Bernanke and Gertler (1989, *American Economic Review*)

ECON 70428: Advanced Macro: Financial Frictions

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Agency Costs, Net Worth, and Business Fluctuations

This is perhaps the seminal modern paper on incorporating financial frictions into a macro model

Firm balance sheets can matter for economic fluctuations

Both be a **source** of fluctuations as well as a **propagation** mechanism

Based on the costly state verification (csv) Townsend (1979)

Key follow-up papers:

1. Carlstrom and Fuerst (1997, *AER*)
2. Bernanke, Gertler, and Gilchrist (1999, *Handbook of Macro*)
3. Christiano, Motto, Rostagno (2014, *AER*)
4. Carlstrom, Fuerst, and Paustian (2016, *AEJ: Macro*)

Model Basics

Two period overlapping generations (OLG)

Within each generation, exogenous assignment of households to one of two types – lenders or entrepreneurs

Entrepreneurs are heterogeneous with respect to their efficiency

They have access to a stochastic capital goods production technology. Uncertain at the idiosyncratic level, but no aggregate uncertainty

Full capital depreciation, so with no frictions model is effectively static – no endogenous propagation

Agency Friction

If entrepreneurs are endowed with insufficient net worth and/or are not sufficiently efficient, they need external funds – i.e. they need to borrow

Lender cannot observe the stochastic realization of the entrepreneur's capital goods production

This gives entrepreneur potential incentive to misreport outcomes to try to keep more for him/herself

Costly Monitoring

To prevent this, lender must promise to monitor outcomes in bad states of the world

Monitoring is costly and involves some deadweight loss – this is the agency friction

Monitoring will be more frequent the more the entrepreneur borrows – i.e. the lower is his/her net worth

The more monitoring there is / the lower is net worth, the less capital accumulation there is

Propagation

A positive technology shock improves entrepreneur balance sheets

This lowers agency costs, allows them to borrow more, and results in more investment

More investment **propagates** the productivity shock in a way that wouldn't happen without the agency cost

In addition, can consider **redistribution** shocks from entrepreneurs to lenders (e.g. debt-deflation)

Exogenous changes in balance sheet conditions can affect investment in ways that wouldn't be true without the underlying agency friction

Model Environment

Two generations each period: "young" and "old." Per capita

- Fraction $\eta \in [0, 1]$ of each generation are entrepreneurs – they have access to investment technology, others don't. $1 - \eta$ are lenders
- Entrepreneurs are heterogeneous, indexed by $\omega \sim U[0, 1]$. Low ω: more efficient

Output in period t can be (i) consumed, (ii) invested in the production of the capital good, which is available for production in $t + 1$, or (iii) stored at gross return r

Labor is supplied inelastically in youth only by both entrepreneurs and lenders: $L_t = 1 = \eta L^e + (1 - \eta) L$

FRICTIONLESS MODEL

Firms

Representative firm produces output according to:

$$Y_t = \theta_t F(K_t, L_t)$$

θ_t is iid with mean θ and $F(\cdot)$ is h.o.d. 1. Factor prices are q_t and w_t. Optimization entails:

$$\begin{aligned} q_t &= \theta_t F_K(K_t, L_t) \\ w_t &= \theta_t F_L(K_t, L_t) \end{aligned}$$

Let $y_t = Y_t / L_t$ and $k_t = K_t / L_t$. Let $f(k_t) = F(K_t / L_t, 1)$ ("per capita" = "per member of generation" since L_t total generation size). Then:

$$\begin{aligned} q_t &= \theta_t f'(k_t) \\ w_t &= \theta_t \left[f(k_t) - k_t f'(k_t) \right] \end{aligned}$$

