# Bank Runs and the Diamond-Dybvig (1983) Model 

ECON 43370: Financial Crises

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## Bank Runs

- Bank runs, broadly construed, are a recurrent theme in economic history
- Financial crises are about runs on short term bank debt
- Situation in which short term liability holders "run" en masse to liquidate their savings in financial intermediaries, forcing intermediaries to engage in asset sales that could leave them insolvent
- Why are runs so prevalent?
- Why do people hold short term bank debt (e.g. deposits) if it is nevertheless susceptible to runs?
- What type of policies can be used to prevent/reduce/mitigate runs?


## Diamond-Dybvig (1983) Model

- The Diamond-Dybvig (1983) model is a celebrated contribution that:

1. Provides a precise definition of liquidity
2. Exposits the benefits of the liquidity transformation that financial intermediaries do
3. Points out the perils of liquidity transformation - susceptibility to runs
4. Provides framework to think about policies

- Will follow Diamond (2007). Focus on deposits but basic idea applies to any short term debt obligations issued by a bank or bank-like financial intermediary


## Model Basics

- There are three periods, indexed by $T: T=0, T=1, T=2$. $T=0$ is the "present" and $T=1,2$ measure the "future"
- There are (many) households who are (ex-ante) identical and are endowed with 1 in $T=0$ and will need to consume in either $T=1$ or $T=2$
- Idiosyncratic uncertainty: individual does not know (at $T=0$ ) whether she will be type 1 (needs to consume in $T=1$ ) or type 2 (can wait to consume until $T=2$ ). Type revealed in $T=1$
- But there is no aggregate uncertainty: a fixed fraction, $t \in[0,1]$, of households will be type 1 and a fixed fraction $1-t$ type 2
- There are two assets:

1. Storage (cash): save 1 in $T=0$, have 1 available to consume in either $T=1$ or $T=2$
2. Illiquid investment opportunity: save 1 in $T=0$, can get $r_{1}$ (gross) if liquidated (sold) in $T=1$, and $r_{2} \geq r_{1}$ if liquidated in $T=2$

## Preferences

- An individual household has utility:

$$
U(c)=1-\frac{1}{c}
$$

- Its expected utility is simply the probability-weighted sum of utility flows depending on which type it ends up being:

$$
\mathbb{E}[U]=t U\left(c_{1}\right)+(1-t) U\left(c_{2}\right)
$$

- Where $c_{1}$ and $c_{2}$ are consumption at each date depending on type
- Consumption allocations: $c_{1}=c_{2}=1$ if storage, $c_{1}=r_{1}$ and $c_{2}=r_{2}$ if invest


## Numerical Example

- Suppose $r_{1}=1, r_{2}=2$ on the investment project
- Suppose $t=\frac{1}{4}$
- The expected return (gross) from investing in the project is:

$$
\mathbb{E}[R]=\frac{1}{4} \times 1+\frac{3}{4} \times 2=\frac{7}{4}>1
$$

- The expected return (gross) on storage is just 1
- Expected utility from storage and investing are:

$$
\begin{aligned}
\mathbb{E}[U]^{\text {store }} & =\frac{1}{4} \times 0+\frac{3}{4} \times 0=0 \\
\mathbb{E}[U]^{\text {invest }} & =\frac{1}{4} \times 0+\frac{3}{4}\left(1-\frac{1}{2}\right)=\frac{3}{8}
\end{aligned}
$$

- Household prefers investment to storage


## Liquidity

- We can think about the liquidity of an asset as the discount one has to pay for "early" liquidation

$$
I=\frac{r_{1}}{r_{2}}
$$

- Since $r_{2} \geq r_{1}$ (by assumption), $l \leq 1$
- The further I is from 1 , the less liquid is the asset
- Cash is perfectly liquid, $I=1$ - you get 1 regardless of when you access it
- Investment is less liquid, $I=\frac{1}{2}$
- Though in this example you still prefer to hold the less liquid investment


## Alternative Example with Less Liquid Investment

- Suppose that early liquidation of the investment incurs a cost of $1-\tau, \tau \geq 0$. So you get $(1-\tau) r_{1}$ for early liquidation
- Liquidity of investment in above example is then:

$$
I=(1-\tau) \frac{1}{2} \leq \frac{1}{2}
$$

- How big must $\tau$ be for household to not want to do the investment?

