# Sims and Wu (2021, JME) <br> ECON 70428: Advanced Macro: Financial Frictions 

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## Evaluating Central Banks' Tool Kit: Past, Present, and

## Future

The paper begins by focusing on differences and similarities between conventional policy (adjusting short-term policy rate via a Taylor rule) and unconventional policies:

1. Negative Interest Rate Policy (NIRP)
2. Forward Guidance (FG)
3. Quantitative Easing (QE)

Allows for NIRP by imposing ZLB on deposit rate but not on interest rate on reserves; implemented via a reserve requirement (basically a tax on intermediaries)

NIRP has two competing effects: FG channel and an offsetting "banking channel" (tax on reserves)

FG allows for imperfect credibility

For this class, I'm going to focus exclusively on QE aspect of this paper

The model borrows features from Carlstrom, Fuerst, and Paustin (2017) - long bonds, segmentation, and a loan in advance constraint - and Gertler and Karadi $(2011,2013)$, in particular the structure of financial intermediaries

Different than both: explicitly model the central bank balance sheet, where assets are financed via reserves

The policy rate is the interest rate on reserves, QE involves creating reserves to purchase private or government long-term bonds

Explicitly model the ZLB, and in particular ask the question: how effective a substitute is QE for conventional policy rate adjustments at the ZLB?

## Results

Postulate a Taylor-type rule for QE that only "turns on" when the economy is at the ZLB

Scale the QE reaction to the endogenous component of a Taylor rule so that an exogenous QE shock has same output effect as exogenous policy rate shock

Look at how endogenous QE mitigates the effects of the ZLB
QE seems to be a very good substitute and largely renders the ZLB moot

Has implications for policies to avoid the ZLB in the first place (e.g. raising inflation target)

The Model

## Agents

1. Household
2. Fiscal authority
3. Central Bank
4. Financial intermediaries (FI)
5. New capital goods producer
6. Labor Market:
(a) Competitive labor packer
(b) Monopolistically competitive labor unions (Calvo wage rigidity)
7. Production:
(a) Competitive final good firm
(b) Monopolistically competitive retailers (Calvo price rigidity)
(c) Wholesale firm

## Household

Preferences:

$$
U_{0}=\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t}\left\{\ln \left(C_{t}-b C_{t-1}\right)-\chi \frac{L_{t}^{1+\eta}}{1+\eta}\right\}
$$

Budget constraint:

$$
P_{t} C_{t}+D_{t} \leq M R S_{t} L_{t}+R_{t-1}^{D} D_{t-1}+D I V_{t}-P_{t} X-P_{t} T_{t}
$$

- $D_{t}$ : one-period deposits; household can only save/borrow via short-term debt
- X: fixed (real) equity transfer to newly born Fls
- $D I V_{t}$ : lump sum payout from non-financial and financial firms
- $M R S_{t}$ : nominal remuneration for supplying labor to unions

These are all completely standard looking:

$$
\begin{gathered}
\mu_{t}=\frac{1}{C_{t}-b C_{t-1}}-\beta b \mathbb{E}_{t} \frac{1}{C_{t+1}-b C_{t}} \\
\chi L_{t}^{\eta}=\mu_{t} m r s_{t} \\
1=R_{t}^{d} \mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{-1}
\end{gathered}
$$

## Labor Market

Rather than the Erceg, Henderson, Levin (2000) heterogeneous households w/perfect insurance setup, a "unions" setup

- A continuum of unions (indexed $h \in[0,1]$ ) purchase labor from the household at $M R S_{t}$ and "repackage" it
- A competitive "labor packer" aggregates union labor into a final labor input available for production via a CES technology
- Union labor is imperfectly substitutable, which generates a downward-sloping demand for union labor
- Unions sell their labor to the labor packer at $W_{t}(h)$, who in turn sells labor to production firms at $W_{t}$
- Unions are subject to Calvo wage rigidity

Just a useful trick to keep heterogeneity out of the household problem (and, among other things, allow for more general preferences, e.g. Schmitt-Grohe and Uribe 2005)