Lenders

Consumption is z_t^y or z_{t+1}^0 in youth and old age

Preferences are quasi-linear over consumption

$$U = U(z_t^y) + \beta\,\mathbb{E}_t\, z_{t+1}^o$$

Savings offers known gross return $r \geq 1$

Budget constraints:

$$z_t^y + S_t = w_t L$$

$$z_{t+1}^o = rS_t$$

Optimization $z_t^y = z_y^*(r)$ and:

$$S_t = w_t L - z_y^*(r)$$

Entrepreneurs

Only get utility from $t+1$ consumption:

$$U^e = c^e_{t+1}$$

- Each is endowed with a discrete project that takes $x(\omega)$ units to conduct, $x'(\omega) > 0$
- Project can generate κ_1 or κ_2 units of capital available for lease to firms in $t+1$, $\kappa_1 < \kappa_2$. Probability of κ_1 is π_1. Let $\kappa = \pi_1\kappa_1 + \pi_2\kappa_2$. But no aggregate uncertainty
- Earns income $w_t L^e$. Can either save this, $S = w_t L^e$ via storage at r or produce new capital. Indifference between storage and undertaking project given occurs at $\bar{\omega}$:

$$q_{t+1}\kappa = rx(\bar{\omega})$$

- $\omega \leq \bar{\omega}$ they invest; $\omega > \bar{\omega}$ they store

Aggregation

So that there is always some storage, assume:

$$\eta S^e + (1-\eta)S > \int_0^{\bar{\omega}} x(\omega) d\omega$$

No aggregate uncertainty. Hence fraction $\bar{\omega}$ of entrepreneurs will do projects, so total number of aggregate projects is:

$$i_t = \bar{\omega}\eta$$

No aggregate uncertainty over outcomes of projects, so new capital stock (full depreciation) is:

$$k_{t+1} = \kappa i_t$$

Capital Supply and Demand

We have:

$$\bar{\omega} = \frac{k_{t+1}}{\kappa\eta}$$

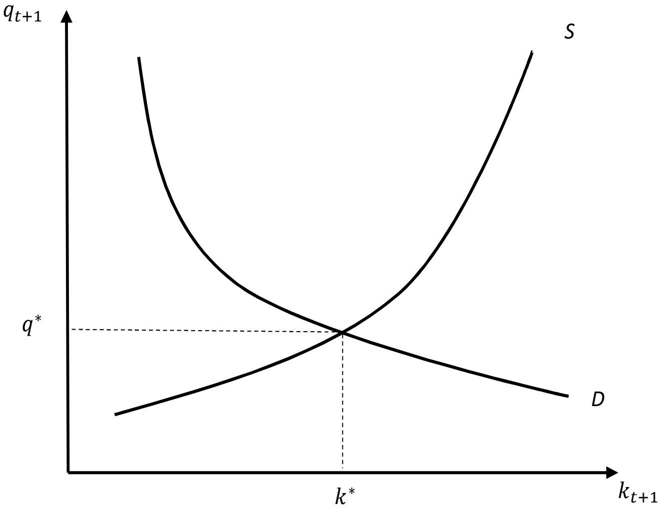
Hence capital supply curve is upward-sloping since $x'(\cdot) > 0$:

$$q_{t+1} = \frac{rx\left(\frac{k_{t+1}}{\kappa\eta}\right)}{\kappa}$$

Capital demand curve is standard:

$$q_{t+1} = \theta f'(k_{t+1})$$

Capital Market-Equilibrium



Equilibrium Dynamics

Because θ_t is i.i.d. and neither demand nor supply depend on endogenous states, $k_{t+1} = k^*$ and $q_{t+1} = q^*$ are constant

Thus, in response to increase in θ_t, there is no change in k_{t+1} (no propagation).

c_t^e will go up (because q_t will be higher, so each entrepreneur gets more for his/her capital)

S_t will go up (because w_t is high), so z_{t+1}^o will go up (because there is more storage) – so some autocorrelation in aggregate consumption

AGENCY COSTS

Asymmetric Information

An entrepreneur who undertakes his/her project gets a stochastic realization – κ_1 or κ_2

If he/she has borrowed from a lender, he/she can underreport idiosyncratic returns and keep surplus

If lender wants to see true realization of κ_j, must pay cost of $\gamma > 0$ of the capital good

So if a fraction h_t of projects are monitored, in the aggregate capital will be:

$$k_{t+1} = (\kappa - \gamma h_t) i_t$$

Think about this in two stages:

1. Partial equilibrium contracting problem
2. Investment decision and GE

Optimal Contract

Assume entrepreneur has $\omega < \bar{\omega}$, but for whom $x(\omega) > S^e = w_t L^e$. He/she needs to borrow

