Carlstrom, Fuerst, and Paustian (2017, *AEJ: Macro*)

ECON 70428: Advanced Macro: Financial Frictions

Eric Sims

University of Notre Dame

Spring 2021

Targeting Long Rates in a Model with Segmented Markets

Introduces long-bonds as generalized consols into a monetary New Keynesian DSGE model

But two twists:

1. Constrained financial intermediaries (FIs) stand between borrowers (long-maturity) and savers (short-maturity)
   - i.e. they "borrow short and lend long" – maturity transformation
   - They are impatient relative to households and subject to what amounts to a binding leverage constraint
2. Bond market is **segmented** by maturity – savers cannot save via long maturity bonds and borrowers can borrow via short maturity

Segmentation plus constrained intermediaries generates long-short spreads and a term premium, even to first order

QE Shock

FIs hold two perfectly substitutable long bonds – privately issued "investment bonds" to finance investment and government bonds

QE shock: reduces the amount of government bonds FIs hold

This "frees" up balance sheet resources to purchase private investment bonds

Pushes long bond prices up, spreads down, and stimulates demand

Caveat: not really modeling this as a monetary shock. Really more like a **fiscal** shock

Normative Implications

In the model, the term premium is present to first order

Essentially the term premium equals the **investment wedge** – distortion showing up in the dynamic Euler equation for capital

QE shocks lower this distortion; credit shocks raise

Other shocks interact with the distortion as well

Points to normative implication: use "balance sheet" policies to neutralize fluctuations in this wedge

i.e. "term premium targeting" is a good idea

The Model

Households

Indexed by $s \in [0, 1]$ and supply differentiated labor, but perfect consumption insurance so identical along all other dimensions (Erceg, Henderson, and Levin 2000)

Save via **short-term** deposits (D) and borrow via **long-term** investment bonds (F), which take the generalized consol form with decay $\kappa \in [0, 1]$

Segmented in the sense that they can't save via the long-term investment bonds

Own the physical capital stock and lease to firms

Are **required** to finance new investment by issuing long-term debt

Preferences and Constraints

Preferences:

$$U_t = \ln(C_t - hC_{t-1}) - B\frac{H_t(s)^{1+\eta}}{1+\eta} + \beta\,\mathbb{E}_t\,U_{t+1}$$

Budget constraint:

$$C_t + p_t^k \widehat{I}_t + \frac{D_t}{P_t} + \frac{F_{t-1}}{P_t} \leq$$

$$w_t(s)H_t(s) + R_t K_t - T_t + \frac{D_{t-1}}{P_t}R_{t-1}^d + \frac{Q_t(F_t - \kappa F_{t-1})}{P_t} + div_t + \Pi_t^y + \Pi_t^k$$

Capital accumulation:

$$K_{t+1} = \widehat{I}_t + (1-\delta)K_t$$

Loan in advance:

$$p_t^k \widehat{I}_t \leq \frac{Q_t(F_t - \kappa F_{t-1})}{P_t}$$

FOC

Let λ_t be the multiplier on accumulation, Λ_t the multiplier on budget, and ϑ_t the multiplier on loan in advance

FOC for consumption, bonds, capital are standard:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h \, \mathbb{E}_t \, \frac{1}{C_{t+1} - hC_t}$$

$$\Lambda_t = \beta \, \mathbb{E}_t \, \Lambda_{t+1} R_t^d \Pi_{t+1}^{-1}$$

$$\lambda_t = \beta \, \mathbb{E}_t [\Lambda_{t+1} R_{t+1} + \lambda_{t+1}(1 - \delta)]$$

FOC for Investment and Bonds

These are impacted by the loan in advance constraint:

$$\lambda_t = p_t^k(\Lambda_t + \vartheta_t)$$

$$(\Lambda_t + \vartheta_t)Q_t = \beta\,\mathbb{E}_t\,\Pi_{t+1}^{-1}\left[(\Lambda_{t+1} + \vartheta_{t+1})\,\kappa Q_{t+1} + \Lambda_{t+1}\right]$$

If $\vartheta_t = 0$, these would be standard for the model with long bonds

Define:

$$M_t = 1 + \frac{\vartheta_t}{\Lambda_t}$$

ϑ_t/Λ_t is the consumption value of relaxing the loan in advance constraint (same idea as q_t . . .)

