Carlstrom and Fuerst (1997, *American Economic Review*)

ECON 70428: Advanced Macro: Financial Frictions

Eric Sims

University of Notre Dame

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Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis

This paper embeds basic mechanism from Bernanke and Gertler (1989) into a canonical RBC model

The agency cost is simpler, too

- More like a bankruptcy cost than a monitoring cost with random auditing (given by optimal auditing probabilities)

Principal conclusions:

1. Net worth becomes relevant endogenous state variable
2. Reallocation of resources from households (lenders) to entrepreneurs (borrowers) is expansionary
3. Responses of output and investment to a productivity shock are hump-shaped, which is in-line with the data

Differences Relative to Bernanke and Gertler (1989)

1. Persistent aggregate uncertainty (not iid productivity shock)
2. Incomplete depreciation (capital is slow-moving state)
3. Infinitely-lived agents
4. Variable labor supply (amplification)

Model is essentially a textbook RBC model with the Bernanke and Gertler (1989) agency cost mechanism

- Though, as noted above, the agency friction is a bit simpler and more like a bankruptcy cost

Partial Equilibrium Contracting Problem

Entrepreneurial Heterogeneity

Continuum of entrepreneurs

An entrepreneur transforms i units of consumption goods into ωi of new capital goods

ω is stochastic and iid across entrepreneurs

It satisfies $\mathbb{E}\,\omega = 1$, where the expectations operator is across entrepreneurs

ω has density $\phi(\omega)$ and distribution function $\Phi(\omega)$; support $(0, \infty)$ (e.g. log-normal)

Lenders can't observe ω. If they want to learn it, they have to pay μi, $\mu > 0$

Optimal contract: entrepreneurs won't misreport ω, but when they default lenders will have to pay the monitoring cost

Intraperiod Loan

Consider an entrepreneur with net worth n who wants to do i of investment; assume $i > n$

Entrepreneur gets an *intraperiod* loan from a lender at r^k

- e.g. borrow $i - n$ in middle of period t to finance i, agreeing to pay $1 + r^k$
- ω is realized and entrepreneur has ωi of new physical capital
- If $\omega i \geq (1 + r^k)(i - n)$, entrepreneur pays back loan
- Otherwise entrepreneur defaults

Cutoff ω, $\bar{\omega}$, satisfies:

$$\bar{\omega} = \frac{(1 + r^k)(i - n)}{i}$$

Expected Outcome: Entrepreneur

Let the price of capital be q. The loan is paid back in units of capital goods, not units of consumption

Expected entrepreneurial income from getting a loan:

$$q\left[\int_{\bar{\omega}}^{\infty}\omega i\phi(\omega)d\omega-(1-\Phi(\bar{\omega}))(1+r^{k})(i-n)\right]$$

Using cutoff $\bar{\omega}$, this can be written:

$$qi\underbrace{\left[\int_{\bar{\omega}}^{\infty}\omega\phi(\omega)d\omega-(1-\Phi(\bar{\omega}))\bar{\omega}\right]}_{f(\bar{\omega})}=qif(\bar{\omega})$$

Expected Outcome: Lender

The "lender" is called a "capital market mutual fund" (CMF)

Lender's expected income from making a loan:

$$q\left[\int_0^{\bar{\omega}} \omega i \phi(\omega) - \underbrace{\Phi(\bar{\omega})\mu i}_{\text{Agency Cost}} + (1-\Phi(\bar{\omega}))(1+r^k)(i-n)\right]$$

Using definition of $\bar{\omega}$, can write this:

$$qi\underbrace{\left[\int_0^{\bar{\omega}} \omega\phi(\omega)d\omega - \Phi(\bar{\omega})\mu + (1-\Phi(\bar{\omega}))\bar{\omega}\right]}_{g(\bar{\omega})} = qig(\bar{\omega})$$

Note:

$$g(\bar{\omega}) + f(\bar{\omega}) = 1 - \Phi(\bar{\omega})\mu$$

Optimal Contract

Need to assume entrepreneur always wants external funds (i.e. can't save too much, otherwise will outgrow need for external funds and hence the agency cost)

Problem is to pick terms of contract to maximize entrepreneur's expected profit, subject to participation constraint that lender gets at least opportunity cost (which is 1, since the loan is intraperiod)

$$\max_{i,\bar{\omega}} \; qif(\bar{\omega})$$

s.t.

$$qig(\bar{\omega}) \geq (i - n)$$

q and n (price of capital and net worth) are taken as given.