Lender has opportunity cost r

Take S^e, r, and q (price of k_{t+1}, so q_{t+1}) as given

Entrepreneur borrows $x(\omega) - S^e$

Lender requires $r(x(\omega) - S^e)$ to make loan

Entrepreneur Problem

Entrepreneurs get a loan and then they learn they have κ_1 or κ_2

They then tell the lender a realization

If they announce κ_1, they get audited with probability p, which entails a monitoring cost

No audit if they announce κ_2 (the good state)

They get c_2 if they announce the good state, c_1 if they announce the bad state and are not audited, and c^a if they announce the bad state and are audited

Lender gets difference between project outcome and entrepreneurial consumption

Problem

$$\max_{p,c_1,c_2,c^a} \quad \pi_1 \left[pc^a + (1-p)c_1\right] + \pi_2 c_2$$

s.t.

$$\pi_1 \left[q\kappa_1 - p(c^a + q\gamma) - (1-p)c_1\right] + \pi_2[q\kappa_2 - c_2] \geq r(x - S^e)$$

$$c_2 \geq (1-p)(q(\kappa_2 - \kappa_1) + c_1)$$

$$c_1 \geq 0$$

$$c^a \geq 0$$

$$0 \leq p \leq 1$$

In Words . . .

Maximize expected entrepreneurial consumption subject to:

1. Lender earns opportunity cost, r
2. Entrepreneur tells the truth (he/she never says κ_1 when he/she gets κ_2)
3. Consumption is positive in all states
4. Auditing probability between 0 and 1

Optimal Contract

There are two regimes:

1. "Full Collateralization:"

$$q\kappa_1 \geq r\left[x(\omega) - S^e\right]$$

- Entrepreneur can always pay regardless of state
- Lender never audits, hence no agency costs

$$c_{fc} = q\kappa - r\left[x(\omega) - S^e\right]$$

- This will occur when S^e is "big"

2. "Incomplete Collateraliztion:"
   - S^e not sufficiently big; entrepreneur can't pay back if he/she realizes κ_1

Insufficient Collateralization

All constraints bind in this case, implying:

$$p = \frac{r\left[x(\omega) - S^e\right] - q\kappa_1}{\pi_2 q(\kappa_2 - \kappa_1) - \pi_1 q\gamma}$$

Assume $\pi_2 q(\kappa_2 - \kappa_1) - \pi_1 q\gamma > 0$, so $p > 0$

p is decreasing in S^e

Agency costs: $\pi_1 pq\gamma$ therefore decreasing in S^e

Intuition:

- Low S^e, lenders require large return in good state, so entrepreneurial consumption in state 2 is low
- With low c_2, entrepreneur is risking less by falsely claiming the bad state. Thus he/she must be audited more

Entrepreneurial Consumption

Noting that $c_2 = (1-p)q(\kappa_2 - \kappa_1)$, and expected consumption is π_2 times this, we have:

$$c_{ic} = \alpha\left[qk - r\left[x(\omega) - S^e\right] - \pi_1 q\gamma\right]$$

where:

$$\alpha = \frac{\pi_2 q(\kappa_2 - \kappa_1)}{\pi_2 q(\kappa_2 - \kappa_1) - \pi_1 q\gamma}$$

Since $\alpha > 1$, we have:

$$\frac{\partial c_{ic}}{\partial S^e} = \alpha r > r$$

Return to "inside" funds exceeds return to "outside" funds (external finance premium)

Investment Decision

For the optimal contract, we assumed that the entrepreneur definitely wanted to undertake a project

This may not be the case

With perfect information, there was one cutoff $\bar{\omega}$: with $\omega \leq \bar{\omega}$, an entrepreneur would invest and with $\omega > \bar{\omega}$ he/she would store

Now we need to worry about three cases: $\underline{\omega}$ and $\bar{\omega}$

$\underline{\omega}$ and $\bar{\omega}$

Good entrepreneurs have $\omega \leq \underline{\omega}$:

$$q\kappa - rx(\underline{\omega}) - q\pi_1\gamma = 0$$

Their efficiency level is good enough that their expected return, net of auditing costs, is positive even if audited with probability 1