Re-Written FOC

In terms of M_t, we could write:

$$p_t^k M_t = \mathbb{E}_t \, m_{t,t+1} \left[R_{t+1} + (1-\delta) p_{t+1}^k M_{t+1} \right]$$

$$Q_t M_t = \mathbb{E}_t \, m_{t,t+1} \Pi_{t+1}^{-1} \left[1 + \kappa Q_{t+1} M_{t+1} \right]$$

Where $m_{t,t+1} = \frac{\beta \Lambda_{t+1}}{\Lambda_t}$ is the real stochastic discount factor

If $M_t = 1$, these would be standard asset pricing conditions

$M_t \neq 1$ distorts these – in particular, it represents an **investment wedge** in the capital Euler equation (Chari, Kehoe, and McGrattan 2007)

Wage-Setting

Labor packer transforms household labor into aggregate labor, H_t. Elasticity of substitution ϵ_w

Households subject to Calvo wage-setting friction (θ_w). Optimality conditions:

$$w_t^{\#} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\widehat{f}_{1,t}}{\widehat{f}_{2,t}}$$

$$\widehat{f}_{1,t} = \left(\frac{w_t}{w_t^{\#}}\right)^{\epsilon_w(1+\eta)} BH_t^{1+\eta} + \theta_w \beta \, \mathbb{E}_t \, \Pi_{t+1}^{\epsilon(1+\eta)} \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w(1+\eta)} \widehat{f}_{1,t+1}$$

$$\widehat{f}_{2,t} = \Lambda_t H_t \left(\frac{w_t}{w_t^{\#}}\right)^{\epsilon_w} + \theta_w \beta \, \mathbb{E}_t \, \Pi_{t+1}^{\epsilon_w - 1} \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w} \widehat{f}_{2,t+1}$$

Production

Continuum of firms subject to Calvo pricing friction (θ_p), facing downward-sloping demand from final good aggregator (elasticity of substitution ϵ_p)

$$Y_t(l) = A_t K_t(l)^{\alpha} H_t(l)^{1-\alpha}$$

Firms are heterogeneous but all hire capital and labor in same *proportion*, face the same factor prices, and face same marginal cost

Factor demands:

$$w_t = mc_t(1-\alpha)A_t \left(\frac{K_t}{H_t}\right)^{\alpha}$$

$$R_t = mc_t \alpha A_t \left(\frac{K_t}{H_t}\right)^{\alpha-1}$$

Pricing Conditions

$$\Pi_t^{\#} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{x_{1,t}}{x_{2,t}}$$

$$x_{1,t} = \Lambda_t mc_t Y_t + \theta_p \beta \, \mathbb{E}_t \, \Pi_{t+1}^{\epsilon_p} x_{1,t+1}$$

$$x_{2,t} = \Lambda_t Y_t + \theta_p \beta \, \mathbb{E}_t \, \Pi_{t+1}^{\epsilon_p - 1} x_{2,t+1}$$

Where $\Pi_t^{\#} = P_t^{\#} / P_t$, i.e. the relative reset price

New Capital Producer

Uses I_t as input (unconsumed output) to produced $\widehat{I_t}$ (new capital) via:

$$\widehat{I_t} = \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

Purchases I_t at P_t and sells $\widehat{I_t}$ at P_t^k, $p_t^k = P_t^k / P_t$

FOC:

$$p_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] = 1 - \beta\,\mathbb{E}_t\,\frac{\Lambda_{t+1}}{\Lambda_t} p_{t+1}^k \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

Financial Intermediaries

FIs can hold (i) private investment bonds and (ii) government bonds, B_t

These are both generalized consols with exactly the same payout structure and no default risk; hence trade at the same price and are perfectly substitutable

FIs finance themselves with deposits and net worth

Balance sheet:

$$\frac{B_t}{P_t} Q_t + \frac{F_t}{P_t} Q_t = \frac{D_t}{P_t} + N_t$$

Profits

Let $R_t^L = \frac{1+\kappa Q_t}{Q_{t-1}}$ be the return on long bonds (either investment or government)

Let $L_t = \frac{Q_t\left(\frac{F_t}{P_t}+\frac{B_t}{P_t}\right)}{N_t}$ be leverage

Assume FI cannot choose its leverage (more on this below)