Looks "weird" to pick $\bar{\omega}$, but this is really picking r^k given i and q, and n

Let Constraint Hold with Equality

Eliminate i by subbing in constraint:

$$\max_{\bar{\omega}} \; qn \left[\frac{f(\bar{\omega})}{1 - qg(\bar{\omega})} \right]$$

Can drop qn and further write this:

$$\max_{\bar{\omega}} \; f(\bar{\omega}) \Big[1 - q \left[1 - \Phi(\bar{\omega})\mu - f(\bar{\omega}) \right] \Big]^{-1}$$

Optimality Condition

FOC is:

$$1 = q\left[1 - \Phi(\bar{\omega})\mu + \Phi'(\bar{\omega})\mu\frac{f(\bar{\omega})}{f'(\bar{\omega})}\right]$$

Note, via Leibniz's rule, $f'(\bar{\omega}) = -(1 - \Phi(\bar{\omega}))$. Hence:

$$1 = q\left[1 - \Phi(\bar{\omega})\mu - \Phi'(\bar{\omega})\mu\frac{f(\bar{\omega})}{1 - \Phi(\bar{\omega})}\right]$$

If $\mu > 0$, then $q > 1$

- Installed capital more valuable than extra consumption due to agency friction associated with creating new capital

Investment Supply

Optimality condition gives us $\bar{\omega}(q)$, where $\bar{\omega}'(q) > 0$

Investment is then implicitly:

$$i(q,n) = \frac{n}{1 - qg(\bar{\omega}(q))}$$

Expected new capital is then:

$$I^s(q,n) = \underbrace{i(q,n)}_{\text{investment}} \underbrace{(1 - \mu\Phi(\bar{\omega}(q)))}_{\text{Agency Cost}} = \frac{1 - \mu\Phi(\bar{\omega}(q))}{1 - qg(\bar{\omega}(q))} n = \Lambda(q) n$$

- Capital supply is (i) increasing in q and (ii) increasing in n (just like in Bernanke and Gertler 1989)
- Further, it is *linear* in n so this will aggregate nicely

GE Model

Basic Setup

Three principal actors: representative production firm, identical households, and entrepreneurs

Households risk averse with respect to consumption, have variable labor supply, and can purchase capital (for rental to the firm). Mass of η

Entrepreneurs are risk-neutral and supply labor inelastically. They discount future more heavily than household. Mass of $1 - \eta$

- Only entrepreneurs can transform consumption goods into new capital goods

Production is via CRTS technology in total capital (which can be leased from either household or entrepreneurs), household labor, and entrepreneurial labor

Representative Household

Solves:

$$\max_{c_t, l_t, k_{c,t}} \; \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \nu(1 - l_t) \right\}$$

s.t.

$$c_t + q_t \left[k_{c,t+1} - (1-\delta) k_{c,t} \right] = r_t k_{c,t} + w_t l_t$$

FOC:

$$\nu = \frac{w_t}{c_t}$$

$$q_t = \mathbb{E}_t \left\{ \beta \frac{c_t}{c_{t+1}} \left[r_{t+1} + (1-\delta) q_{t+1} \right] \right\}$$

Firm

Output produced according to:

$$Y_t = \theta_t K_t^{\alpha_1} H_t^{\alpha_2} H_{e,t}^{1-\alpha_1-\alpha_2}$$

FOC:

$$\begin{aligned}
r_t &= \alpha_1 \theta_t K_t^{\alpha_1 - 1} H_t^{\alpha_2} H_{e,t}^{1-\alpha_1-\alpha_2} \\
w_t &= \alpha_2 \theta_t K_t^{\alpha_1} H_t^{\alpha_2 - 1} H_{e,t}^{1-\alpha_1-\alpha_2} \\
x_t &= (1 - \alpha_1 - \alpha_2) \theta_t K_t^{\alpha_1} H_t^{\alpha_2} H_{e,t}^{-\alpha_1-\alpha_2}
\end{aligned}$$

Entrepreneurs

Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_{e,t}$$

- $\gamma < 1$: extra discounting (so they don't outgrow need for external funds)
- Risk-neutral
- Supply one unit of labor inelastically