Fair entrepreneurs $\underline{\omega} < \omega \leq \bar{\omega}$:

$$q\kappa - rx(\bar{\omega}) = 0$$

Their efficiency level makes investing only profitable if there is no chance of being audited

Poor entrepreneurs, $\omega > \bar{\omega}$, will just want to store

Both $\bar{\omega}$ and $\underline{\omega}$ are increasing functions of q

Full-Collateralization Level of Savings

For any ω, define $S^*(\omega)$ as the level of savings above which the entrepreneur can repay regardless of the realization of κ_1 or κ_2

This satisfies:

$$q\kappa_1 = r\left[x(\omega) - S^*\right]$$

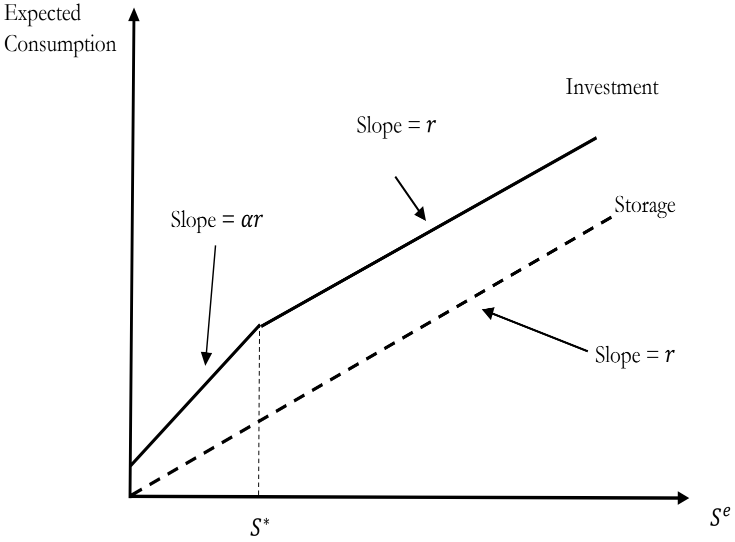
Or:

$$S^*(\omega) = x(\omega) - \frac{q}{r}\kappa_1$$

Note this is a decreasing function of q

Note also $S^*(\omega)$ will be lower for lower values of ω

Storage vs. Investment: Good Entrepreneurs



Outcomes for Good Entrepreneur

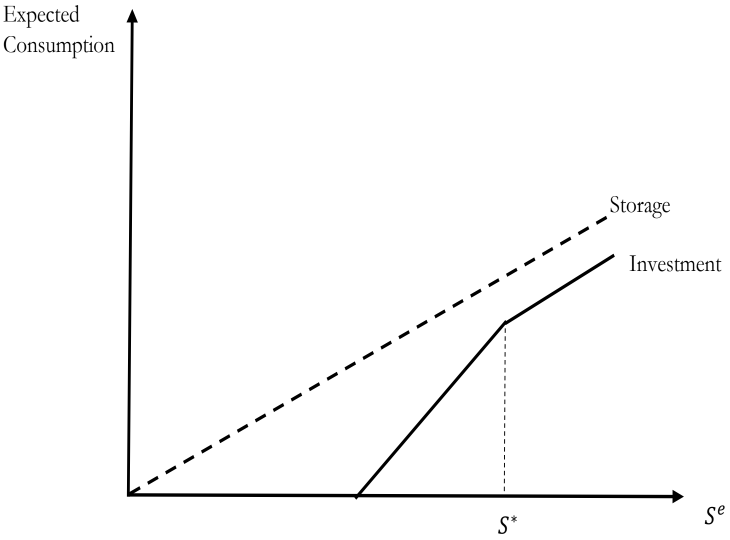
Good entrepreneur always wants to invest

Below S^*, return to internal funds (more saving) is $\alpha r > r$ since $\alpha > 1$

Above S^*, return to internal funds and external funds is the same, r – there are no agency cost, so he or she is indifferent to internal vs. external funds

Poor entrepreneurs (next slide), in contrast, will never find it optimal to invest. They will just store