## Labor Packer

Let $L_{d, t}$ be final labor available for production; $L_{d, t}(h)$ labor supplied by union $h$ :

$$
L_{d, t}=\left(\int_{0}^{1} L_{d, t}(h)^{\frac{\epsilon_{W}-1}{\epsilon_{w}}} d h\right)^{\frac{\epsilon_{w}}{\epsilon_{w}-1}}
$$

Profit maximization gives rise to downward-sloping to demand for union labor and a wage index:

$$
\begin{aligned}
L_{d, t}(h) & =\left(\frac{W_{t}(h)}{W_{t}}\right)^{-\epsilon_{w}} L_{d, t} \\
W_{t}^{1-\epsilon_{w}} & =\int_{0}^{1} W_{t}(h)^{1-\epsilon_{w}} d h
\end{aligned}
$$

## Labor Unions

A union simply repackages labor it purchases from the household:

$$
L_{d, t}(h)=L_{t}(h)
$$

Its nominal flow profit using the demand function is:

$$
\begin{aligned}
& D I V_{L, t}(h)=W_{t}(h)\left(\frac{W_{t}(h)}{W_{t}}\right)^{-\epsilon_{w}} L_{d, t}- \\
& M R S_{t}\left(\frac{W_{t}(h)}{W_{t}}\right)^{-\epsilon_{w}} L_{d, t}
\end{aligned}
$$

With no other frictions, $W_{t}(h)$ would be a markup over $M R S$ due to market power

Unions face Calvo wage-setting friction: $1-\phi_{w}$ can change wage any period

## Optimal Wage-Setting

Updating firms choose same wage to maximize PDV of real profits, where discounting is by household SDF and probability a wage chosen today is relevant in future:

$$
\begin{gathered}
w_{t}^{\#}=\frac{\epsilon_{w}}{\epsilon_{w}-1} \frac{f_{1, t}}{f_{2, t}} \\
f_{1, t}=m r s_{t} w_{t}^{\epsilon_{w}} L_{d, t}+\phi_{w} \mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{\epsilon_{w}} f_{1, t+1} \\
f_{2, t}=w_{t}^{\epsilon_{w}} L_{d, t}+\phi_{w} \mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{\epsilon_{w}-1} f_{2, t+1}
\end{gathered}
$$

The aggregate nominal wage evolves via:

$$
W_{t}^{1-\epsilon_{w}}=\left(1-\phi_{w}\right)\left(W_{t}^{\#}\right)^{1-\epsilon_{w}}+\phi_{w} W_{t-1}^{1-\epsilon_{w}}
$$

## New Capital Goods

New capital goods, $\widehat{l}_{t}$, are produced from unconsumed output, $I_{t}$, via:

$$
\widehat{I}_{t}=\left[1-S\left(\frac{I_{t}}{I_{t-1}}\right)\right] I_{t}
$$

Where $S(1)=0, S^{\prime}(1)=0$, and $S^{\prime \prime}(1)=\psi_{k} \geq 0$
Nominal dividend:

$$
D I V_{k, t}=P_{t}^{k}\left[1-S\left(\frac{I_{t}}{I_{t-1}}\right)\right] I_{t}-P_{t} I_{t}
$$

FOC:

$$
\begin{aligned}
1=p_{t}^{k}\left[1-S\left(\frac{I_{t}}{I_{t-1}}\right)-S^{\prime}\right. & \left.\left(\frac{I_{t}}{I_{t-1}}\right) \frac{I_{t}}{I_{t-1}}\right]+ \\
& \mathbb{E}_{t} \Lambda_{t, t+1} p_{t+1}^{k} S^{\prime}\left(\frac{I_{t+1}}{I_{t}}\right)\left(\frac{I_{t+1}}{I_{t}}\right)^{2}
\end{aligned}
$$

## Final Goods Production

Competitive final goods firm transforms output of a continuum of retailers (indexed by $f \in[0,1]$ into final output good:

$$
Y_{t}=\left(\int_{0}^{1} Y_{t}(f)^{\frac{\varepsilon_{p}-1}{\epsilon_{p}}} d f\right)^{\frac{\epsilon_{p}}{\varepsilon_{p}-1}}
$$

