Bernanke, Gertler, and Gilchrist (1999, *Handbook of Macroeconomics*)

ECON 70428: Advanced Macro: Financial Frictions

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Financial Accelerator in a Quantitative Business Cycle Framework

One of the most celebrated papers in modern macro

Basic idea:

- Asset price fluctuations influence balance sheet condition of firms
- Higher asset prices $\rightarrow$ better balance sheets
- But better balance sheets $\rightarrow$ better access to credit because of agency friction
- Better balance sheets $\rightarrow$ more investment and aggregate demand
- More aggregate demand $\rightarrow$ higher asset prices

A feedback loop, multiplier effect, or **accelerator** effect

Basic Framework

Underlying agency friction is similar to Carlstrom and Fuerst (1997)

Model is New Keynesian (sticky prices) with endogenously fluctuating price of capital (adjustment cost)

Agency friction applies to **entire** capital stock (as opposed to production of new investment goods in Carlstrom and Fuerst 1997)

Gets you **amplification** of shocks (as opposed to **propagation** in Carlstrom and Fuerst 1997)

LOG-LINEARIZED MODEL

Aggregate Demand Block

1. Resource constraint:

$$y_t = (C/Y)c_t + (I/Y)i_t + (G/Y)g_t + (C^e/Y)c_t^e$$

2. Euler equation bonds (IS):

$$c_t = -r_t + \mathbb{E}_t\, c_{t+1}$$

3. Euler equation capital:

$$\mathbb{E}_t\, r_{t+1}^k = (1-\epsilon)\,\mathbb{E}_t(y_{t+1} - k_{t+1} - x_{t+1}) + \epsilon\,\mathbb{E}_t\, q_{t+1} - q_t$$

4. Price of capital (adjustment cost):

$$q_t = \varphi(i_t - k_t)$$

5. Entrepreneur consumption:

$$c_t^e = n_t$$

6. **Lending spread**:

$$\mathbb{E}_t\, r_{t+1}^k - r_t = -\nu(n_t - (q_t + k_{t+1}))$$

Lending Spread and the Accelerator

The key to the model is the last condition:

$$\mathbb{E}_t\, r^k_{t+1} - r_t = -\nu(n_t - (q_t + k_{t+1}))$$

In a standard model, $\nu = 0$, so $\mathbb{E}_t\, r^k_{t+1} = r_t$

- Arbitrage equates the return to bonds and capital

N_t is entrepreneur net worth, Q_t is price of capital, K_{t+1} is new capital, $L_t = \frac{Q_t K_{t+1}}{N_t}$ is leverage

So $l_t = q_t + k_{t+1} - n_t$ is leverage log-linearized

If $L_t > 1$ (entrepreneur is levered), then $\uparrow Q_t \rightarrow \downarrow L_t$ holding everything else fixed

$\uparrow Q_t$ therefore $\rightarrow \downarrow E_t r^k_{t+1} - r_t$, which results in investment boom

Which leads to further $\uparrow Q_t$: multiplier/accelerator effect

Aggregate Supply Block

1. Production:
$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t$$
2. Labor market-clearing:
$$y_t - h_t - x_t - c_t = \eta^{-1} h_t$$
3. Phillips Curve:
$$\pi_t = -\kappa x_t + \beta\, \mathbb{E}_t\, \pi_{t+1}$$

- x_t is the linearized price markup (equivalently, inverse real marginal cost)
- Ω is the household labor share ($1-\Omega$ is entrepreneurial labor share)

Evolution of State Variables, Policy, and Exogenous Processes

1. Capital:

$$k_{t+1} = \delta i_t + (1-\delta) k_t$$

2. Net worth:

$$n_t = \frac{\gamma RK}{N}(r_t^k - r_{t-1}) + r_{t-1} + n_{t-1}$$

3. Taylor rule:

$$r_t^n = \rho r_{t-1}^n + \zeta \pi_{t-1} + \varepsilon_t^{rn}$$

4. Fisher relationship:

$$r_t^n = r_t - \mathbb{E}_t \, \pi_{t+1}$$

5. Productivity:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

6. Government spending:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g$$

MODEL DETAILS

Agents

1. Households (standard)
2. Wholesale firms (standard)
3. Retailers (this is where price stickiness comes in)
4. Capital goods producers (fairly standard, this is where you get adjustment cost and $Q_t \neq 1$)
5. **Entrepreneurs**: they accumulate physical capital subject to *idiosyncratic returns*, ω_{t+1}, financed via interperiod loans from a risk-neutral intermediary (effectively owned by household)
    - They have to liquidate and finance the *entire* capital stock each period
    - A fraction γ die off each period
    - Continuing entrepreneurs accumulate net worth and exiting ones consume it