Gross nominal profit is:

$$PROF_t = (1+\kappa Q_t)\left[F_{t-1}+B_{t-1}\right] - R_{t-1}^d D_{t-1}$$

Using balance sheet condition, we get:

$$prof_t = \Pi_t^{-1}\left[\left(R_t^L - R_{t-1}^d\right)L_{t-1} + R_{t-1}^d\right]N_{t-1}$$

Dividends and Net Worth Evolution

Law of motion for real dividend, div_t, is:

$$div_t + N_t[1 + f(N_t)] \leq \Pi_t^{-1}\left[\left(R_t^L - R_{t-1}^d\right)L_{t-1} + R_{t-1}^d\right]N_{t-1}$$

What this says is that net profits are split between dividends and the change in net worth

To see this, let $R_t^d = 1 + r_t^d$ (gross versus net). Ignoring the $f(N_t)$ term, we'd have:

$$N_t - \Pi_t^{-1}N_{t-1} \leq \Pi_t^{-1}\left[\left(R_t^L - R_{t-1}^d\right)L_{t-1} + r_{t-1}^d\right]N_{t-1} - div_t$$

Net Worth Adjustment Cost

$f(N_t)$ is a net worth adjustment cost

$f(N) = f'(N) = 0$, where N is steady state net worth

$$f(N_t) = \frac{\psi_N}{2}\left(N_t/N - 1\right)^2$$

This is very similar to the notion of the dividend adjustment cost in Jermann and Quadrini (2012)

Problem Taking Leverage as Given

Problem is:

$$\max_{N_t} \; \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\zeta)^t \Lambda_t div_t$$

s.t.

$$div_t + N_t[1 + f(N_t)] \leq \Pi_t^{-1} \left[\left(R_t^L - R_{t-1}^d \right) L_{t-1} + R_{t-1}^d \right] N_{t-1}$$

Where $\zeta \leq 1$ is **additional discounting**

FOC:

$$\Lambda_t \left[1 + f(N_t) + f'(N_t) N_t \right] = \beta\zeta \, \mathbb{E}_t \, \Lambda_{t+1} \Pi_{t+1}^{-1} \left[\left(R_{t+1}^L - R_t^d \right) L_t + R_t^d \right]$$

Extra Discounting and the Adjustment Cost

If $\zeta = 1$, in SS this FOC would be:

$$1 = \beta \left[\left(R^L - R^d \right) L_t + R^d \right]$$

To be consistent with HH FOC for deposits, would have to have $R^L = R^d$

- Extra discounting keeps FI from arbitraging away lending spread and gets you a steady state spread, $R^L > R^d$
- The net worth adjustment costs prevent this arbitraging from happening in a dynamic sense

Leverage

Problem above assumes FI takes leverage, L_t, as given, not something it can choose

Holdup problem / limited enforcement:

$$\mathbb{E}_t\, V_{t+1} \geq \Psi_t L_t N_t\, \mathbb{E}_t\, \Lambda_{t+1} R^L_{t+1} \Pi^{-1}_{t+1}$$

LHS: enterprise value of being a FI, where V_t is the FI value function

RHS: what FI can take by defaulting – $L_t N_t$ is total assets multiplied by gross return

Ψ_t: fraction FI can abscond with

Basic idea: depositors only allow FI to lever up to the point where FI will not want to default in equilibrium

Value Function

If the adjustment cost were not there, value function would be linear in net worth:

$$V_t = \Lambda_t X_t N_{t-1}$$

Where:

$$X_t = \Pi_t^{-1} \left[\left(R_t^L - R_{t-1}^d \right) L_{t-1} + R_{t-1}^d \right]$$

With net worth adjustment cost, value function includes an additional ter:

$$V_t = \Lambda_t X_t N_{t-1} + g_t$$

One can show g_t is independent of N and 0 in steady state

Trick

If Ψ_t were an exogenous random variable (as we normally would have), the constraint binding in steady state would imply that L_t would be decreasing in net worth

- i.e. we cannot treat L_t as given by the FI

Trick: specify function for Ψ_t that undoes this effect, with Ψ_t decreasing in N_t in a particular way:

$$\Psi_t = \Phi_t \left[1 + \frac{1}{N_t} \frac{\mathbb{E}_t \, g_{t+1}}{\mathbb{E}_t \, \Lambda_{t+1} X_{t+1}} \right]$$