Retail output purchased at $P_{t}(f)$ and sold at $P_{t}$. Generates downward-sloping demand curve and price index:

$$
\begin{aligned}
& Y_{t}(f)=\left(\frac{P_{t}(f)}{P_{t}}\right)^{-\epsilon_{p}} Y_{t} \\
& P_{t}^{1-\epsilon_{p}}=\int_{0}^{1} P_{t}(f)^{1-\epsilon_{p}} d f
\end{aligned}
$$

## Retailers

Retailers purchase wholesale output, $Y_{w, t}$, for $P_{w, t}$, and repackage it, $Y_{t}(f)=Y_{w, t}(f)$, for sale at $P_{t}(f)$
Nominal dividend:

$$
D I V_{R, t}(f)=P_{t}(f)\left(\frac{P_{t}(f)}{P_{t}}\right)^{-\epsilon_{p}} Y_{t}-P_{w, t}\left(\frac{P_{t}(f)}{P_{t}}\right)^{-\epsilon_{p}} Y_{t}
$$

Absent other frictions, $P_{t}(f)$ would be a fixed markup of wholesale price, $P_{w, t}$

Retailers face Calvo price-setting friction: $1-\phi_{p}$ can change price any period

## Optimal Price-Setting

Updating retailers choose same price to maximize PDV of real profits, where discounting is by household SDF and probability a price chosen today is relevant in future:

$$
\begin{gathered}
\Pi_{t}^{\#}=\frac{\epsilon_{p}}{\epsilon_{p}-1} \frac{x_{1, t}}{x_{2, t}} \\
x_{1, t}=p_{w, t} Y_{t}+\phi_{p} \mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{\epsilon_{p}} x_{1, t+1} \\
x_{2, t}=Y_{t}+\phi_{p} \mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{\epsilon_{p}-1} x_{1, t+1}
\end{gathered}
$$

Where $\Pi_{t}^{\#}=P_{t}^{\#} / P_{t}$. Price level evolves according to:

$$
P_{t}^{1-\epsilon_{w}}=\left(1-\phi_{p}\right)\left(P_{t}^{\#}\right)^{1-\epsilon_{p}}+\phi_{p} P_{t-1}^{1-\epsilon_{p}}
$$

## Wholesale Firm

Produces output according to:

$$
Y_{w, t}=A_{t}\left(u_{t} K_{t}\right)^{\alpha} L_{d, t}^{1-\alpha}
$$

$L_{d, t}$ : purchased from labor packer at $W_{t}$
Owns its own capital stock and makes utilization, $u_{t}$ decisions. Law of motion is:

$$
K_{t+1}=\widehat{l}_{t}+\left(1-\delta\left(u_{t}\right)\right) K_{t}
$$

Where $\delta(1)=\delta_{0}, \delta^{\prime}(1)=\delta_{1}$, and $\delta^{\prime \prime}(1)=\delta_{2}$
$\widehat{I}_{t}$ purchased from new capital producer at $P_{t}^{k}$

## Long-Bonds

Wholesaler can issue long term bonds with decaying coupon payments (decay parameter $\kappa$ )

Let $F_{w, t-1}$ denote coupon payment due in $t$ from past issuances, and $Q_{t}$ the price of new issues

As in Carlstrom, Fuerst, and Paustian (2017), subject to loan in advance constraint:

$$
\psi P_{t}^{k}{\widehat{l_{t}}} \leq Q_{t}\left(F_{w, t}-\kappa F_{w, t-1}\right)
$$

Must finance a fraction, $\psi$, of total investment by issuing long-term bonds

## Dividends

Nominal dividends are:

$$
\begin{aligned}
D I V_{w, t}=P_{w, t} A_{t}\left(u_{t} K_{t}\right)^{\alpha} L_{d, t}^{1-\alpha} & -W_{t} L_{d, t}-P_{t}^{k} \widehat{I}_{t} \\
& -F_{w, t-1}+Q_{t}\left(F_{w, t}-\kappa F_{w, t-1}\right)
\end{aligned}
$$