Storage vs. Investment: Poor Entrepreneurs



Fair Entrepreneurs

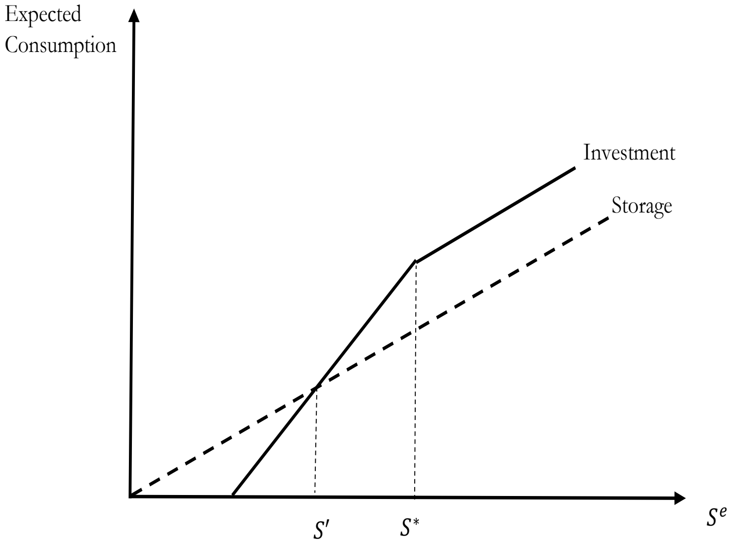
This case is trickier

If $S(\omega) < S'$, then the fair entrepreneur will store

If $S' \leq S(\omega) < S^*$, entrepreneur will invest but faces positive auditing probability

If $S(\omega) \geq S^*$ there is full collateralization

Storage vs. Investment: Fair Entrepreneurs



Lotteries

Fair entrepreneur with less than S^* would like to take a gamble

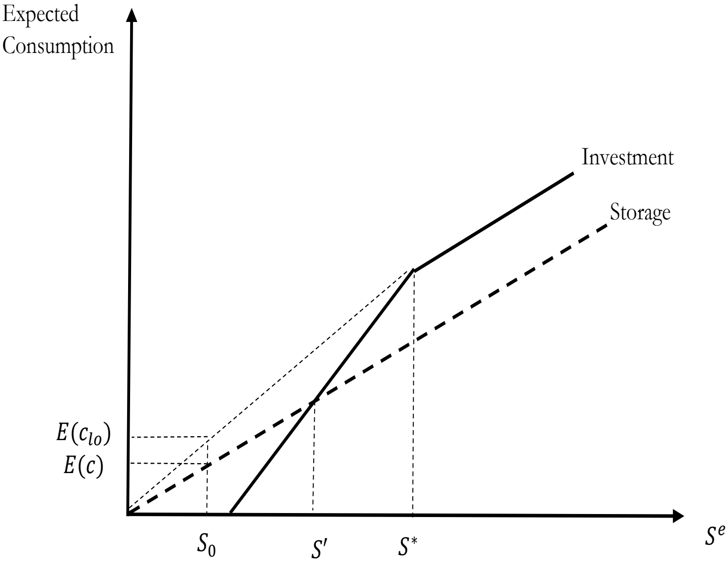
Risk $S(\omega)$ for a lottery that pays S^* with probability S^e/S^* and zero otherwise

Convexity of upper envelope of dashed (storage) and solid (investment) lines between 0 and S^* ensures that expected consumption from taking the lottery exceeds expected outcome of storage or investment