Timing and Middle of Period Net Worth

Wake up with $k_{e,t}$ units of capital; rent to firm at r_t

Supply one unit of labor inelastically to firm at x_t *After* production

takes place, but before investment decision, have $(1-\delta)$ physical capital left over, which is worth q_t

Middle of period net worth, n_t, satisfies:

$$n_t = x_t + [r_t + (1-\delta)q_t]\, k_{e,t}$$

Investment Decision

In middle of period, entrepreneur then borrows $i_t - n_t$ from CMF (effectively, from household) at $1 + r_t^k$

Contracting problem and FOC same as above, just with time subscripts:

$$i_t = \frac{1}{1 - q_t g(\bar{\omega}_t)} n_t$$

$$q_t = \left[1 - \Phi(\bar{\omega}_t) - \Phi'(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{1 - \Phi(\bar{\omega}_t)}\right]^{-1}$$

- After the choices implied by these FOC, an entrepreneur draws ω_t. If $\omega_t < \bar{\omega}_t$, the entrepreneur defaults
- In default, $c_{e,t} = k_{e,t+1} = 0$

Solvent Entrepreneurs

Entrepreneurs drawing $\omega_t \geq \bar{\omega}_t$ are solvent.

They then make a consumption savings-decision

Budget constraint is:

$$c_{e,t} + q_t k_{e,t+1} = y_t^e = \omega_t i_t - (1 + r_t^k)(i_t - n_t)$$

From perspective of middle of period, the RHS is predetermined at this point

Expected budget constraint in $t + 1$:

$$c_{e,t+1} + q_{t+1} k_{e,t+2} = \\ [x_{t+1} + k_{e,t+1}(r_{t+1} + q_{t+1}(1 - \delta))] \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})}$$

Optimization Problem for Solvent Entrepreneurs

$$\max_{k_{e,t+1}} = y_t^e - q_t k_{e,t+1} +$$

$$\beta\gamma\,\mathbb{E}_t\left[\left[x_{t+1} + k_{e,t+1}(r_{t+1} + q_{t+1}(1-\delta))\right]\frac{q_{t+1}f(\bar{\omega}_{t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1})} - q_{t+1}k_{e,t+2}\right]$$

FOC is:

$$q_t = \beta\gamma\,\mathbb{E}_t\left\{\left[r_{t+1} + q_{t+1}(1-\delta)\right]\frac{q_{t+1}f(\bar{\omega}_{t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1})}\right\}$$

Aggregation

New capital production by entrepreneurs is:

$$i_t \int_0^{\infty} \omega_t \phi(\omega_t) d\omega_t - \mu i_t \int_0^{\bar{\omega}_t} \phi(\bar{\omega}_t) d\omega_t = i_t(1 - \mu \Phi(\bar{\omega}_t))$$

Total capital stock used in production is weighted sum of capital across households and entrepreneurs:

$$K_t = (1 - \eta) k_{c,t} + \eta k_{e,t}$$

Physical capital depreciates at δ. Aggregate capital evolves according to:

$$K_{t+1} = \underbrace{\eta i_t (1 - \mu \Phi(\bar{\omega}_t))}_{= I_t (1 - \mu \Phi(\bar{\omega}_t))} + (1 - \delta) K_t$$

Like an adjustment cost

Other Aggregate Conditions

$$C_t = (1-\eta)c_t + \eta c_{e,t}$$
$$H_t = (1-\eta)l_t$$
$$H_{e,t} = \eta$$
$$\theta_t = (1-\rho_\theta) + \rho_\theta \theta_{t-1} + s_\theta e_{\theta,t}$$
$$c_{e,t} + q_t k_{e,t+1} = q_t f(\bar{\omega}_t) i_t$$

- The above for $c_{e,t}$ is **average** entrepreneurial consumption and next period capital; a bit of abuse of notation
- Resource constraint is standard:

$$Y_t = (1-\eta)c_t + \eta c_{e,t} + I_t$$

Log-Normal Stuff

ω_t is distributed log-normally, $\ln \omega_t \sim N(\mu, \sigma^2)$
Expected value:

$$\mathbb{E}[\omega_t] = \exp\left[\mu + \frac{1}{2}\sigma^2\right]$$

Let NN be the normal cdf. For a log-normal variable, we have its cdf is:

$$\Phi(\bar{\omega}_t) = NN\left(\frac{\ln \bar{\omega}_t - \mu}{\sigma}\right)$$