Entrepreneurs

The problem is frankly not very well-specified by BGG

More complete expositions are in Christiano, Motto, and Rostagno (2014) and Carlstrom, Fuerst, and Paustian (2016)

Basic idea (dropping entrepreneur indexes):

1. Entrepreneurs wake up in period t with some physical capital chosen previously, K_t, via a loan from an intermediary
2. They receive an *idiosyncratic* return to capital into efficiency units, $\omega_t K_t$. $\mathbb{E}[\omega_t] = 1$
3. They lease this to production firms, earning rental rate RR_t (which equals the MPK of capital accounting for price markup), and are left over with $(1-\delta)$ of their capital
4. If return is bigger than interest payment, they liquidate all of capital stock and pay back intermediary; otherwise they default
5. Continuing intermediaries then borrow from an intermediary to purchase next period's capital, K_{t+1}

Return to Capital

The *aggregate* return to capital (so no ω) going from t to $t+1$ is:

$$\mathbb{E}_t \, R^k_{t+1} = \mathbb{E}_t \left\{ \frac{\left(\frac{\alpha Y_{t+1}}{K_{t+1} X_{t+1}} \right) + (1-\delta) Q_{t+1}}{Q_t} \right\}$$

Where $RR_{t+1} = \frac{\alpha Y_{t+1}}{K_{t+1} X_{t+1}}$, δ is the depreciation rate, Q_t is what they pay for capital in t, and Q_{t+1} is what it's worth in $t+1$

The *idiosyncratic return* (again, dropping entrepreneur indexes for ease of exposition) is:

$$\mathbb{E}_t \, \omega_{t+1} R^k_{t+1}$$

Loan Contract

An entrepreneur liquidates his/her capital stock each period and has to finance purchase of new capital stock each period via an intratemporal loan from an intermediary

Entrepreneur wakes up with net worth N_t – composed of accumulated returns from past capital investments plus wage from supplying labor inelastically, W_t^e

Must borrow $Q_t K_{t+1} - N_t$; N_t is net worth, $Q_t K_{t+1}$ is value of capital it is purchasing

Gross loan rate is Z_{t+1}

Realized return to a particular entrepreneur:

$$\omega_{t+1} R_{t+1}^k Q_t K_{t+1} - Z_{t+1} \left(Q_t K_{t+1} - N_t \right)$$

Default Cutoff

An entrepreneur will default if $\omega_{t+1} < \bar{\omega}_{t+1}$, defined via:

$$\bar{\omega}_{t+1} R^k_{t+1} Q_t K_{t+1} = Z_{t+1} \left(Q_t K_{t+1} - N_t \right)$$

Define leverage as $L_t = Q_t K_{t+1} / N_t$. This implies:

$$Z_{t+1} = \bar{\omega}_{t+1} R^k_{t+1} \frac{L_t}{L_t - 1}$$

Let ω_{t+1} be distributed log-normal, with CDF $\Phi(\cdot)$ and density $\phi(\cdot)$

Expected Entrepreneurial Return

The entrepreneur's expected outcome from getting a loan is:

$$R^k_{t+1} Q_t K_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} \omega_{t+1} \phi(\omega)_{t+1} d\omega_{t+1} - \\ (1 - \Phi(\bar{\omega}_{t+1})) Z_{t+1} (Q_t K_{t+1} - N_t)$$

Can eliminate Z_{t+1} using the cutoff value, to get:

$$f(\bar{\omega}_{t+1}) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega_{t+1} \phi(\omega_{t+1}) d\omega_{t+1} + (1 - \Phi(\bar{\omega}_{t+1})) \bar{\omega}_{t+1}$$

Entrepreneur's expected return (expressed relative to net worth) is therefore:

$$R^k_{t+1} L_t f(\bar{\omega}_{t+1})$$

Expected Lender Outcome

Lender's expected outcome is:

$$(1-\mu)R^k_{t+1}Q_tK_{t+1}\int_0^{\bar{\omega}_{t+1}}\omega_{t+1}\phi(\omega_{t+1})d\omega_{t+1}+ \\ (1-\Phi(\bar{\omega}_{t+1}))Z_{t+1}(Q_tK_{t+1}-N_t)$$