Problem: pick $u_{t}, L_{d, t}, \widehat{I}_{t}, K_{t+1}$, and $F_{w, t}$ to maximize PDV of real profits (discounted by SDF), taking $P_{w, t}, W_{t}, P_{t}^{k}$, and $Q_{t}$ as given, subject to law of motion for capital and loan in advance constraint

Let $v_{1, t}$ be multiplier on accumulation equation and $v_{2, t}$ be the multiplier on loan in advance

$$
\begin{gathered}
w_{t}=(1-\alpha) p_{w, t} A_{t}\left(u_{t} K_{t}\right)^{\alpha} L_{d, t}^{-\alpha} \\
v_{1, t} \delta^{\prime}\left(u_{t}\right)=\alpha p_{w, t} A_{t}\left(u_{t} K_{t}\right)^{\alpha-1} L_{d, t}^{1-\alpha} \\
\left(1+\psi v_{2, t}\right) p_{t}^{k}=v_{1, t}
\end{gathered}
$$

$$
v_{1, t}=\mathbb{E}_{t} \Lambda_{t, t+1}\left[\alpha p_{w, t+1} A_{t+1}\left(u_{t+1} K_{t+1}\right)^{\alpha-1} u_{t+1} L_{d, t+1}^{1-\alpha}+\right.
$$

$$
\left.v_{1, t+1}\left(1-\delta\left(u_{t+1}\right)\right)\right]
$$

$$
\left(1+v_{2, t}\right) Q_{t}=\mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{-1}\left[1+\left(1+v_{2, t+1}\right) \kappa Q_{t+1}\right]
$$

## Change of Variables

Define $M_{2, t}=1+v_{2, t}$ and $M_{1, t}=1+\psi v_{2, t}$
Eliminate $\nu_{1, t}=p_{t}^{k} M_{2, t}$
Then write:

$$
p_{t}^{k} M_{2, t} \delta^{\prime}\left(u_{t}\right)=\alpha p_{w, t} A_{t}\left(u_{t} K_{t}\right)^{\alpha-1} L_{d, t}^{1-\alpha}
$$

$$
\begin{gathered}
p_{t}^{k} M_{2, t}=\mathbb{E}_{t} \Lambda_{t, t+1}\left[\alpha p_{w, t+1} A_{t+1}\left(u_{t+1} K_{t+1}\right)^{\alpha-1} u_{t+1} L_{d, t+1}^{1-\alpha}+\right. \\
\left.\left(1-\delta\left(u_{t+1}\right)\right) p_{t+1}^{k} M_{2, t+1}\right] \\
Q_{t} M_{1, t}=\mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{-1}\left[1+\kappa Q_{t+1} M_{1, t+1}\right] \\
\frac{M_{1, t}-1}{M_{2, t}-1}=\psi
\end{gathered}
$$

## Discussion

Define:

$$
R_{t}^{F}=\frac{1+\kappa Q_{t}}{Q_{t-1}}
$$

Then bond Euler equation can be written:

$$
M_{1, t}=\mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{-1}\left[R_{t+1}^{F}+\frac{\kappa Q_{t+1}}{Q_{t}}\left(M_{1, t+1}-1\right)\right]
$$

If $v_{2, t}=0$ (constraint not binding), then $M_{1, t}=M_{1, t+1}=1$, and this is standard, and to first order approximation we would have $\mathbb{E}_{t} r_{t+1}^{f}=r_{t}^{d}$

As in Carlstrom, Fuerst, and Paustian (2017), $v_{2, t}$ also distorts capital Euler equation. Like a tax on capital - the relevant capital price for the firm is:

$$
p_{t}^{k} M_{2, t}=p_{t}^{k}\left(1+v_{2, t}\right)
$$

Here, it also distorts the utilization decision

## Financial Intermediaries

Technically indexed by $i$ but will behave identically (scaled versions of one another)

Begin with startup net worth $X$
Accumulate net worth through retained earnings
Optimal to retain all earnings until death if there are excess returns and household cannot directly hold the long-term assets

