Quadrini (2011)

ECON 70428: Advanced Macro: Financial Frictions

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Readings

Quadrini (2011)

Overview

Paper provides a unifying “toy” model

We can use the model to think about three different scenarios

1. No financial frictions
2. Costly state verification (CSV)
3. Collateral constraint

Environment

Two periods: x and x'

Two types of agents: workers and entrepreneurs (unit mass of both). Both risk neutral

Workers consume in both periods and work in the first

Entrepreneurs don't work, but can create new capital goods

Preferences

Workers:

$$\mathbb{E}\left\{c - \frac{h^2}{2} + \delta c'\right\}$$

Entrepreneurs:

$$\mathbb{E}\left\{c_e + \beta c_e'\right\}$$

Production

Entrepreneurs endowed with K of capital. Capital does not depreciate

B_e of debt owed to workers

So workers endowed with B of assets, $B = B_e$)

Output produced in period 1 by entrepreneurs using capital and labor from workers

Output produced in period 2 using just capital; can be produced by either entrepreneurs or workers

Production Period 1

Two stages:

- Stage 1 (intermediates)

$$y = AK^{\theta}h^{1-\theta}$$

- Stage 2: consumption and capital goods

$$\begin{aligned} y &= c + i \\ k_e' &= \omega i \end{aligned}$$

ω realized *after* choice of i

Only entrepreneurs can produce new capital

Idiosyncratic uncertainty, but no aggregate uncertainty. Either $\mathbb{E}\,\omega = 1$ or $\mathbb{E}\,\omega = 0$ (across mass of entrepreneurs). CDF is $\Phi(\omega)$ with density $\phi(\omega)$

Production Period 2

Entrepreneurial output:

$$y_e' = A' k_e'$$

Residual worker output:

$$y' = A' G(k')$$

- $G'(\cdot) > 0$, $G''(\cdot) < 0$, $G'(0) = 1$
- Means MPK of entrepreneurial capital higher than residual output

Additional assumptions: $\delta = \beta$ (only two periods) and $\beta A' > 1$ (no uncertainty over A'; want to postpone consumption through investment)

Worker

Worker problem is the same whether there are frictions or not

$$\max_{c,c',h,k',b'} \quad c - \frac{h^2}{2} + \beta c'$$

s.t.

$$B + wh = c + \frac{b'}{R} + qk'$$

$$A'G(k') + b' = c'$$

$$c \geq 0, \;\; c' \geq 0$$

Second inequality will never bind

Worker FOC

$$h = w(1 + \lambda_1)$$

$$(1 + \lambda_1)q = \beta A' G'(k')$$

$$1 + \lambda_1 = \beta R$$

Where $\lambda_1 \geq 0$ is multiplier on first period non-negativity constraint

Entrepreneur Problem

Will look different with frictions. But here frictionless

$$\max_{h, c_e, c_e', i, k_e', b_e'} \ \mathbb{E}\left\{ c_e + \beta c_e' \right\}$$

s.t.

$$AK^{\theta} h^{1-\theta} - wh + qK + q[\mathbb{E}\,\omega] i + \frac{b_e'}{R} = B_e + c_e + i + qk_e'$$

$$A' k_e' = b_e' + c_e'$$

$$c_e \geq 0, \;\; c_e' \geq 0, \;\; i \geq 0$$

Non-negativity constraint on second period consumption will never bind

Entrepreneur FOC

$$w = \theta A K^{\theta} h^{-\theta}$$

$$q(1 + \lambda_1^e) = \beta A'$$

$$1 + \lambda_1^e = \beta R$$

$$(1 + \lambda_1^e)(q\,\mathbb{E}\,\omega - 1) = -\lambda_2^e$$

Where λ_1^e and λ_2^e are the multipliers on non-negativity constraints for first period consumption and investment, respectively

A Couple of Notes

First, $\lambda_1 = \lambda_1^e = \lambda$: non-negativity constraint on first period consumption binds for both or for neither

Second, can solve for h, and hence entrepreneurial production in period 1:

$$h = (1-\theta)^{\frac{1}{1+\theta}} A^{\frac{1}{1+\theta}} K^{\frac{\theta}{1+\theta}} (1+\lambda)^{\frac{1}{1+\theta}}$$

$$Y^e = AK^\theta h^{1-\theta} - \frac{h^2}{1+\lambda}$$

Two Cases

Consider two cases:

1. Case 1: $\mathbb{E}\,\omega = 1$: there will be capital accumulation
2. Case 2: $\mathbb{E}\,\omega = 0$: there will not be capital accumulation

$\mathbb{E}\,\omega = 1$

Will have $\lambda_2^e = 0$ ($i > 0$), so $q = 1$

$q = 1$ implies $\lambda > 0$ (since $\beta A' > 1$ by assumption): $c = c_e = 0$

$q = 1$ plus $\lambda = 0$ implies $k' = 0$ (all capital ends up with entrepreneurs)

Means $R = A'$ and $1 + \lambda = \beta A'$

Summed across entrepreneurs, there is no uncertainty, so:

$$k_e' = K + i$$

So $y = i$ and $y' = A' k_e' = c' + c_e'$, and:

$$b' = R[B + wh] = c'$$

$\mathbb{E}\,\omega = 1$: Effects of Technology Shocks

$\uparrow A$:

- h goes up (amplification), y goes up, hence i goes up
- Higher i generates more y' and hence more consumption in the future for both types of agents (propagation)

$\uparrow A'$:

- h goes up, y goes up, hence i goes up (propagation)
- Hence y' goes up more than the direct effect of higher A' (amplification)

$\mathbb{E}\,\omega = 0$

Will have $i = 0$: $\lambda_2^e > 0$

Must have $\lambda = 0$, $c, c_e > 0$ (because with $i = 0$ someone has to have positive consumption, so they both have to be positive)

But then $q = \beta A' > 1$

b' undetermined

$\mathbb{E}\,\omega = 0$: Effects of Technology Shocks

$\uparrow A$:

- h goes up, y goes up (amplification), c and c_e go up
- But no propagation because no capital accumulation

$\uparrow A'$:

- No effect on period 1 variables
- Causes y' to go up one-for-one (no labor supply, so no amplification)
- No intertemporal effects because no capital accumulation

CSV Model

Assume $\mathbb{E}\,\omega = 1$, so there will be capital accumulation

Only entrepreneurs can observe their draw of ω

Other agents can observe, but at cost μi (cost proportional to the amount of investment)

Townsend (1979): optimal for a standard debt contract

Worker side is the same

Entrepreneur: Timing

Before choosing i, entrepreneur has net worth:

$$n = qK + Y^e - B$$

Recall $Y^e = AK^\theta h^{1-\theta} - wh$, and is predetermined because choice of h is independent of investment decision

Entrepreneur most borrow $i - n$ at intratemporal rate $1 + r^k$, denominated in units of k (i.e. q units of consumption)

After this decision, ω is realized

Entrepreneur: Default Decision

Entrepreneur will default if $\omega i \leq (1 + r^k)(i - n)$

Cutoff $\bar{\omega}$:

$$\bar{\omega} = (1 + r^k)\frac{i - n}{i}$$

$\bar{\omega}$ increasing in loan rate and leverage ratio, $(i - n)/i$ –
$\bar{\omega} = \bar{\omega}(n, i, r^k)$

Lender

The lender itself isn't interesting – "capital market mutual fund" acting in place of worker

Lender must break-even (incentive compatability):

$$q\left[\int_0^{\bar{\omega}(n,i,r^k)}(\omega-\mu)i\phi(\omega)d\omega+\int_{\bar{\omega}(n,i,r^k)}^{\infty}(1+r^k)(i-n)\phi(\omega)d\omega\right] = i-n$$

Implicitly defines:

$$r^k = r^k(n,i,q)$$
$$\bar{\omega} = \bar{\omega}(n,i,q)$$

Entrepreneur

Entrepreneur picks i to maximize:

$$\max_i \; q \int_{\bar{\omega}(n,i,q)}^{\infty} \left[\omega i - (1 + r^k(n,i,q))(i-n) \right] \phi(\omega) d\omega$$

Effectively internalize how choice of i affects $\bar{\omega}$ and r^k. Let solution by $i = i(n,q)$

- Can't really see it here, but will have $i = F(q)n$, so proportional to n

Entrepreneur Continued

Net worth *after* production is:

$$\pi(n, q, \omega) = \max\left\{0, q\left[\omega i(n, q) - (1 + r^k(n, i(n, q), q))(i(n, q) - n)\right]\right\}$$