$$
\mathbb{E}[U]^{\text {invest }}=\frac{1}{4}\left(1-\frac{1}{1-\tau}\right)+\frac{3}{4}\left(1-\frac{1}{2}\right)<0
$$

- Can show $\tau>\frac{3}{5}$ makes investment undesirable relative to storage


## Example with $\tau=\frac{2}{3}$

- Expected utility from storage versus investment:

$$
\begin{aligned}
\mathbb{E}[U]^{\text {store }} & =0 \\
\mathbb{E}[U]^{\text {invest }} & =\frac{1}{4} \times\left(1-\frac{1}{\frac{1}{3}}\right)+\frac{3}{4} \times\left(1-\frac{1}{2}\right)=-\frac{1}{8}
\end{aligned}
$$

- Now household prefers storage to investment!
- Even though expected (gross) return to investment is higher:

$$
\begin{aligned}
\mathbb{E}[R]^{\text {store }} & =1 \\
\mathbb{E}[R]^{\text {invest }} & =\frac{1}{4} \times \frac{1}{3}+\frac{3}{4} \times 2=\frac{19}{12}>1
\end{aligned}
$$

- If project is sufficiently illiquid and/or household is sufficiently risk averse (i.e. $u^{\prime \prime}(C)<0$ ), then household may not want to directly invest in positive net return projects


## A Bank and Liquidity Transformation

- A mutual bank (no equity, not trying to make profit for itself) can potentially step in and make households better off regardless of whether households would directly fund the investment project or not
- How? In essence by exploiting law of large numbers and engaging in what amounts to provision of insurance
- An individual household is uncertain about when she will need to consume: this gives rise to a preference for liquidity
- But in the aggregate there is no uncertainty - exactly the fraction $t$ of households will be type 1 and $1-t$ will be type 2
- Bank can pool resources from many households exploiting this lack of aggregate uncertainty and offer households an asset that is more liquid than the investment project that the household prefers to both direct investment and storage


## Deposits

- Same setup as earlier: $r_{1}=1, r_{2}=2, t=\frac{1}{4}$, and $\tau=0$
- Suppose bank offers household an asset with the following payout structure: $r_{1}^{d}=1.28$ and $r_{2}^{d}=1.813$ (gross return depending on date of liquidation)
- This is more liquid than the investment opportunity: $\frac{1.28}{1.813}=0.706>\frac{1}{2}$
- How does this work? Suppose there are 100 households and exactly 25 will need to withdraw in period $T=1$
- Bank takes 100 in $T=0$ and puts it into 100 units of the investment (assume $r_{1}$ and $r_{2}$ independent of amount invested)
- Will need to liquidate $25 \times 1.28=32$ units of the investment to raise necessary funds in $T=1$, leaving 68 invested
- These 68 will generate 136 in income in $T=2$, which can be distributed to remaining 75 deposit holders for $r_{2}=\frac{136}{75}=1.813$


## Which Does Household Prefer?

- An individual household has three options: storage (expected gross return 1), deposits (expected gross return 1.68), or direct investment (expected gross return 2)
- Which does it prefer? Expected utilities:

$$
\begin{aligned}
\mathbb{E}[U]^{\text {store }} & =0 \\
\mathbb{E}[U]^{\text {invest }} & =\frac{3}{8} \\
\mathbb{E}[U]^{\text {deposit }} & =\frac{1}{4} \times\left(1-\frac{1}{1.28}\right)+\frac{3}{4} \times\left(1-\frac{1}{1.813}\right)=0.391>\frac{3}{8}
\end{aligned}
$$

- A household's best option is deposits!
- Household willing to tolerate a lower expected return on deposits because of higher liquidity of deposits relative to direct investment
- Can make this even starker if $\tau>0$


## Consumption Smoothing and Preference for Liquidity

- We have assumed household is risk averse and uncertain about when it will need to consume
- Given risk aversion ( $\left.U^{\prime \prime}(c)<0\right)$, household has incentive to smooth consumption across states (i.e. type 1 or type 2)
- If it directly invests in project, marginal utility $\left(U^{\prime}(C)\right)$ is high if type 1 (gets comparatively low return) and low if type 2 (gets comparatively high return)
- Would like to potentially reallocate some consumption from type 2 state (low marginal utility) to type 1 state (higher marginal utility) - i.e. would like something more liquid
- Would even be willing to sacrifice some expected return to get this