$\mu \geq 0$ is a bankruptcy cost – lender loses a fraction in bankruptcy

Eliminate Z_{t+1} and define:

$$g(\bar{\omega}_{t+1}) = (1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega_{t+1}\phi(\omega_{t+1})d\omega_{t+1} + (1-\Phi(\bar{\omega}_{t+1}))\bar{\omega}_{t+1}$$

Lender's expected return is therefore:

$$g(\bar{\omega}_{t+1})R^k_{t+1}\frac{L_t}{L_t-1}$$

Formal Contracting Problem

Lender is risk-neutral and has opportunity cost of funds of R_t, the (gross) risk-free rate

Contracting problem is to maximize expected entrepreneurial income subject to participation constraint for lender:

$$\max_{\bar{\omega}_{t+1}, L_t} \ \mathbb{E}_t \, R^k_{t+1} f(\bar{\omega}_{t+1}) L_t$$

s.t.

$$R^k_{t+1} g(\bar{\omega}_{t+1}) \frac{L_t}{L_t - 1} \geq R_t$$

Note: lender's required return is **predtermined** at t (R_t)

This means $\bar{\omega}_{t+1}$ is state-contingent on realization of R^k_{t+1}

FOC

The FOC are (letting Λ_{t+1} denote the multiplier on the lender's participation constraint):

$$\bar{\omega}_{t+1}: \; \mathbb{E}_t \left\{ R^k_{t+1} f'(\bar{\omega}_{t+1}) + \Lambda_{t+1} R^k_{t+1} g'(\bar{\omega}_{t+1}) \right\} = 0$$

$$L_t: \; \mathbb{E}_t \left\{ R^k_{t+1} f(\bar{\omega}_{t+1}) + \Lambda_{t+1} \left[R^k_{t+1} g(\bar{\omega}_{t+1}) - R_t \right] \right\} = 0$$

$$\Lambda_{t+1}: \; R^k_{t+1} g(\bar{\omega}_{t+1}) L_t = (L_t - 1) R_t$$

Linearized FOC

The FOC linearized about steady state (ignoring expectations operators):

$$\Psi\widehat{\omega}_{t+1} = \lambda_{t+1}$$

$$r^k_{t+1} - r_t + l_t + \Theta_f\widehat{\omega}_{t+1} = \lambda_{t+1}$$

$$r^k_{t+1} - r_t + \Theta_g\widehat{\omega}_{t+1} = \frac{1}{L-1} l_t$$

Where $\lambda_{t+1} = \ln\Lambda_{t+1} - \ln\Lambda$, $\widehat{\omega}_{t+1} = \ln\bar{\omega}_{t+1} - \ln\bar{\omega}$, and $l_t = \ln L_t - \ln L$, and:

$$\Psi = \frac{\bar{\omega}f''(\bar{\omega})}{f'(\bar{\omega})} - \frac{\bar{\omega}g''(\bar{\omega})}{g'(\bar{\omega})}$$

$$\Theta_g = \frac{\bar{\omega}g'(\bar{\omega})}{g(\bar{\omega})}$$

$$\Theta_f = \frac{\bar{\omega}f'(\bar{\omega})}{f(\bar{\omega})}$$

Combining Altogether

One gets:

$$\mathbb{E}_t\, r^k_{t+1} - r_t = \frac{\Psi}{\Psi(L-1) - \Theta_f L} l_t$$
$$= -\nu\left(n_t - (q_t + k_{t+1})\right)$$

Note that:

$$f(\bar{\omega}) + g(\bar{\omega}) = 1 - \mu \int_0^{\bar{\omega}} \omega\phi(\omega)d\omega$$

If $\mu = 0$ (no bankruptcy cost), then $f(\bar{\omega}) = -g(\bar{\omega})$, which then implies $\Psi = 0$

So $\mu > 0 \rightarrow \nu > 0$

CALIBRATION AND IRFS

Calibration

I'm not going to attempt to very closely recreate their calibration, which is not very well laid out

