	1.0651	0.0168	0.2822	0.9044	0.0041	-0.2878
Welfare	-2.3321	1.5799	0.4990	3.1068	0.6278	0.0042
Cons Eq	-0.1414	0.0942	0.4504	0.3210	0.0249	-0.0786
<i>Regime 5</i>						
Output	1.0650	0.0171	0.3195	0.8966	0.0040	-0.2420
Welfare	-2.3491	1.6167	0.5202	3.1242	0.6242	-0.0021
Cons Eq	-0.1412	0.0966	0.4697	0.3217	0.0245	-0.0731

Note: this table is structured similarly to Table 4, but considers five different spending response regimes. All government finance is from lump sum taxes and all other parameters are set to their baseline values. In Regime 1, government consumption responds to deviations of output from steady state and government investment does not (with $\gamma_{G_C}^Y = 0.05$ and $\gamma_{G_I}^Y = 0$). Regime 2 is similar but features no response of government consumption and a positive response of government investment. Regimes 3 and 4 are similar but with opposite signs – government spending reacts negatively to deviations of output from steady state. In Regime 5, both government consumption and investment react negatively to the deviation of output from steady state, with $\gamma_{G_C}^Y = \gamma_{G_I}^Y = -0.05$.

F Equilibrium Conditions with Rule of Thumb Consumers

This Appendix lists the full set of equilibrium conditions for the version of our model augmented to include rule of thumb (ROT) households. This model is described in Section 5.3 of the text. In what follows, we use o subscripts to demarcate variables pertinent to optimizing households and r subscripts for variables chosen by ROT households.

F.1 Optimizing Household Optimality Conditions

The optimality conditions for an optimizing household are identical to the baseline model. They are listed here again for convenience.

$$(F.1) \quad (1 + \tau_t^C) \lambda_{o,t} = v_t \frac{1}{\widehat{C}_{o,t}} \phi_G(C_{o,t} - bC_{o,t-1})^{-\frac{1}{\nu}} - \beta b E_t v_{t+1} \frac{1}{\widehat{C}_{o,t+1}} \phi_G(C_{o,t+1} - bC_{o,t})^{-\frac{1}{\nu}}$$

$$(F.2) \quad \widehat{C}_{o,t} = \phi_G(C_{o,t} - bC_{o,t-1})^{\frac{\nu-1}{\nu}} + (1 - \phi_G) G_{C,t}^{\frac{\nu-1}{\nu}}$$

$$(F.3) \quad (1 - \tau_t^K) \lambda_{o,t} R_t = \mu_{o,t} (\delta_1 + \delta_2(u_{o,t} - 1))$$

$$(F.4) \quad \lambda_{o,t} = \mu_{o,t} Z_t \left[1 - \frac{\kappa}{2} \left(\frac{I_{o,t}}{I_{o,t-1}} - 1 \right)^2 - \kappa \left(\frac{I_{o,t}}{I_{o,t-1}} - 1 \right) \frac{I_{o,t}}{I_{o,t-1}} \right] + \beta E_t \mu_{o,t+1} Z_{t+1} \kappa \left(\frac{I_{o,t+1}}{I_{o,t}} - 1 \right) \left(\frac{I_{o,t+1}}{I_{o,t}} \right)^2$$

$$(F.5) \quad \mu_{o,t} = \beta E_t \lambda_{o,t+1} (1 - \tau_{t+1}^K) R_{t+1} u_{o,t+1} + \beta E_t \mu_{o,t+1} \left(1 - \delta_0 - \delta_1(u_{o,t+1} - 1) - \frac{\delta_2}{2} (u_{o,t+1} - 1)^2 \right)$$

$$(F.6) \quad \lambda_{o,t} = \beta (1 + i_t) E_t \lambda_{o,t+1} (1 + \pi_{t+1})^{-1}$$

$$(F.7) \quad w_{o,t}^\# = \frac{\epsilon_w}{\epsilon_w - 1} \frac{F_{1,t}}{F_{2,t}}$$

$$(F.8) \quad F_{1,t} = v_t \xi_t \left(\frac{w_{o,t}^\#}{w_t} \right)^{-\epsilon_w(1+\chi)} N_{o,t}^{1+\chi} + \beta \theta_w E_t \left(\frac{w_{o,t}^\#}{w_{o,t+1}^\#} \frac{(1 + \pi_t)^{\zeta_w}}{1 + \pi_{t+1}} \right)^{-\epsilon_w(1+\chi)} F_{1,t+1}$$