Stochastically exit with probability $1-\sigma$

## Balance Sheet and Net Worth Evolution

Hold private bonds, government bonds (same maturity structure) and reserves on account with central bank

Financed via deposits and net worth

$$
Q_{t} F_{t}+Q_{B, t} B_{t}+R E_{t}=D_{t}+N_{t}
$$

Net worth conditional on surviving across periods is:

$$
\begin{array}{r}
N_{t}=\left(R_{t}^{F}-R_{t-1}^{d}\right) Q_{t-1} F_{t-1}+\left(R_{t}^{B}-R_{t-1}^{d}\right) Q_{B, t-1} B_{t-1}+ \\
\left(R_{t-1}^{r e}-R_{t-1}^{d}\right) R E_{t-1}+R_{t-1}^{d} N_{t-1}
\end{array}
$$

$R_{t}^{r e}$ is the gross interest rate on reserves (the policy rate)

## Enforcement Constraint

At the end of the period, FI can default and abscond with $\theta_{t}$ of its private bonds and $\theta_{t} \Delta$ of its government bonds
$\theta_{t}$ is a credit shock in language of Jermann and Quadrini (2012): the bigger it is, the greater the incentive to default, and the more creditors will limit FI leverage to prevent default
$0 \leq \Delta \leq 1$ means it is "harder" to run away with government bonds than private bonds

Note we assume reserves can be fully recovered by creditors in the event of default

Let $V_{t}$ be the value function. Enforcement constraint is:

$$
V_{t} \geq \theta_{t}\left(Q_{t} f_{t}+\Delta Q_{B, t} b_{t}\right)
$$

## Value Function

Value function at end of period $t$ written recursively is:

$$
V_{t}=(1-\sigma) \mathbb{E}_{t} \Lambda_{t, t+1} n_{t+1}+\sigma \mathbb{E}_{t} \Lambda_{t, t+1} V_{t+1}
$$

$(1-\sigma)$ : probability of exit after $t+1$
$\sigma$ : probability of continuation
Lagrangian, with $\lambda_{t}$ multiplier on constraint:

$$
\begin{aligned}
\mathbb{L}=\left(1+\lambda_{t}\right)\left[(1-\sigma) \mathbb{E}_{t} \Lambda_{t, t+1} n_{t+1}\right. & \left.+\sigma \mathbb{E}_{t} \Lambda_{t, t+1} V_{t+1}\right] \\
& -\lambda_{t} \theta_{t}\left(Q_{t} f_{t}+\Delta Q_{B, t} b_{t}\right)
\end{aligned}
$$

## FOC

The FOC are:

$$
\begin{gathered}
\mathbb{E}_{t} \Lambda_{t, t+1}\left(R_{t+1}^{F}-R_{t}^{d}\right) \Pi_{t+1}^{-1} \Omega_{t+1}=\frac{\lambda_{t}}{1+\lambda_{t}} \theta_{t} \\
\mathbb{E}_{t} \Lambda_{t, t+1}\left(R_{t+1}^{B}-R_{t}^{d}\right) \Pi_{t+1}^{-1} \Omega_{t+1}=\frac{\lambda_{t}}{1+\lambda_{t}} \Delta \theta_{t} \\
\mathbb{E}_{t} \Lambda_{t, t+1}\left(R_{t}^{r e}-R_{t}^{d}\right) \Pi_{t+1}^{-1} \Omega_{t+1}=0
\end{gathered}
$$

Where:

$$
\Omega_{t+1}=1-\sigma+\sigma \frac{\partial V_{t+1}}{\partial n_{t+1}}
$$

Guess (and verify) that the value function is linear in net worth:
$V_{t}=a_{t} n_{t}$
We get $a_{t}=\phi_{t} \theta_{t}$
$\phi_{t}$ is a modified leverage ratio