Where now Φ_t is the exogenous component of this constraint – i.e. the "credit shock"

Leverage

Given they are extra impatient, the constraint will be bind in the steady state – FI will borrow as much as it can given existence of positive spreads

Thus, we can focus on case where the constraint always binds

Given this assumption on Ψ_t, we get an expression for leverage that is indeed independent of any FI choices:

$$L_t = \frac{\mathbb{E}_t \Lambda_{t+1}\Pi_{t+1}^{-1}}{\mathbb{E}_t \Lambda_{t+1}\Pi_{t+1}^{-1} + (\Phi_t - 1)\,\mathbb{E}_t \Lambda_{t+1}\Pi_{t+1}^{-1}\frac{R_{t+1}^L}{R_t^d}}$$

This validates finding the FOC for N_t taking L_t as given

The Really Cool Thing . . .

If you log-linearize this expression for L_t about the steady state, you get:

$$\mathbb{E}_t\, r^L_{t+1} - r^d_t = \nu_1 l_t + \nu_2 \phi_t$$

This **exactly** the same as the condition in Bernanke, Gertler, and Gilchrist (1999)

The lending spread depends positively on leverage, $l_t = \ln L_t$

It also has an exogenous component given by $\phi_t = \ln \Phi_t$, the credit shock

Modeling QE

There is kind of an incomplete modeling of QE

What matters in the model is **how much government bonds the FI has to hold**

- Given exogenous (to the FI) leverage, having to hold fewer government bonds allows FI to buy more private investment bonds, which allows them to arbitrage away some of the spread
- QE experiment in their paper is reducing government bonds held by intermediaries, thereby effectively freeing up balance sheet space

Let $b_t = B_t / P_t$. Assume:

$$\ln b_t = (1 - \rho_{1,b} - \rho_{2,b}) \ln b + \rho_{1,b} \ln b_{t-1} + \rho_{2,b} \ln b_{t-2} + s_b \varepsilon_{b,t}$$

Negative shock to $\varepsilon_{b,t}$ is a positive QE shock . . . but there is also a fiscal interpretation here

Bond Yields and the Term Premium

R_t^L is the holding period return on long bonds

This is *not* the yield to maturity (except in SS)

Gross yield, $R_{y,t}$, satisfies:

$$Q_t = \frac{1}{R_{y,t}} + \frac{\kappa}{R_{y,t}^2} + \frac{\kappa^2}{R_{y,t}^3} + \dots$$

Or:

$$R_{y,t} = Q_t^{-1} + \kappa$$

EH Bond and the Term Premium

Introduce a hypothetical expectations hypothesis bond with the same decay parameter, but priced according to short-term safe rate, not stochastic discount factor:

$$Q_t^{EH} = \frac{1 + \kappa \, \mathbb{E}_t \, Q_{t+1}^{EH}}{R_t^d}$$

Yield to maturity:

$$R_{y,t}^{EH} = \left(Q_t^{EH} \right)^{-1} + \kappa$$

Gross term premium is thus:

$$TP_t = \frac{R_{y,t}}{R_{y,t}^{EH}}$$

Calibration and SS

Will not go into details here They target a term premium of 100 basis points annualized (1.0025 quarterly) and a leverage ratio of $L = 6$

This implies a value of $\zeta = 0.9852$ (with $\beta = 0.99$)

They estimate some parameters, and some of them are weird

I'm just going to use standard parameters where possible (e.g. price and wage rigidity)

Log-Linearization

It is useful for intuition to log-linearize a few conditions

Log-linearized capital Euler equation:

$$p_t^k + m_t = \sum_{j=0}^{\infty} [\beta(1-\delta)]^j \, \mathbb{E}_t \left[(1-\beta(1-\delta))\, r_{t+j+1} - \left(r_{t+j}^d - \pi_{t+j+1} \right) \right]$$

$m_t = \ln M_t$ is basically an **investment wedge** in the capital Euler equation

Recall $M_t = 1 + \vartheta_t / \Lambda_t$

$\vartheta_t \geq 0$ measures "tightness" of loan in advance constraint

Linearized Bond Prices and Returns

Bond prices are just PDVs of returns

$$q_t^{EH} = -\sum_{j=0}^{\infty} (\beta\kappa)^j r_{t+j}^d$$

$$q_t = -\sum_{j=0}^{\infty} \left(\frac{\beta\kappa}{TP} \right)^j r_{t+1+j}^L$$