$$(F.9) \quad F_{2,t} = \lambda_{o,t} (1 - \tau_t^N) \left(\frac{w_{o,t}^\#}{w_t} \right)^{-\epsilon_w} N_{o,t} + \beta \theta_w E_t \left(\frac{w_{o,t}^\#}{w_{o,t+1}^\#} \right)^{-\epsilon_w} \left(\frac{(1 + \pi_t)^{\zeta_w}}{1 + \pi_{t+1}} \right)^{1-\epsilon_w} F_{2,t+1}$$

$$(F.10) \quad K_{o,t+1} = Z_t \left[1 - \frac{\kappa}{2} \left(\frac{I_{o,t}}{I_{o,t-1}} - 1 \right)^2 \right] I_{o,t} + \left(1 - \delta_0 - \delta_1(u_{o,t} - 1) - \frac{\delta_2}{2}(u_{o,t} - 1)^2 \right) K_{o,t}$$

F.2 Rule of Thumb Household Optimizing Conditions

Optimization for the ROT household is characterized by the following four conditions:

$$(F.11) \quad (1 + \tau_t^C)C_{r,t} = (1 - \tau_t^N)w_t N_{r,t} - T_{r,t}$$

$$(F.12) \quad v_t \xi_t N_{r,t}^X = \lambda_{r,t} (1 - \tau_t^N) w_t$$

$$(F.13) \quad (1 + \tau_t^C) \lambda_{r,t} = v_t \frac{1}{\widehat{C}_{r,t}} \phi_G(C_{r,t} - bC_{r,t-1})^{-\frac{1}{\nu}} - \beta b E_t v_{t+1} \frac{1}{\widehat{C}_{r,t+1}} \phi_G(C_{r,t+1} - bC_{r,t})^{-\frac{1}{\nu}}$$

$$(F.14) \quad \widehat{C}_{r,t} = \phi_G(C_{r,t} - bC_{r,t-1})^{\frac{\nu-1}{\nu}} + (1 - \phi_G) G_{C,t}^{\frac{\nu-1}{\nu}}$$

F.3 Firm Optimality Conditions

Optimality conditions for firms are the same as in our baseline model. The only minor modification necessary is that firms use the stochastic discount factor of optimizing households to discount future profit flows.

$$(F.15) \quad w_t = mc_t (1 - \alpha) A_t K_{G,t}^\varphi \left(\frac{\widehat{K}_t}{N_t} \right)^\alpha$$

$$(F.16) \quad R_t = mc_t \alpha A_t K_{G,t}^\varphi \left(\frac{\widehat{K}_t}{N_t} \right)^{\alpha-1}$$

$$(F.17) \quad 1 + \pi_t^\# = \frac{\epsilon_p}{\epsilon_p - 1} (1 + \pi_t) \frac{X_{1,t}}{X_{2,t}}$$

$$(F.18) \quad X_{1,t} = \lambda_{o,t} m c_t Y_t + \theta_p \beta E_t (1 + \pi_t)^{-\zeta_p \epsilon_p} (1 + \pi_{t+1})^{\epsilon_p} X_{1,t+1}$$

$$(F.19) \quad X_{2,t} = \lambda_{o,t} Y_t + \theta_p \beta E_t (1 + \pi_t)^{\zeta_p (1 - \epsilon_p)} (1 + \pi_{t+1})^{\epsilon_p - 1} X_{2,t+1}$$

F.4 Government

The law of motion for government capital and exogenous process for government consumption and investment are:

$$(F.20) \quad K_{G,t+1} = G_{I,t} + (1 - \delta_G) K_{G,t}$$

$$(F.21) \quad \ln G_{C,t} = (1 - \rho_{G_C}) \ln G_C + \rho_{G_C} \ln G_{C,t-1} + s_{G_C} \varepsilon_{G_C,t}$$

$$(F.22) \quad \ln G_{I,t} = (1 - \rho_{G_I}) \ln G_I + \rho_{G_I} \ln G_{I,t-1} + s_{G_I} \varepsilon_{G_I,t}$$