## Liquidity Transformation is Like Insurance

- The bank is engaging in liquidity transformation
- It is creating an asset (deposit, which is a liability to bank but asset to household) that is more liquid than the underlying asset it is investing in
- In so doing, it can make households better off
- This is essentially functioning just like insurance - give up some consumption in "good states" (low marginal utility, type 2) by paying a "premium" to get some extra consumption in "bad states" (high marginal utility, type 1)
- The bank can offer this, just like an insurance company, by playing law of large numbers
- With aggregate uncertainty things would be more complicated but the basic gist would be the same


## Nash Equilibrium

- With many households and a mutual bank, the outcome described above is a Nash Equilibrium
- Everyone is behaving optimally given beliefs about how others are going to play $\Rightarrow$ no incentive to deviate
- If I wake up in $T=1$ and am revealed type 2 , I do worse by withdrawing in $T=1\left(r_{1}^{d}=1.28\right)$ than by waiting until $T=2\left(r_{2}^{d}=1.813\right)$
- Provided I think other type 2 s are going to wait, it's optimal to wait, all type $2 s$ will do this, and then beliefs are self-fulfilling
- When would it make sense to withdraw in $T=1$ even if I don't have to?
- Only if I think I will get back less than 1.28 in $T=2$
- I think (enough) other type 2's are going to withdraw "early"
- I'm worried the bank's investments are going to go bad


## Good vs. Bad Equilibrium

- Let's not worry about the "bank's investments are going to go bad" reason for withdrawing early - this requires some aggregate uncertainty we don't want to worry about, though important in the real world
- Let's focus on multiplicity of equilibria with no aggregate uncertainty
- Good equilibrium: what we just described
- Bad equilibrium: type 2 's withdraw early in $T=1$ because they expect other type 2's to withdraw early as well, which will cause the bank to fail and make everyone (weakly) worse off


## Early Withdrawals

- Let $f$ denote the fraction of depositors who withdraw in $T=1 ; f \geq t$
- In our example, withdrawals in $T=1$ are due $r_{1}^{d}=1.28$. With 100 deposits, bank must liquidate $128 \times f$ of the investment to meet this withdrawal demand
- This leaves $(100-128 f)$ invested, which itself earns a return of 2 which can be distributed to the remaining $(1-f) 100$ depositor holds in $T=2$ :

$$
r_{2}^{d}=\frac{2(100-128 f)}{(1-f) 100}
$$

- When $f=t=1 / 4$, then we get $r_{2}^{d}=1.813$
- But if $f>t$ (some type $2 s$ withdraw), then $r_{2}^{d}<1.813$.


## Expectations

- Let $\widehat{f}$ be the expectation of each household about what $f$ will be (i.e. the fraction who will withdraw in $T=1$ )
- Suppose $\widehat{f}=\frac{1}{2}$ - you think half the population is going to withdraw, or $1 / 3$ of the type 2 's withdraw earlier than needed
- Is this expectation self-fulfilling? If $\widehat{f}=\frac{1}{2}$, then:

$$
\widehat{r}_{2}^{d}=\frac{2(100-128 \widehat{f})}{(1-\widehat{f}) 100}=1.44
$$

- This is less than what was promised ( $r_{2}^{d}=1.813$ ), but nevertheless better than what you get for withdrawing today
- So it cannot be optimal for type 2 s to withdraw early given this forecast (they're better off waiting), so $\widehat{f}=\frac{1}{2}$ is not self-fulfilling, because even if it is believed by everyone, only $f=t=\frac{1}{4}$ will withdraw, so not an equilibrium
- So $\widehat{f}=f=t=\frac{1}{4}$ is a Nash Equilibrium


## The Bad Equilibrium

- Suppose instead that $\widehat{f}=\frac{3}{4}$. Then people will believe they will get:

$$
\widehat{r}_{2}^{d}=\frac{2(100-128 \widehat{f})}{(1-\widehat{f}) 100}=0.32
$$