## Binding Constraint

Assuming the constraint binds:

$$
a_{t} n_{t}=\theta_{t}\left(Q_{t} f_{t}+\Delta Q_{B, t} b_{t}\right)
$$

Define:

$$
\phi_{t}=\frac{Q_{t} f_{t}+\Delta Q_{B, t} b_{t}}{n_{t}}
$$

Hence $a_{t}=\phi_{t} \theta_{t}$
Through manipulation of law of motion for net worth, get:

$$
\phi_{t}=\frac{\mathbb{E}_{t} \Lambda_{t, t+1} \Pi_{t+1}^{-1} \Omega_{t+1} R_{t}^{d}}{\theta_{t}-\mathbb{E}_{t} \Lambda_{t, t+1} \Omega_{t+1} \Pi_{t+1}^{-1}\left(R_{t+1}^{F}-R_{t}^{d}\right)}
$$

If constraint were not binding, would have $a_{t}=1-$ net worth as valuable inside an intermediary as out

## Central Bank

Notional (i.e. desired) short-term rate follows a Taylor rule:

$$
\begin{aligned}
& \ln R_{t}^{t r}=\left(1-\rho_{r}\right) \ln R^{t r}+\rho_{r} \ln R_{t-1}^{t r}+ \\
& \quad\left(1-\rho_{r}\right)\left[\phi_{\pi}\left(\ln \Pi_{t}-\ln \Pi\right)+\phi_{y}\left(\ln Y_{t}-\ln Y_{t-1}\right)\right]+s_{r} \varepsilon_{r, t}
\end{aligned}
$$

Interest rate on reserves (the policy rate) is the max of this and 1 (so zero net), imposing a ZLB:

$$
R_{t}^{r e}=\max \left\{1, R_{t}^{t r}\right\}
$$

From looking at FI optimality, will get that $R_{t}^{d}=R_{t}^{r e}$ in equilibrium
So Taylor rule controls the economically relevant short-term rate, $R_{t}^{d}$

## Central Bank Balance Sheet

Central bank can hold either private or public bonds
These are financed via reserves (the economy is cashless, so this constitutes the monetary base). Central bank can freely choose reserves

$$
Q_{t} F_{c b, t}+Q_{B, t} B_{c b, t}=R E_{t}
$$

Central bank earns something on its assets ( $R_{t}^{F}$ and $R_{t}^{B}$ ) and pays something for its liabilities ( $R_{t}^{r e}$ ). It returns the surplus to the fiscal authority via a transfer:

$$
P_{t} T_{c b, t}=R_{t}^{F} Q_{t-1} F_{c b, t-1}+R_{t}^{B} Q_{B, t-1} B_{c b, t-1}-R_{t-1}^{r e} R E_{t-1}
$$

Real bond holdings, $f_{c b, t}$ and $b_{c b, t}$, follows exogenous $\operatorname{AR}(1)$ (for now); reserves adjust automatically to make balance sheet hold

## Fiscal Policy

Flow budget constraint in nominal terms is:

$$
P_{t} G_{t}+B_{G, t-1}=P_{t} T_{t}+P_{t} T_{c b, t}+Q_{B, t}\left(B_{G, t}-\kappa B_{G, t-1}\right)
$$

Assume $G_{t}$ follows exogenous $\mathrm{AR}(1)$ process
Ricardian Equivalence does not hold because of the frictions on intermediaries, so sequence of debt is not innocuous

- Assume $B_{G, t}$ follows an exogenous process
- Given this, $T_{t}$ automatically adjusts to make this hold


## Market-Clearing and Aggregation

Bonds issued (by the wholesale firm or government) must be held (either by Fls or the central bank):

$$
\begin{aligned}
f_{w, t} & =f_{t}+f_{c b, t} \\
b_{G, t} & =b_{t}+b_{c b, t}
\end{aligned}
$$

Aggregate net worth evolves according to:

$$
\begin{array}{r}
n_{i, t}=\sigma \Pi_{t}^{-1}\left[\left(R_{t}^{F}-R_{t-1}^{d}\right) Q_{t-1} f_{t-1}+\left(R_{t}^{B}-R_{t-1}^{d}\right) Q_{B, t-1} b_{t-1}\right. \\
\left.+\left(R_{t-1}^{r e}-R_{t-1}^{d}\right) r e_{t-1}+R_{t-1}^{d} n_{t-1}\right]+X
\end{array}
$$