Let nn be the normal density. The derivative of the log-normal cdf, $\Phi'(\cdot) = \phi(\cdot)$ (i.e. the log-normal density), is:

$$\Phi'(\bar{\omega}_t) = nn\left(\frac{\ln \bar{\omega}_t - \mu}{\sigma}\right)(\bar{\omega}\sigma)^{-1}$$

Steady State Targets

Note:

$$g(\bar{\omega}_t) = \Phi\left(\frac{\ln \omega_t - \mu - \sigma^2}{\sigma}\right) - \Phi(\bar{\omega}_t) + (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t$$

$$f(\bar{\omega}_t) = 1 - \Phi(\bar{\omega}_t)\mu - g(\bar{\omega}_t)$$

Target a **credit spread** of:

$$(1 + r^k)q - 1 = 0.0187/4$$

- Since loan is intratemporal, opportunity cost is just 1. $(1 + r^k)q$ is what you get from making a loan; 1 is the opportunity cost

Target a **bankruptcy rate** of:

$$\Phi(\bar{\omega}) = 0.0097$$

For both Euler equations to hold, must have:

$$\frac{\gamma q f(\bar{\omega})}{1 - q g(\bar{\omega})} = 1$$

Other Parameters and Steady State Values

σ and γ are chosen to hit the steady state financial targets:
$\sigma = 0.205$ and $\gamma = 0.947$

Fix $\beta = 0.99$, $\alpha_1 = 0.36$, $\delta = 0.2$, $\eta = 0.1$, and $\mu = 0.25$

Implies $c_e/n = 0.067$, $n/i = 0.38$, and $q = 1.024$

θ_t is AR(1) with $\rho = 0.95$

Net Worth Shock

It's not part of stochastic process, but we can consider reallocating resources from households to entrepreneurs

Essential friction: entrepreneurs don't have enough net worth in steady state (as evidenced by $q > 1$)

Typical entrepreneur's budget constraint:

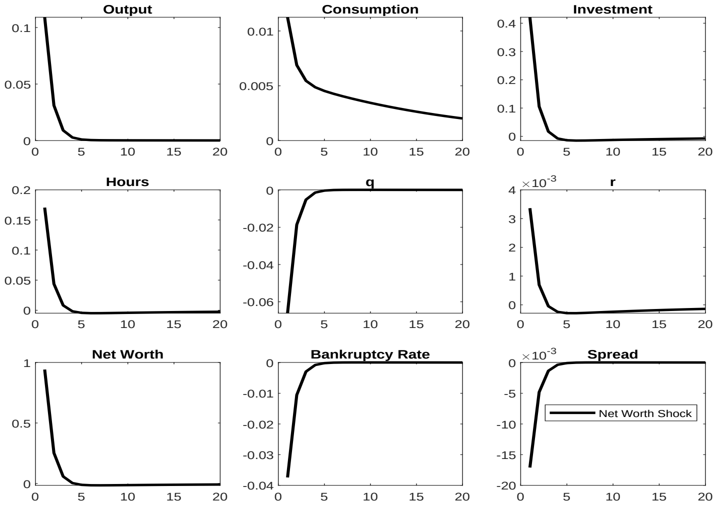
$$n_t = x_t + [r_t + q_t(1-\delta)]\, k_{e,t} + s_n e_{n,t}$$

Typical household's budget constraint:

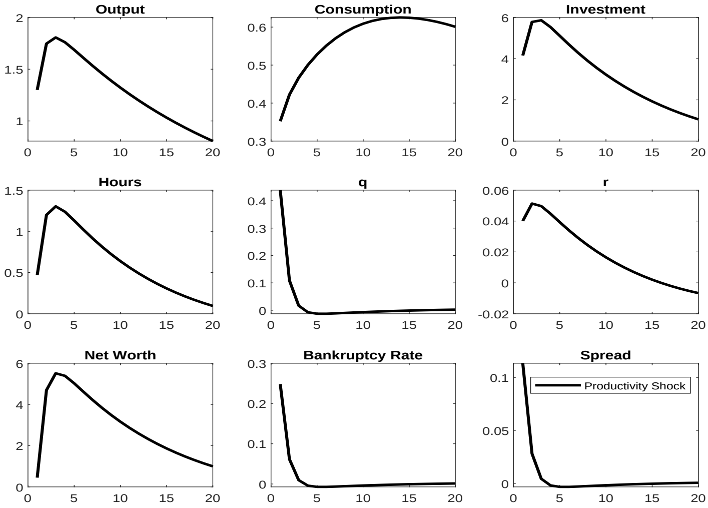
$$c_t + q_t(k_{c,t+1} - k_{c,t}) = r_t k_{c,t} + w_t l_t - s_n e_{n,t}$$