As noted in the text, we assume that the government balances its budget with lump sum taxes each period. This means that $\tau_t^C = \tau_t^K = \tau_t^N$ and that $b_{g,t} = 0$. This significantly simplifies the government's budget constraint, which can be written: $G_{C,t} + G_{I,t} = T_t$. We assume that lump sum taxes for each type of household are proportional to the population weights:

$$(F.23) \quad T_t = T_{o,t} + T_{r,t}$$

$$(F.24) \quad T_{o,t} = (1 - \Phi) (G_{C,t} + G_{I,t})$$

$$(F.25) \quad T_{r,t} = \Phi (G_{C,t} + G_{I,t})$$

Monetary policy is conducted according to the same Taylor rule as in the baseline model:

$$(F.26) \quad i_t = (1 - \rho_i)i + \rho_i i_{t-1} + (1 - \rho_i) [\phi_\pi \pi_t + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_i \varepsilon_{i,t}$$

F.5 Exogenous Processes

Other exogenous processes in the model are identical to our baseline model. These are given by:

$$(F.27) \quad \ln A_t = (1 - \rho_A) \ln A + \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t}$$

$$(F.28) \quad \ln Z_t = \rho_Z \ln Z_{t-1} + s_Z \varepsilon_{Z,t}$$

$$(F.29) \quad \ln v_t = \rho_v \ln v_{t-1} + s_v \varepsilon_{v,t}$$

$$(F.30) \quad \ln \xi_t = (1 - \rho_\xi) \ln \xi + \rho_\xi \ln \xi_{t-1} + s_\xi \varepsilon_{\xi,t}$$

F.6 Aggregate Conditions

The aggregate market-clearing conditions of the model augmented to include a fraction of ROT households are:

$$(F.31) \quad Y_t = C_t + I_t + G_{C,t} + G_{I,t}$$

$$(F.32) \quad v_t^p Y_t = A_t K_{G,t}^\varphi \widehat{K}_t^\alpha N_t^{1-\alpha} - F$$

$$(F.33) \quad v_t^p = (1 + \pi_t)^{\epsilon_p} \left[(1 - \theta_p) (1 + \pi_t^\#)^{-\epsilon_p} + \theta_p (1 + \pi_{t-1})^{-\zeta_p \epsilon_p} v_{t-1}^p \right]$$

$$(F.34) \quad \widehat{K}_t = (1 - \Phi)\widehat{K}_{o,t}$$

$$(F.35) \quad \widehat{K}_{o,t} = u_{o,t}K_{o,t}$$

$$(F.36) \quad I_t = (1 - \Phi)I_{o,t}$$

$$(F.37) \quad C_t = (1 - \Phi)C_{o,t} + \Phi C_{r,t}$$

$$(F.38) \quad N_t = (1 - \Phi)N_{o,t} + \Phi N_{r,t}$$

$$(F.39) \quad (1 + \pi_t)^{1-\epsilon_p} = (1 - \theta_p)(1 + \pi_t^\#)^{1-\epsilon_p} + \theta_p(1 + \pi_{t-1})^{\zeta_p(1-\epsilon_p)}$$

$$(F.40) \quad w_t^{1-\epsilon_w} = (1 - \theta_w)w_{o,t}^{\#,1-\epsilon_w} + \theta_w \left(\frac{(1 + \pi_{t-1})^{\zeta_w}}{1 + \pi_t} w_{t-1} \right)^{1-\epsilon_w}$$

F.7 Equilibrium

Expressions (F.1)-(F.40) comprise forty equations in forty variables: $\{C_{o,t}, I_{o,t}, \widehat{C}_{o,t}, \lambda_{o,t}, \mu_{o,t}, u_{o,t}, K_{o,t}, \widehat{K}_{o,t}, w_{o,t}^\#, N_{o,t}, F_{1,t}, F_{2,t}, C_{r,t}, N_{r,t}, \lambda_{r,t}, \widehat{C}_{r,t}, m_{c,t}, w_t, R_t, i_t, \pi_t, \pi_t^\#, X_{1,t}, X_{2,t}, \widehat{K}_t, N_t, Y_t, G_{C,t}, G_{I,t}, K_{G,t}, T_t, T_{o,t}, T_{r,t}, I_t, C_t, v_t^p, A_t, Z_t, v_t, \xi_t\}$.

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