## Why does QE Work?

In Carlstrom, Fuerst, and Paustian (2017) and Gertler and Karadi (2011, 2013), QE works by first reducing the asset holdings of intermediaries. But this reduces their total leverage, which allows them to expand their balance sheet back up by buying private securities, which pushes prices up and yields down, stimulating investment

Similar here, but it's really more an asset swap
The central bank is swapping assets that affect the enforcement constraint (private or government bonds) for reserves, which do not show up in the constraint

This makes the constraint less binding, which then in turn allows intermediaries to expand their balance sheets by buying more privately issued assets

## Calibration

See my notes for details
Target interest rate spreads and leverage ratios - gives steady state values of $\theta$ and $X$, taking a value of $\sigma$ as given

The rest of the calibration is reasonably straightforward.

## Impulse Responses

## Private QE Shock



## Public QE Shock



## Credit Shock











## Government Debt Shock



## Similarity of all These Shocks

These four shocks - public and private QE, credit, and government debt - all have very similar effects

In model without constrained intermediaries, they would all be irrelevant

They directly affect FI balance sheets:

- QE through asset swap, which loosens constraint
- Credit shock directly tightens constraint
- Government debt shock forces intermediaries to hold more public debt, which tightens their balance sheet constraint and crowds out private debt purchases


## Government Spending Shock



## Productivity Shock



## Conventional Monetary Shock









— Monetary Shock

## The ZLB

## QE and the ZLB

QE was implemented in the US and other countries as a substitute for conventional policy at the ZLB

Implement the ZLB following Guerrieri and Iacoviello (2015) "occbin" toolkit
"occbin" has two .mod files:

- Standard file and constrained file, constrained file features $R_{t}^{r e}=1$ and kicks in when $R_{t}^{t r} \leq 1$
- Uses a piecewise solution; determinacy properties governed by standard file
- Pick a sequence of shocks to drive $R_{t}^{t r} \leq 1$ for a desired period of time (I use credit shocks, specific shock doesn't matter)
- The ZLB binds for about 10 quarters ( 2.5 years), which is roughly how long people expected ZLB to last (obviously lasted much longer ex-post)


## Private QE Shock, ZLB








## Public QE Shock, ZLB








## Credit Shock, ZLB



## Government Spending Shock, ZLB








## Productivity Shock, ZLB








## Endogenous QE as a Policy Substitute at ZLB

## Endogenous QE

When $R_{t}^{t r}>1$, policy rate follows Taylor rule, $R_{t}^{r e}=R_{t}^{t r}$.
At ZLB, an endogenous component of QE kicks in:

$$
\begin{aligned}
f_{c b, t}= & \left(1-\rho_{f}\right) f_{c b}+\rho_{f} f_{c b, t-1} \cdots \\
& -\Psi_{f}\left(1-\rho_{f}\right)\left[\phi_{\pi} \ln \Pi_{t}+\phi_{y}\left(\ln Y_{t}-\ln Y_{t-1}\right)\right]+s_{f} \varepsilon_{f, t}
\end{aligned}
$$

Just like the Taylor rule
Pick $\Psi_{f}$ so that an exogenous QE shock has $\approx$ same effect on output as exogenous monetary policy shock

Sims and $W u(2021)$ use $\Psi_{f}=7$, but $\rho_{f}=0.8$
I use $\rho_{f}=0.97$. I pick $\Psi_{f}\left(1-\rho_{f}\right)$ to be the same as in Sims and Wu (2020), implying a significantly higher $\Psi_{f}$

## Credit Shock, ZLB w/ Endogenous QE



## Government Spending Shock, ZLB w/ Endogenous QE



## Productivity Shock, ZLB w/ Endogenous QE








## QE as a Substitute for Conventional Policy

Endogenous QE serves an excellent substitute for conventional policy and largely renders the ZLB moot

The output and investment responses are basically the same to each shock as when there is no ZLB

Larger differences for consumption, but this makes sense: consumption not directly affected by QE, which targets long-term rates

Suggests ZLB may not be such a problem