In aggregate, the $e_{n,t}$ vanishes in the resource constraint – just a one-time reallocation of wealth from households to entrepreneurs (e.g. a tax)

Net Worth Shock



Productivity Shock



Results

Key thing is that net worth becomes a relevant endogenous state variable (not so in RBC model without frictions)
Because of this:

1. An iid redistribution shock has persistent effects
2. Responses of output and investment to productivity shock are **hump-shaped**
   - This matches the autocorrelation of output and investment growth in the data (Cogley and Nason, 1995)
   - Isomorphic to an investment adjustment cost specification
   - Why? It takes time for net worth to accumulate in response to productivity shock (slow-moving state)
   - Optimally delay investment until period when agency costs are lowest (when net worth peaks)

Intuition for Hump-Shape

To fix intuition, think about $\delta = 1$ (like Bernanke and Gertler 1989)

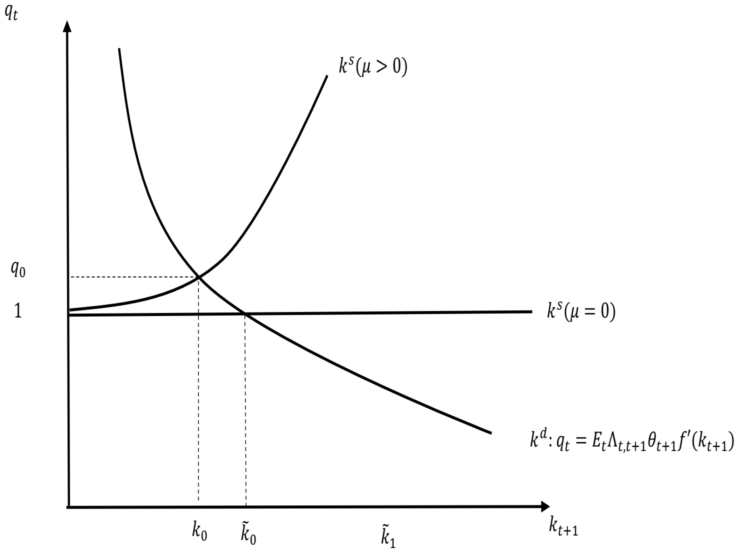
Capital demand:

$$q_t = \mathbb{E}_t \, \Lambda_{t,t+1}\theta_{t+1} f'(k_{t+1})$$

Capital supply:

1. If $\mu = 0$, then RBC world: $q_t = 1$ perfectly elastic
2. If $\mu > 0$, then agency cost world; $q_t > 1$, capital supply upward-sloping and shifts with net worth

Initial Equilibrium: RBC vs. Agency Friction



Responses to a Persistent Productivity Shock: Impact

k_{t+1} goes up because a persistent productivity shock shifts capital demand right

- If net worth doesn't react a ton (here I've assumed it doesn't move at all within period), then k_{t+1} goes up more in the RBC case than when capital supply is upward-sloping

Could get big enough kick that net worth goes up so much that capital supply shifts right at the same time as demand, but not what we typically see (see also discussion from Quadrini 2011)