The term premium is a function of differences in the long bond price and the hypothetical expectations hypothesis price

$$tp_t = -(1 - \kappa\beta / TP) q_t + (1 - \kappa\beta) q_t^{EH}$$

The Term Premium and the Investment Wedge

Combining these, we get:

$$tp_t = (1 - \kappa\beta / TP) \sum_{j=0}^{\infty} \left(\frac{\beta\kappa}{TP} \right)^j r^L_{t+1+j} - (1 - \kappa\beta) \sum_{j=0}^{\infty} (\beta\kappa)^j r^d_{t+j}$$

But m_t is approximately ($TP \approx 1$):

$$m_t = \mathbb{E}_t \sum_{j=0}^{\infty} (\kappa\beta)^j \, \mathbb{E}_t \left(\kappa\beta q_{t+j+1} - q_{t+j} - r^d_{t+j} \right) \approx$$

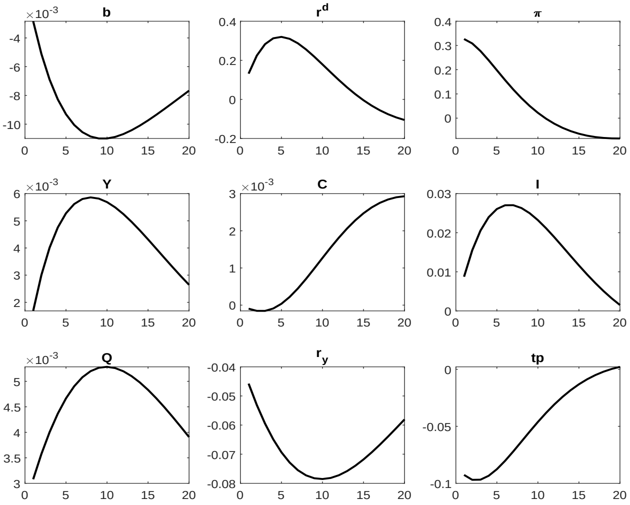
$$\mathbb{E}_t \sum_{j=0}^{\infty} (\kappa\beta)^j \, \mathbb{E}_t (r^L_{t+j+1} - r^d_{t+j})$$

Therefore:

$$m_t \approx \frac{tp_t}{1 - \beta\kappa}$$

Term premium is essentially the investment wedge

QE Shock



QE Shock Intuition

Recall linearized leverage condition:

$$\mathbb{E}_t \, r^L_{t+1} - r^d_t = \nu_1 l_t + \nu_2 \phi_t$$

Immediate, partial equilibrium effect of lower b_t is to lower l_t

This results in lower lending spread

But PDV of that is the term premium / investment wedge

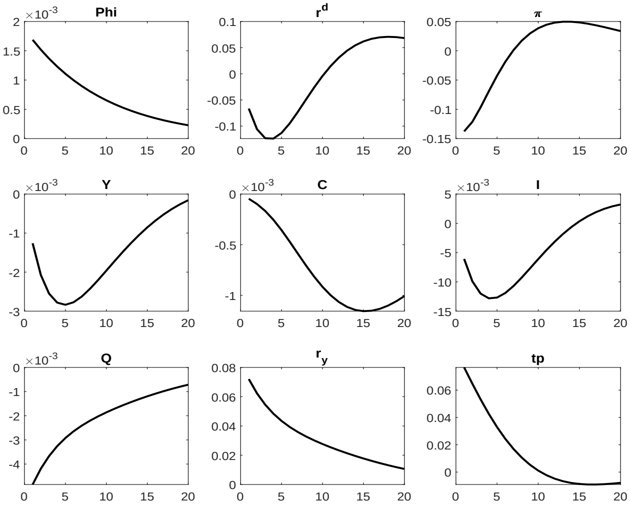
That wedge stimulates aggregate demand

Alternative intuition: less b_t frees up space on FI balance sheet to buy f_t, which raises q_t and stimulates investment

