Sims, Wu, and Zhang (2021, Working Paper)

ECON 70428: Advanced Macro: Financial Frictions

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New Keynesian Model

The textbook three equation model (Woodford 2003, Gali 2008) is ubiquitous in macro and monetary economics

A number of important insights that have been put into practice by policymakers:

1. Divine coincidence
2. Inflation targeting
3. Discretion vs. commitment
4. Importance of the natural rate of interest, or "r-star"
5. Price level targeting, flexible inflation targeting, the importance of signaling future policy paths
6. Forward guidance

Drawbacks

No role for financial frictions / credit shocks

- Often appended in reduced-form way as IS curve residuals

No role for quantitative easing / large scale asset purchases (QE/LSAPs)

Model seems *too* forward-looking

- e.g. the forward guidance puzzle

The Four Equation Model

Sims, Wu, and Zhang (2021) set out to incorporate the frictions and insights concerning quantitative easing from Sims and Wu (2020) into a framework that is as close as possible to the three equation model

The end result is the four equation model

Main insights:

1. QE should be used to offset credit shocks all the time, not just at ZLB

2. QE an effective, yet imperfect, substitute for conventional policy at ZLB

Results are not new per se, but are more transparent in a small-scale model that facilitates analytical solutions

Textbook Three Equation Model

Demand (IS), supply (NKPC), and policy (Taylor rule) equations:

$$x_t = \mathbb{E}_t\, x_{t+1} - \frac{1}{\sigma}\left(r_t^s - \mathbb{E}_t\, \pi_{t+1} - r_t^f\right)$$

$$\pi_t = \lambda x_t + \beta\, \mathbb{E}_t\, \pi_{t+1}$$

$$r_t^s = \rho_R r_{t-1}^s + (1-\rho_R)\left(\phi_\pi \pi_t + \phi_x x_t\right) + s_R \varepsilon_{R,t}$$

Optimal policy is rather straightforward: achieve $\pi_t = x_t = 0$ (Divine Coincidence) $\Rightarrow$ implies $r_t^s = r_t^f$

But ZLB is quite costly: fluctuations in r_t^f lead to large changes in π_t and x_t if $r_t^s = 0$

Four Equation Model

Demand (IS), supply (NKPC), policy rule for short-rate (Taylor rule), and rule for central bank balance sheet, $\widehat{qe}_t$:

$$x_t = \mathbb{E}_t\, x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \mathbb{E}_t\, \pi_{t+1} - r_t^f\right) - z\left(\frac{b^{FI}}{b}\left(\mathbb{E}_t\, \theta_{t+1} - \theta_t\right) + \frac{b^{cb}}{b}\left(\mathbb{E}_t\, \widehat{qe}_{t+1} - \widehat{qe}_t\right)\right)$$

$$\pi_t = \gamma\zeta x_t - \frac{\sigma\gamma z}{1-z}\left(\frac{b^{FI}}{b}\theta_t + \frac{b^{cb}}{b}\widehat{qe}_t\right) + \beta\, \mathbb{E}_t\, \pi_{t+1}$$

$$r_t^s = \rho_R r_{t-1}^s + (1-\rho_R)\left[\phi_\pi \pi_t + \phi_x x_t\right] + s_R \varepsilon_{R,t}$$

$$\widehat{qe}_t = \rho_q \widehat{qe}_{t-1} + s_q \varepsilon_{q,t}$$

$z \in [0,1]$: fraction of impatient/borrower households

θ_t credit shock (allows FIs to take on more leverage), $\widehat{qe}_t$ denotes central bank balance sheet

$z = 0$ reduces to standard model

Alternative IS Expression

Arguably more intuitive:

$$y_t = \mathbb{E}_t\, y_{t+1} - \frac{1}{\sigma}\left(r_t^s - \mathbb{E}_t\, \pi_{t+1}\right) - \frac{z}{\sigma}\left(\mathbb{E}_t\, r_{t+1}^b - r_t^s\right)$$

$\mathbb{E}_t\, r_{t+1}^b$: expected return on long-term bond

This is nice because the IS residual is a credit spread

Downside is it requires keeping track of more variables – can't get the system down to just four equations