- This is (significantly) worse than $r_{1}^{d}$ - given this belief, best to "get out now"
- But then $\widehat{f}=\frac{3}{4}$ is not self-fulfilling: if that's what everyone believes, then everyone should withdraw
- So $\widehat{f}=f=1$ is another Nash Equilibrium
- Note it is completely rational (from the perspective of a type 2 household) to withdraw in $T=1$ given this belief


## The Bad Equilibrium Continued

- If everyone withdraws, the bank will fail
- It can at most come up with $\$ 100$ in $T=1$, so it can't even meet the promised $r_{1}^{d}$
- Typically there is a "first come, first served" aspect - the first 78 people to line up ( $100 / 1.28 \approx 78$ ) are "made whole" and get $r_{1}^{d}=1.28$, but the last 22 get nothing
- This increases incentive to withdraw and withdraw early - you lose out by not being first in line
- Two equilibria: good (no run) and bad (run)


## Equilibrium Selection

- How do we know which equilibrium will be "played"?
- We don't
- There will exist a cutoff $\bar{f}$ above which any $\widehat{f} \rightarrow 1$ (run) and below which $\widehat{f} \rightarrow t$ (no run)
- In this example, $\bar{f}=0.5625$
- As long as this is pretty far above $t$, will spend most of time in "good" equilibrium
- Would take a big event that is widely observed to move beliefs enough to switch to the run equilibrium
- Sometimes referred to as sunspots: big and easily observed by everyone


## Dealing with Runs

- Financial intermediation (i.e. "borrow short, lend long") is structurally subject to runs because of liquidity transformation
- Given that runs can occur, what kind of policies can be instituted to deal with runs once they start?
- Key point: a policy which effectively deals with runs ought not to really need to be used in practice
- Knowledge of an effective policy once a run has started decreases the likelihood of a run happening in the first place
- e.g. if I know my deposits are safe no matter how many type 2s withdraw early, I have no reason to withdraw early, and we stay in the "good" equilibrium


## Suspension of Convertibility

- Prior to a well-organized central bank in the US, private banks dealt with (recurrent) runs internally via clearinghouses (consortiums of banks in a location, e.g. New York)
- Principal means by which this was done was suspension of convertibility
- Simply refuse (temporarily) to honor demands for conversion of bank debt (e.g. deposits) into cash
- Banks did this together (effectively banding together as one large bank rather than many small banks for the duration of the crisis)
- Would lift suspension when panic was over
- In practice was economically costly and didn't stop runs from happening, but was pretty effective at preventing liquidity crises to force banks into insolvency
- Key difficulty: some people really do need their funds at short notice. How do you decide how much conversion to do before suspending? How do you make sure the cash gets into the appropriate hands?


## Lender of Last Resort

- Federal Reserve was in large part brought into existence to attempt to more efficiently deal with the crises and subsequent suspensions that had plagued US banking for much of the 19th century
- Idea: central bank can create all the reserves it wants to
- If banks run out of cash to meet withdrawal demands, could instead go to the central bank to get requisite cash (the discount window) - Bagehot's rule
- People thought this would put an end to crises
- It didn't (US Great Depression) for a variety of reasons:

1. Stigma: banks didn't want to borrow from Fed for fear of exposing themselves as weak
2. Fed itself didn't understand its role and its powers (Friedman and Schwartz)

## Deposit Insurance

- In response to bank failures of early 1930s, Federal Deposit Insurance Corporation (FDIC) was established in 1933
- Promised full value of deposits at member institutions up to a certain limiting value (originally $\$ 2,500$, now $\$ 250,000$ ) in the event that the bank failed
- Who pays for this insurance? Banks pay a (small) fee to be members (like an insurance premium)
- In practice this has more or less eliminated traditional banking panics - people know deposits are safe, so no reason to run, and we stay in the good equilibrium


## 2007-2009

- If we had deposit insurance and we had the Fed, why was there a run in 2007-2009?
- The banking system changed
- New kinds of short term bank debt were introduced and rose to prominence
- No insurance on these new types of debt, and was unclear extent to which the Fed could or would serve as lender of last resort to financial intermediaries that were not formally banks (deposit-taking institutions)