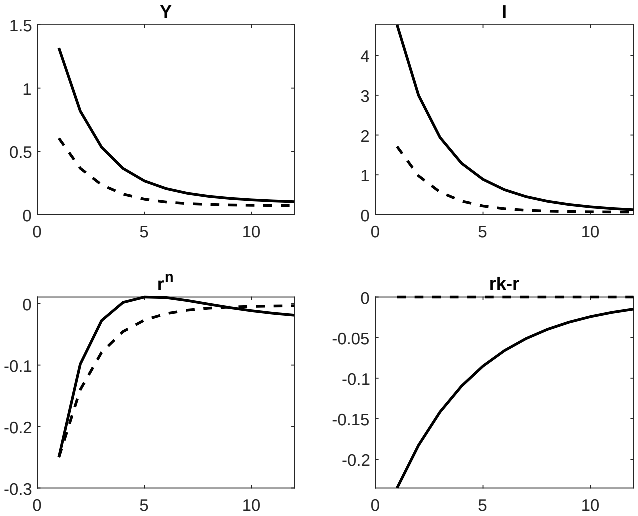
The key parameter to focus on is ν

I'm going to set it to $\nu = 0.2$; this roughly replicates the IRFs they report in the paper (see Figures 3 and 4)

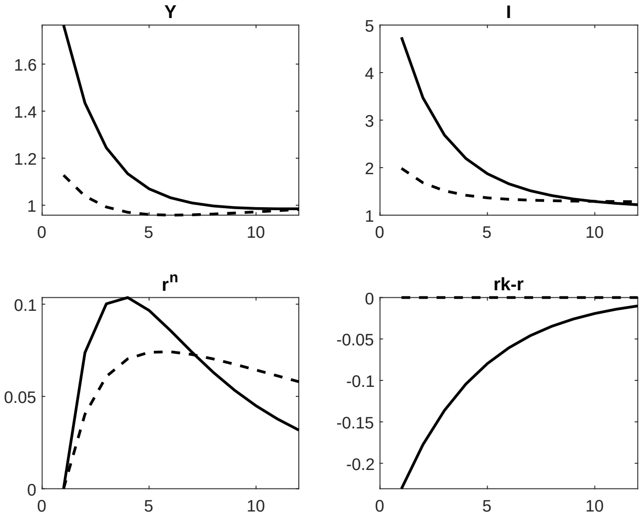
One issue – they use $\rho_a = 1$, so productivity follows a random walk. This turns out to be important for amplification

I'm not going to consider extensions – investment delays and heterogeneous firms

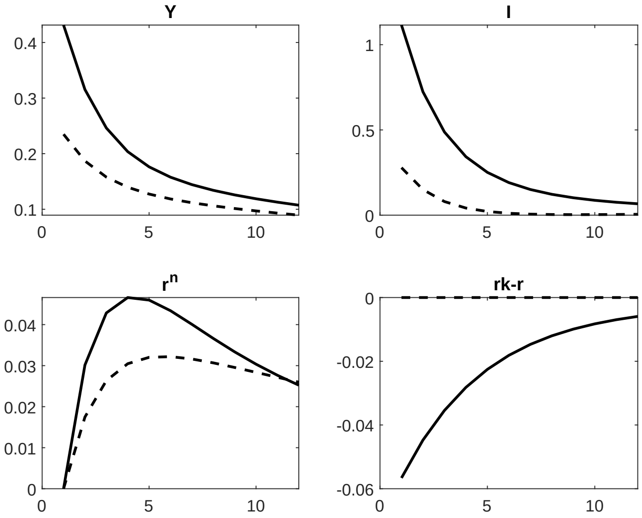
Monetary Policy Shock



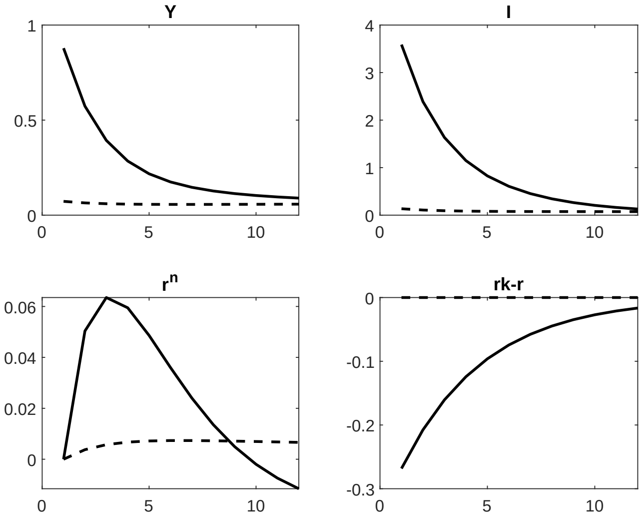
Productivity Shock



Government Spending Shock



Net Worth Shock



Observations

In all these cases, the agency friction **amplifies** the effects of the shock

The interest rate spread declines and output goes up more (solid lines) when $\nu > 0$ compared to when $\nu = 0$