Persistent Productivity Shock Continued

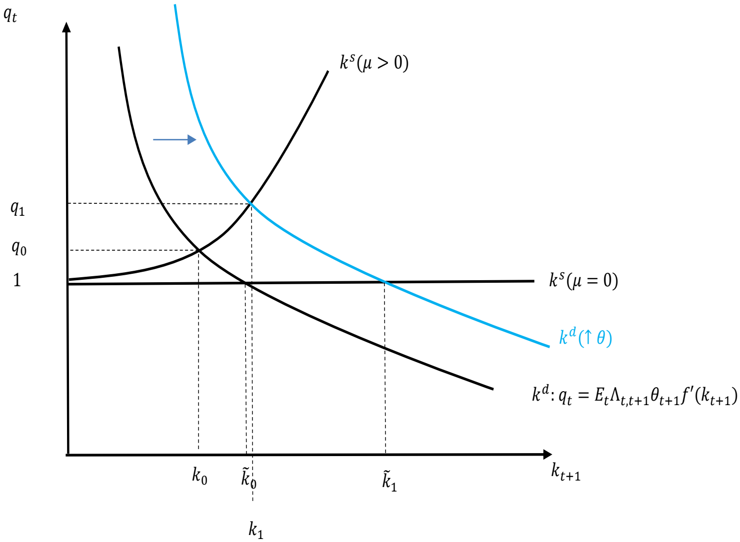
After impact, dynamics kick in:

- Since the productivity shock is mean-reverting, demand starts to shifting back in immediately
  - If capital supply is elastic, then the biggest response of investment is **on impact**

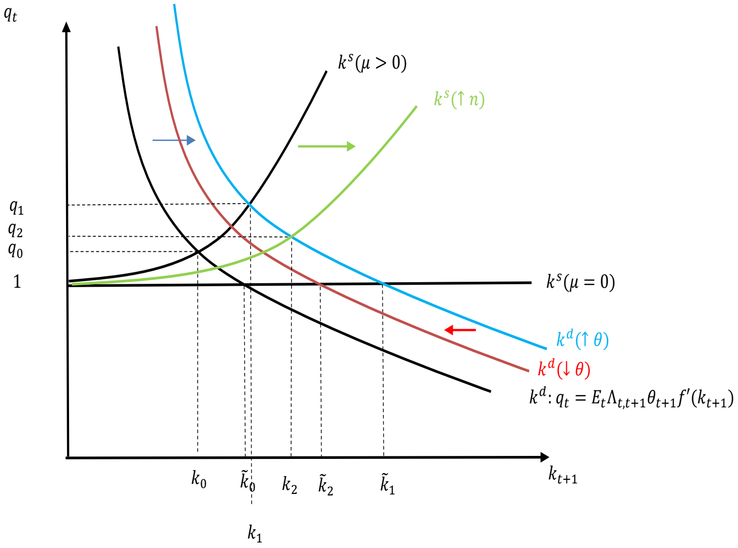
  But if there are agency costs, net worth goes up over time, so capital supply shifts out over time

  This can overcome the inward shift of capital demand, and generate a hump-shaped investment response like we see in the quantitative model

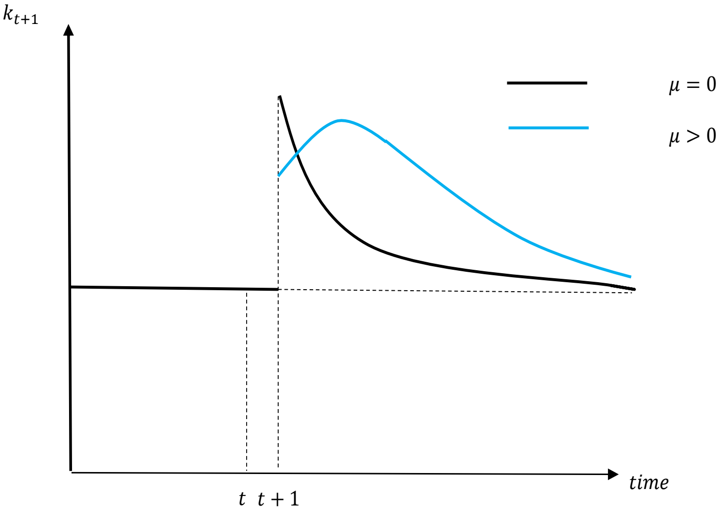
Initial Effect of Productivity Shock: RBC vs. Agency Friction



Dynamic Effects of Productivity Shock: RBC vs. Agency Friction



Investment IRF: RBC vs. Agency Friction



Drawbacks

The model has a couple of drawbacks

In particular, cyclicality of bankruptcy rate and credit spreads

Positive productivity shock $\rightarrow$ increase in q $\rightarrow$ increases in bankruptcy rate, $\Phi(\bar{\omega}_t)$, and risk premia, $(1 + r_t^k)q_t - 1$

- Both of which are counterfactual