Alternative Phillips Curve

Written in terms of marginal cost, the **same** as in standard model

$$\pi_t = \gamma mc_t + \beta \, \mathbb{E}_t \, \pi_{t+1}$$

Credit/QE shocks change relationship between marginal cost and output

Good credit conditions reallocate resources from savers (workers) to borrowers (who don't work)

This puts downward pressure on wage, for a given level of output, and shows up as a cost-push wedge in the Phillips Curve

Immediate Policy Implications

First, use conventional policy to deal with natural rate shocks: $r_t^s = r_t^f$

Second, use QE to deal with credit shocks: $\widehat{qe}_t = -\frac{b^{FI}}{b^{cb}}\theta_t$

- Same idea as in Carlstrom, Fuerst, and Paustian (2017), but more transparent

Third, ZLB poses no problem for credit shocks – just use balance sheet policies as you normally would

Fourth, QE can serve as an (imperfect) substitute for conventional policy at the ZLB in response to natural rate shocks

Model Details

Agents

1. Patient household (parent)
2. Impatient household (child)
3. Financial intermediary
4. Final goods firm
5. Retailers (continuum)
6. Wholesale producer
7. Central bank

As in textbook model, no physical capital

Patient Household

Objective:

$$V_0 = \max_{C_t, L_t, S_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} \right]$$

The flow budget constraint in nominal terms is:

$$\begin{aligned} & P_t C_t + S_t \leq \\ & \quad W_t L_t + R_{t-1}^s S_{t-1} + P_t D_t + P_t D_t^{FI} + P_t T_t - P_t X_t^b - P_t X_t^{FI} \end{aligned}$$

X_t^{FI}: equity transfer to financial intermediary

X_t^b: transfer to impatient household (child)

- Both time-varying but not choices

FOC

Completely standard:

$$\psi L_t^{\chi} = w_t C_t^{-\sigma}$$

$$\Lambda_{t-1,t} = \beta \left(\frac{C_{t-1}}{C_t} \right)^{\sigma}$$

$$1 = \mathbb{E}_t \, \Lambda_{t,t+1} R_t^s \Pi_{t+1}^{-1}$$

Impatient Household

Objective:

$$V_0^b = \max_{C_{b,t}} \quad \mathbb{E}_t \sum_{t=0}^{\infty} \beta_b^t \left\{ \frac{C_{b,t}^{1-\sigma}}{1-\sigma} \right\}$$

Impatient: $\beta_b < \beta$

Budget constraint:

$$P_t C_{b,t} + B_{t-1} \leq Q_t(B_t - \kappa B_{t-1}) + P_t X_t^b$$

B_{t-1}: coupon liability on past long-bond issues; coupons decay at $\kappa \in [0, 1]$, new issues trade at Q_t

FOC

Standard-looking:

$$\Lambda_{b,t-1,t} = \beta_b \left(\frac{C_{b,t-1}}{C_{b,t}} \right)^{\sigma}$$

$$Q_t = \mathbb{E}_t \, \Lambda_{b,t,t+1} \Pi_{t+1}^{-1} (1 + \kappa Q_{t+1})$$

Bond return, $R_{b,t}$:

$$R_t^b = \frac{1 + \kappa Q_t}{Q_{t-1}}$$

Alternative Euler equation:

$$1 = \mathbb{E}_t \, \Lambda_{b,t,t+1} R_{t+1}^b \Pi_{t+1}^{-1}$$

Financial Intermediary

Born at beginning of period t and receive startup net worth

Choose asset holdings to carry into $t+1$

Die and return net worth back to household in $t+1$, who then gives these assets back to a new FI

Balance sheet constraint:

$$Q_t B_t^{FI} + RE_t^{FI} = S_t^{FI} + P_t X_t^{FI}$$

$P_t X_t^{FI}$ is startup net worth

Startup Net Worth

Startup net worth consists of two components: fixed new equity transfer and value of outstanding long bonds:

$$P_t X_t^{FI} = P_t \bar{X}^{FI} + \kappa Q_t B_{t-1}^{FI}$$

They get new equity transfer, $\bar{X}^{FI}$, and existing value of long bonds held by previous FIs

Since $B_t = CB_t + \kappa B_{t-1}$, we could equivalently write the balance sheet:

$$Q_t CB_t + RE_t^{FI} = S_t^{FI} + P_t \bar{X}^{FI}$$

The outstanding long bonds are, in a sense, held by the FI in a custodial capacity – FI manages existing long bonds