Period 1 consumption/saving decision then governed by the budget constraint:

$$\pi(n, q, \omega) = c_e + qk_e' - \frac{b_e'}{R}$$

Two Equilibria

If net worth is sufficiently big (i.e. large initial K or low initial B), then model is like the frictionless case

Not sufficient net worth: $i(n, q) < AK^{\theta} h^{1-\theta}$

Then $\lambda = 0$ (i.e. $c > 0$, $c_e > 0$), so $q = \beta A' > 1$

Focus on this case because otherwise agency friction doesn't matter

Effects of Technology Shock

$\uparrow A$: results in increase in net worth, $n = qK + Y^e - B$, but only through Y^e since q does not change

This generates an increase in i, just like frictionless model

In frictionless model, i increases same amount as output

In this model, response of i is proportional to n

To get i to go up more than y, need n to respond more than y, which requires $qK - B < 0$

Effects of Future Technology Shock

$\uparrow A'$: q increases, which increases net worth

This has two effects:

- Higher net worth makes them want to increase in investment
- But higher q makes them want to do so more than one-to-one
- This raises cost of external finance, so bankruptcy rate rises

CSV Assessment

Difficult to get much amplification from technology shocks in a real model, and it makes counterfactual implications about cyclicality of bankruptcy rates (procyclical). Nominal rigidities help w/ amplification

But does generate more persistence – in a way, isomorphic to adjustment costs (Carlstrom and Fuerst 1997)

- Desire to increase investment exacerbates agency costs in the present, dampening investment
- But more future net worth reduces future agency costs, which generates more persistence and has potential to generate hump-shaped responses
- More basically: ties investment to a slow-moving state variable (net worth), generating investment persistence instead of jumps

Collateral Constraint Model

Follows Kiyotaki and Moore (1997)

Assume $\mathbb{E}\,\omega = 0$, so there is no investment at all

Fixed aggregate stock of capital, $\bar{K}$

Aggregate efficiency would require that $k_e' = \bar{K}$ (i.e. $A' = A' G'(\bar{K} - k_e')$)

But entrepreneurs are subject to a borrowing constraint due to limited enforceability of debt contracts

Limited Enforcement

Based on Hart and Moore (1994)

Entrepreneur cannot be forced to produce after he/she reneges on debt

In case of default, lender can recover $\xi < 1$ of value of assets. $1 - \xi$ like a bankruptcy cost

The fraction ξ of liquidated capital is to residual sector, so $q' = \xi A' G(\bar{K} - k'_e)$. Since $G'(\cdot) \leq 1$ and $\xi < 1$, the value of capital is smaller for lenders than entrepreneurs. This is the limiting factor on entrepreneurs' ability to borrow

Constraint

Constraint takes form:

$$b_e' \leq \xi q' k_e'$$

Entrepreneur's Problem

Deterministic since no capital accumulation and A' is known

$$\max_{c_e, c_e', h, b_e', k_e'} \quad c_e + \beta c_e'$$

s.t.

$$c_e = q\bar{K} + A\bar{K}^{\theta} h^{1-\theta} - wh - B_e + \frac{b_e'}{R} - qk_e'$$

$$\xi q' k_e' \geq b_e'$$

$$c_e' = A' k_e' - b_e'$$

$$c_e \geq 0, \;\; c_e' \geq 0$$

FOC

$$(1-\theta)A\bar{K}^{\theta}h^{-\theta} = w$$

$$(1+\gamma)q = \beta A' + \mu\xi q'$$

$$1+\gamma = (\beta+\mu)R$$

γ is the multiplier on non-negativity of c_e; μ is multiplier on borrowing constraint

FOC Continued

Using results from worker ($\lambda = 0$, or $c > 0$), we get:

$$\mu = \frac{\beta[1 - G'(\bar{K} - k_e')]}{(1-\xi)G'(\bar{K} - k_e')}$$

Note: $k_e' = \bar{K}$ means $\mu = 0$. $\mu > 0 \rightarrow k_e' < \bar{K} \rightarrow k' > 0$

- $\mu = 0$ only possible if B_e sufficiently low

Binding Collateral Constraint

When the constraint binds, $\mu > 0$, so $\gamma > 0$ with $c_e = 0$. Then we have:

$$\left(q - \frac{\xi q'}{R}\right) k_e' = q\bar{K} + Y^e - B_e$$

Cost of more capital is q. This can be financed via:

- Net worth
- $\frac{\xi q'}{R}$ units of debt

Effectively, $\left(q - \frac{\xi q'}{R}\right) k_e'$ is the down payment entrepreneur has to make to get capital

How much entrepreneur can put down depends on net worth

Effects of Shocks

When constraint binds:

$$k_e' = \frac{1}{1-\xi}\left(\bar{K} - \frac{B - Y^e}{q}\right)$$

$\uparrow A$:

- "Direct": $\uparrow Y^e \rightarrow \uparrow k_e'$
- "Indirect": $\uparrow q \rightarrow$ further $\uparrow k_e$ provided $B > Y^e$ (entrepreneur sufficiently levered) amplification and propagation even though this economy has no endogenous capital accumulation

$\uparrow A'$:

- No effect on output in first period
- But $\uparrow q$ triggers $\uparrow k_e$, which generates an amplification of the second period output response

Quantitative Assessment

Amplification effects of collateral constraints are typically quantitatively rather weak

Two related reasons:

1. Investment vs. labor: friction affects investment, only indirectly impacting production inputs (labor and the slow-moving state variable capital). Thus don't get tons of output action

2. Asset price volatility: model needs volatile relative price of capital, q. Macro models typically don't do a great job of generating enough asset price volatility

Labor Wedge

The labor wedge (Chari, Kehoe, and McGrattan 2007) is defined as the deviation from the efficiency condition for labor in a planner's problem, which would be to equate the MRS between consumption and labor to the marginal product of labor. The wedge is defined as the log difference:

$$wedge = mrs - mpl$$

In the data, the wedge is highly volatile (consistent with labor input being volatile); it is as though there is a countercyclical tax on labor income

To get financial frictions to be more relevant for output, move away from just focusing on the investment channel and have frictions impact labor: **working capital**

Working Capital Collateral Constraint

Need to pay labor *before* production via an intraperiod loan (no interest):

$$b_e' + wh \leq \xi q' k'$$

Problem otherwise the same

New FOC for labor:

$$(1-\theta) A \bar{K}^{\theta} h^{-\theta} = w(1+\mu)$$

Tighter constraint $\rightarrow \uparrow \mu \rightarrow$ less labor demand

Wage and Entrepreneurial Income

With worker FOC $h = w$, we get:

$$w(\mu) = \left(\frac{1-\theta}{1+\mu} \right)^{\frac{1}{1+\theta}} A^{\frac{1}{1+\theta}} \bar{K}^{\frac{\theta}{1+\theta}}$$

Tighter constraint means lower w

Similarly, net worth is:

$$Y^e(\mu) = A\bar{K}^{\theta} h^{-\theta} - wh = \frac{(1+\mu)A\bar{K}^{\theta} h^{1-\theta} - (1-\theta)A\bar{K}^{\theta} h^{1-\theta}}{1+\mu}$$

Technology Shocks

$\uparrow A$:

- Holding μ fixed, results in higher $Y^e \rightarrow \uparrow k'_e$
- Generates higher q, which further increases k'_e provided $B < Y^e$
- Higher k'_e means μ is lower: labor wedge is lower, so labor goes up directly because of the friction
- Gets amplification of both current and future output (unlike the no working capital case)

$\uparrow A'$:

- Similarly, generates higher q, higher k'_e, and lower μ
- But lower μ generates increase in labor demand, resulting in higher output in present

Credit Shocks

In collateral constraint model, make ξ_t stochastic

- Higher ξ_t, tighter constraint, less capital allocated to more efficient entrepreneurial sector
- Can also exacerbate the labor wedge, and therefore affect current production, in version of the model in which there is a working capital constraint

In the CSV framework, can think of shocks to σ_t, where σ is the variance of entrepreneurial ω draws. "Risk shocks" Christiano, Motto, and Rostagno (2014)

- Qualitatively similar to time-varying collateral constraint

Alternative Collateral Constraint

Problem: collateral constraint above implies volatility of debt, b_e', is bigger than volatility of asset price, q'

This is counterfactual

Alternative specification:

$$b_e' + wh \leq \xi k_e'$$

Constraint depends on **book value** of capital, not market value

(not multiplied by q')

Generates plausible debt and asset price dynamics (Jermann and Quadrini 2011)

- At the expense of losing the amplification effects of asset prices on the constraint