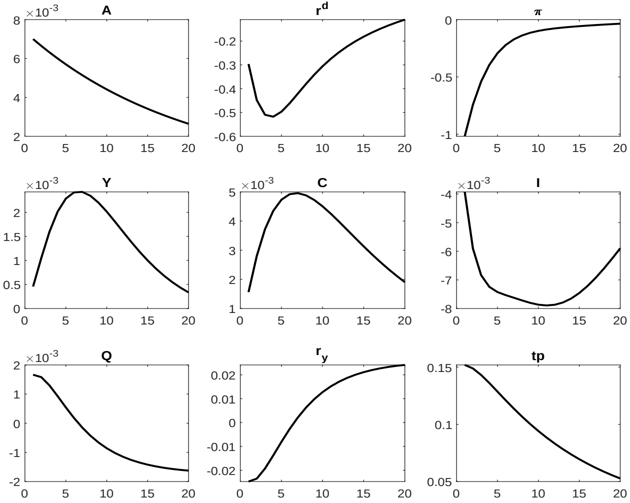
Note net worth adjustment cost is key:

- If $\psi_n = 0$, then net worth adjusts to shock in such a way as to not affect leverage (FI just pays out as dividend proceeds from selling government bonds)

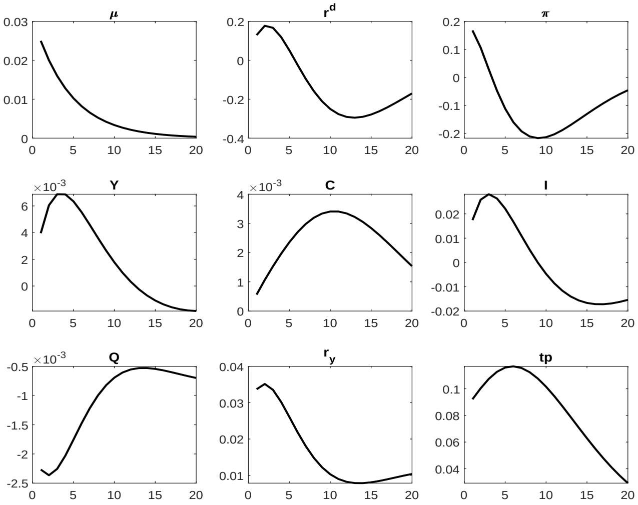
Credit Shock



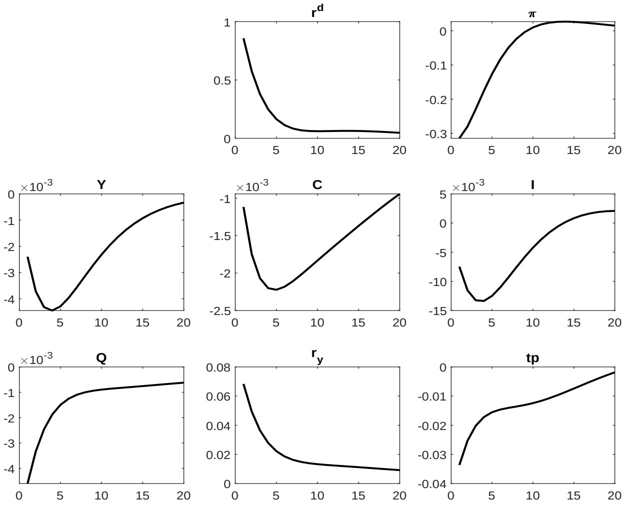
Productivity Shock



Investment Shock



Monetary Shock



Term Premium Targeting

Recall from above: $m_t \approx \frac{tp_t}{1-\beta\kappa}$

Optimal policy is about **undoing** distortions/wedges

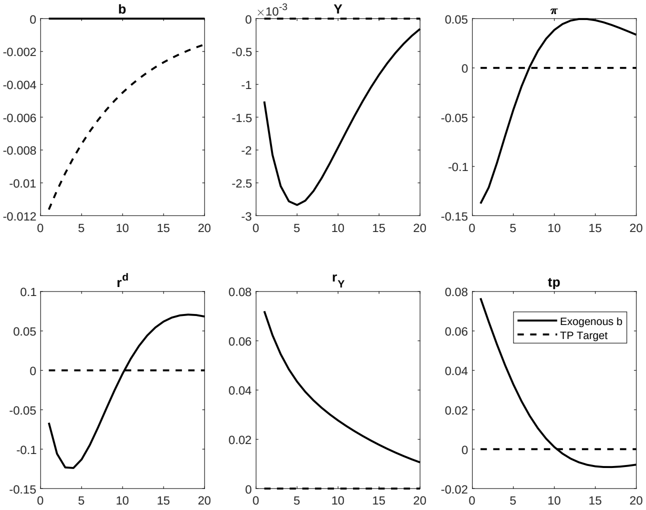
Suggests it might be optimal to engage in **term premium targeting**