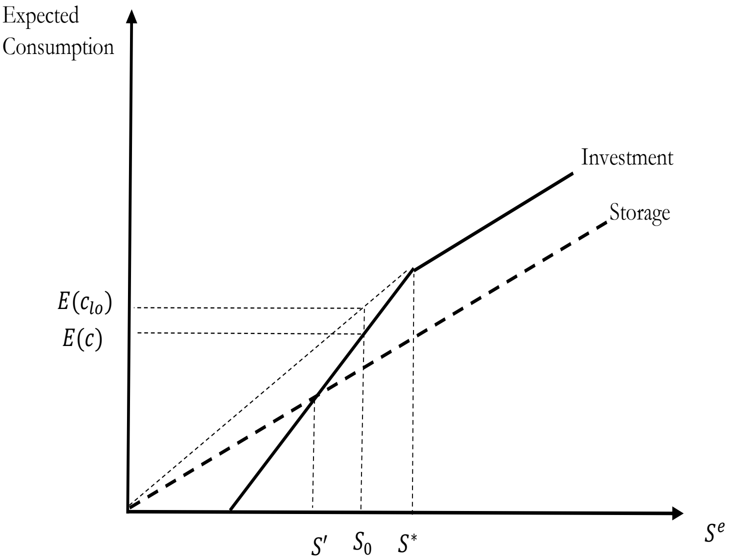
For each ω, a fraction $g(\omega) = S^e/S^*$ win the lottery and become fully collateralized. The $1 - g(\omega)$ rest get zero consumption.

See next two slides

Lottery: Low S^e



Lottery: Higher S^e



Constrained vs. Unconstrained: Basic Idea

In frictionless world, every entrepreneur with $\omega \leq \bar{\omega}$ invests and all others store

In frictionless world, all good entrepreneurs always invest

In asymmetric information world:

- All good entrepreneurs invest, but with positive expected agency costs
- Poor entrepreneurs do not invest
- Only a fraction of fair entrepreneurs invest

Effectively, investment by fair entrepreneurs is restricted by "internal equity" (i.e. how close S^e is to S^*)

Equilibrium

The current capital stock, k_t, is predetermined

The realization of θ_t in conjunction with inelastic labor supply completely determines y_t, S_t, and S_t^e

Demand for next period's capital, k_{t+1}, is the same as the frictionless case:

$$q_{t+1} = \theta f'(k_{t+1})$$

Capital supply

Capital supply is more complicated. Comes from three sources:

1. Good entrepreneurs (with $\omega \leq \underline{\omega}$) who are not audited
2. Good entrepreneurs who are audited
3. Fair entrepreneurs (with $\underline{\omega} < \omega \leq \bar{\omega}$) who win the lottery (no agency costs for them because if they get lottery they go to full collateralization)

Supply from Good Entrepreneurs

All good entrepreneurs invest. There are $\underline{\omega}$ of them in aggregate, and they produce κ each

But some of them are audited. The auditing probability is:

$$p(\omega) = \max \left\{ \frac{rx(\omega) - q_{t+1}\kappa_1 - rS^e}{q_{t+1}\left[\pi_2(\kappa_2 - \kappa_1) - \pi_1\gamma\right]}, 0 \right\}$$

Note $p(\omega) = 0$ if $S^e \geq S^*$

Hence capital output from good entrepreneurs is:

$$\kappa\underline{\omega} - \pi_1\gamma \int_0^{\underline{\omega}} p(\omega) d\omega$$

Supply from Fair Entrepreneurs

Recall fair entrepreneurs take a lottery

$g(\omega)$ invest, where:

$$g(\omega) = \min\left\{ \frac{rS^e}{rx(\omega) - q_{t+1}\kappa_1}, 1 \right\}$$

Note if $S^e \geq S^*$ we have $g(\omega) = 1$

Their supply of capital is:

$$\kappa \int_{\underline{\omega}}^{\bar{\omega}} g(\omega) d\omega$$

Total Capital Supply

Remember, mass of entrepreneurs is η:

$$k_{t+1} = \left[\kappa\underline{\omega} - \pi_1\gamma\int_0^{\underline{\omega}} p(\omega)d\omega\right]\eta + \left[\kappa\int_{\underline{\omega}}^{\bar{\omega}} g(\omega)d\omega\right]\eta$$

Note that:

$$\int_{\underline{\omega}}^{\bar{\omega}} g(\omega) + \int_{\underline{\omega}}^{\bar{\omega}} (1 - g(\omega))d\omega = \bar{\omega} - \underline{\omega}$$

So we can write:

$$k_{t+1} = \left\{\kappa\underline{\omega} - \int_0^{\underline{\omega}} \pi_1\gamma p(\omega)d\omega + \kappa(\bar{\omega} - \underline{\omega}) - \int_{\underline{\omega}}^{\bar{\omega}} \kappa(1 - g(\omega))d\omega\right\}\eta$$