The basic mechanism is the relationship between asset prices (the price of capital, Q_t), net worth, and the interest rate spread

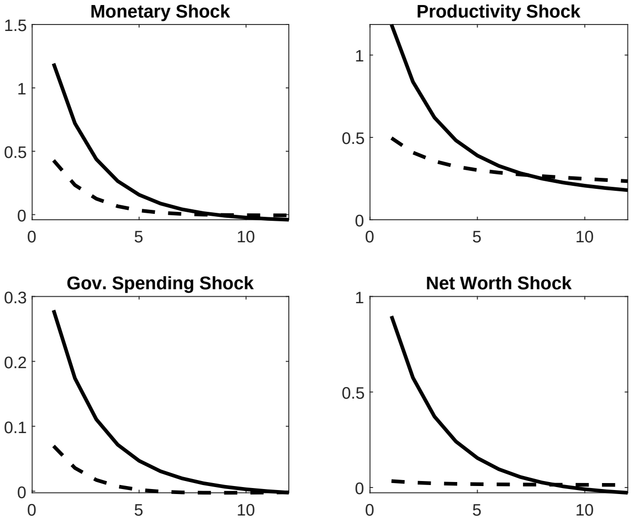
Observations Continued

The model is specified (and calibrated) wherein all of the shocks would cause Q_t to rise (in a model without agency frictions); this is because of capital adjustment costs, not because of the agency friction itself (as in Carlstrom and Fuerst 1997)

But the asset price increase, other factors held constant, lowers $\mathbb{E}_t \, r_{t+1}^k - r_t$ if $\nu > 0$.

But this triggers more investment (and more aggregate demand), resulting in more increases in Q_t – a “multiplier” or “accelerator” effect

Responses of Q to Shocks



Less Persistent Productivity Shock

Their result that the financial friction amplifies the productivity shock is sensitive to the assumed persistence

Even slightly smaller values of ρ_a reverse things

Intuition: with sticky prices, output is (partially) demand determined

How much aggregate demand reacts to a productivity shock depends on persistence

Extreme Case Intuition

Easiest to see with an exogenous money rule than a Taylor rule

Suppose quantity theory holds:

$$M_t V_t = P_t Y_t$$

Suppose $V_t = \bar{V}$ fixed, and $P_t = \bar{P}$ fixed in short run

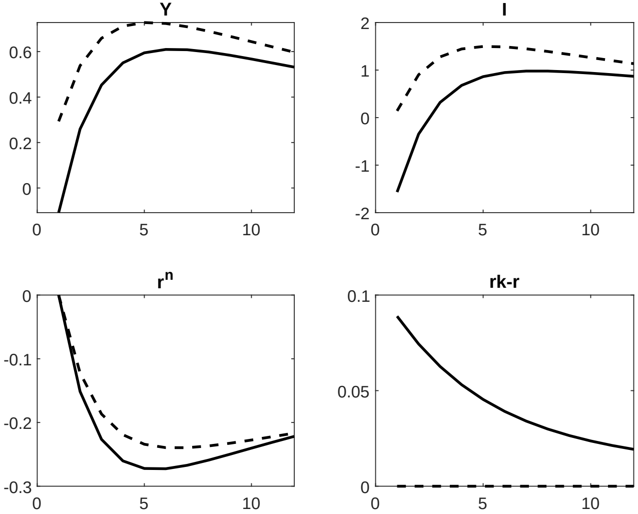
Then a productivity shock can't change output in short run without a money supply response

But because of positive wealth effect, $C_t \uparrow$. With increase in C_t and no change in Y_t, $\downarrow I_t$

But this means $\downarrow Q_t$ and friction **dampens** investment demand

Typical result: financial frictions **amplify** effects of demand shocks but **dampen** effects of supply shocks

Productivity Shock with $\rho_a = 0.95$



Differences Relative to Carlstrom and Fuerst (1997)

Underlying friction is very similar, but results are quite different

Principal differences:

1. NK (BGG) vs. RBC (CF)
2. Friction applies to whole capital stock (BGG) vs. production of new investment goods (CF)
3. Price of capital fluctuates because of adjustment costs (BGG) vs. endogenously due to agency friction (CF)

In CF, amount of investment you can do is tied down to net worth, which is slow-moving

Similar force at play in BGG, but net worth jumps a lot more because Q moves due to adjustment costs. This, plus friction applying to bigger component of production (entire capital stock) gets more amplification