Adjust b_t to target $tp_t = 0$ ($TP_t = TP$)

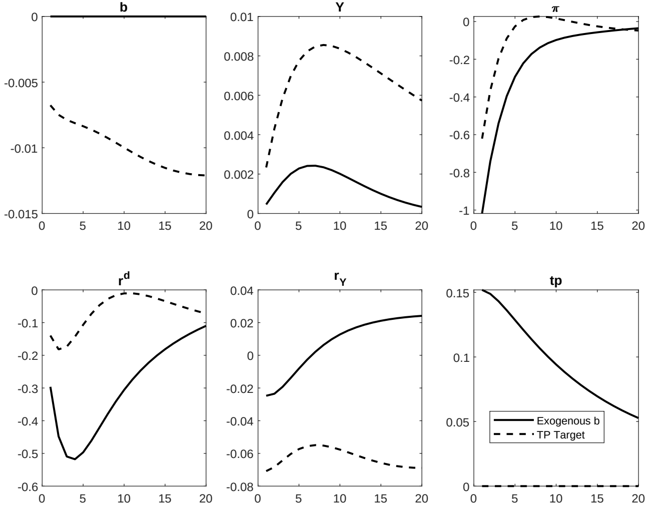
1. This will **completely neutralize** credit shocks – similar result in Sims and Wu (2021b)

2. Results in "better" responses to other shocks as well

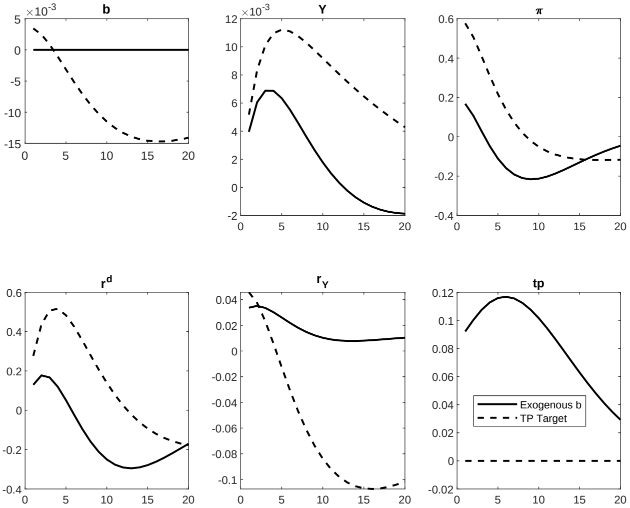
Credit Shock, Term Premium Target



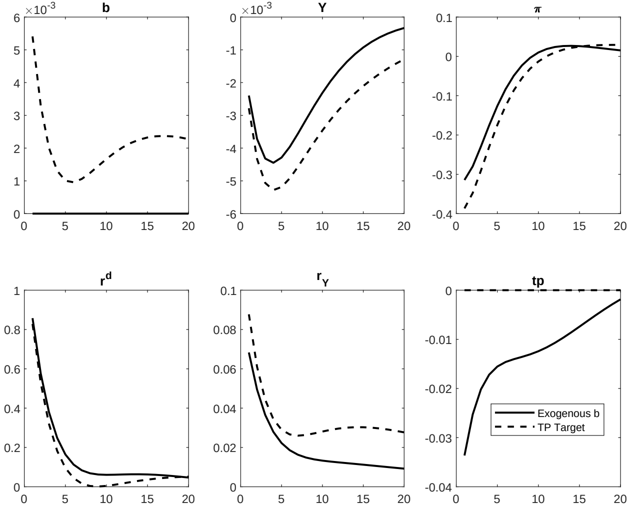
Productivity Shock, Term Premium Target



MEI Shock, Term Premium Target



Monetary Shock, Term Premium Target



What this Paper Doesn't Do

This paper doesn't:

1. Really model QE as a monetary shock – in a sense it's a fiscal shock

2. Doesn't speak to substitutability of QE for conventional monetary policy at the ZLB

That's what we'll focus on next – Sims and Wu (2021a, 2021b)