Capital Supply

This works out to:

$$k_{t+1} = \left\{ \kappa\bar{\omega} - \left[\int_0^{\underline{\omega}} \pi_1 \gamma p(\omega) d\omega + \int_{\underline{\omega}}^{\bar{\omega}} \kappa(1 - g(\omega)) d\omega \right] \right\} \eta$$

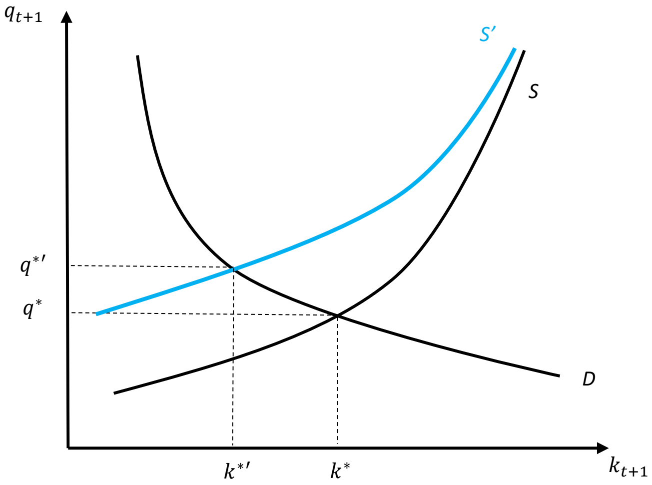
This generalizes to what we had in frictionless case when $\gamma = 0$: $k_{t+1} = \bar{\omega}\kappa\eta$

But when $\gamma > 0$, we have $k_{t+1} < \bar{\omega}\kappa\eta$ – **agency friction reduces supply of capital**

Slope: q_{t+1} affects both $\underline{\omega}$ and $\bar{\omega}$, as well as $p(\omega)$ and $g(\omega)$

- k_{t+1} is upward-sloping in q_{t+1}
- As q_{t+1} gets sufficiently large, $p(\omega) \to 0$ and $g(\omega) \to 1$, so we get $k_{t+1} = \bar{\omega}\kappa\eta$

Capital Supply-Demand



Equilibrium

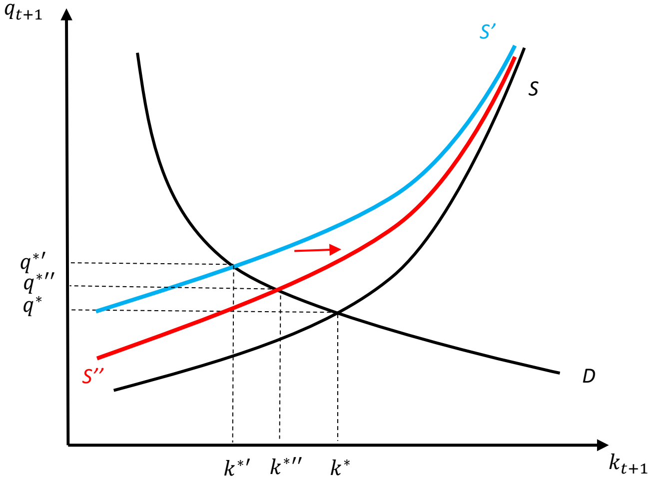
In equilibrium, new capital formation is too low, and the price of capital too high, relative to the model without agency costs $(\gamma = 0)$

- When S^e is higher, then $p(\omega)$ is lower and $g(\omega)$ is higher
- Both of these shift the capital supply curve out, closer to the unconstrained case
- When will S^e be relatively high?
  - When w_t is high (θ_t high)
  - If we redistributed labor endowments from lenders, L, to entrepreneurs, L^e

This all introduces **persistence** into the model where there was none in the frictionless case

- e.g. $\uparrow \theta_t \rightarrow \uparrow k_{t+1}$ and hence $\uparrow w_{t+1}$
- But $\uparrow w_{t+1} \rightarrow \uparrow k_{t+2}$
- And so on

Productivity Shock



"Debt-Deflation": Reallocation from L^e to L