FI really just choosing how many new issuances to buy

FI does not internalize how its choice of how many long bonds to buy affects future FIs

Dividend

After choosing long bonds and reserves in period t, the FI earns returns and gives everything to the patient household in $t+1$. Its dividend is:

$$P_{t+1}D_{t+1}^{FI} = (R_{t+1}^{b} - R_t^s)Q_t B_t^{FI} + (R_t^{re} - R_t^s)RE_t^{FI} + R_t^s P_t X_t^{FI}$$

Subject to a leverage constraint:

$$Q_t B_t^{FI} \leq \Theta_t P_t \bar{X}^{FI}$$

Total value of long bond portfolio cannot exceed a multiple of *new* equity

Θ_t is a credit shock: increases in Θ_t allow for more leverage and will be expansionary

FOC

These are

$$\mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1} (R_{t+1}^b - R_t^s) = \Omega_t$$

$$R_t^{re} = R_t^s$$

Steady state spreads require $\Omega_t > 0$

FI problem really not that interesting – don't need either of these conditions to solve the model if you don't want to keep track of Ω_t

Production

Two sector setup: wholesale and retail

Wholesale output and labor demand:

$$Y_{w,t} = A_t L_t$$

$$w_t = p_{w,t} A_t$$

$p_{w,t}$ relative price of wholesale output to final output; interpretable as real marginal cost / equivalently inverse price markup of retail over wholesale goods

Price-Setting

Price-setting stuff for the retailer is standard:

$$\Pi_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2,t}}$$

$$x_{1,t} = p_{w,t} Y_t + \phi \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} x_{1,t+1}$$

$$x_{2,t} = Y_t + \phi \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} x_{2,t+1}$$

Central Bank

Balance sheet:

$$Q_t B_{cb,t} = RE_t$$

Call $RE_t = QE_t$, they can freely set this and choose holdings of long bonds issued by the child.

Set interest rate on reserves according to Taylor rule:

$$\ln R_t^{re} = (1 - \rho_R) \ln R^{re} + \rho_R \ln R_{t-1}^{re} + (1 - \rho_R)\phi_\pi \ln \Pi_t + s_R \varepsilon_{R,t}$$

Assume that qe_t (the real value of the bond portfolio), obeys some process for now:

$$\ln qe_t = (1 - \rho_q) \ln qe + \rho_q \ln qe_{t-1} + s_q \varepsilon_{q,t}$$

Returns operating profit to patient household lump sum:

$$P_t T_t = R_t^b Q_{t-1} B_{cb,t-1} - R_{t-1}^{re} RE_{t-1}$$

Aggregation and "Full Bailout"

Resource constraint and bond market-clearing:

$$Y_t = C_t + C_{b,t}$$

$$B_t = B_{cb,t} + B_t^{FI}$$

Full bailout: assume parent transfer satisfies:

$$P_t X_t^b = (1 + \kappa Q_t) B_{t-1}$$

Each period, transfer fully pays off outstand ing coupon payments plus principal on child's debt

Implication of Full Bailout

This assumption makes the child's consumption effectively static:

$$P_t C_{b,t} = Q_t B_t$$

But then using bond market-clearing:

$$P_t C_{b,t} = Q_t B_t^{FI} + Q_t B_{cb,t}$$

But then using leverage constraint and definition of QE_t, we get:

$$P_t C_{b,t} = \Theta_t P_t \bar{X}^{FI} + QE_t$$

Child consumption is pinned down; only depends on Θ_t and QE_t

Simplifies model: in the end, don't need to keep track of bonds at all

Parameterization

$\sigma = \chi = 1$, $\beta = 0.995$ and $\beta_b = 0.985$

$\Theta = 5$ and $\kappa = 1 - 40^{-1}$

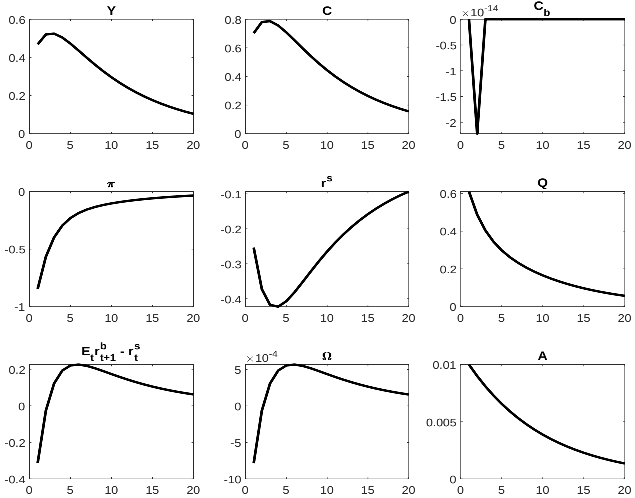
$z = 1/3$ (share of child's consumption)

$\phi = 0.75$

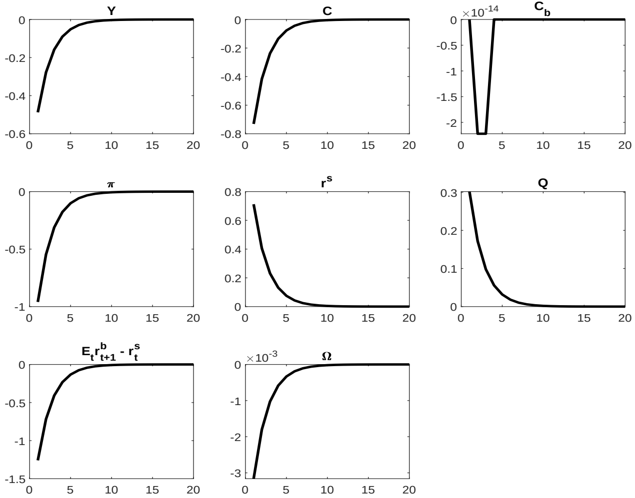
Taylor rule: $\rho_R = 0.8$, $\phi_\pi = 1.5$, $s_R = 0.0025$

Other shocks are AR(1) with AR parameters 0.9 and shock standard deviations of 1 percent

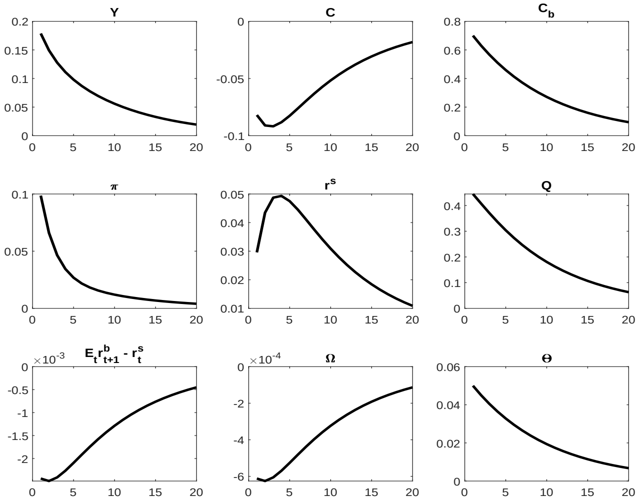
Productivity Shock



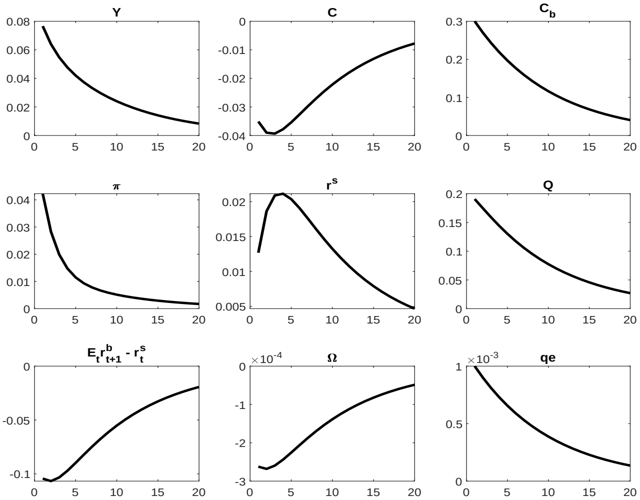
Monetary Shock



Credit Shock



QE Shock



Comments

Responses to productivity and monetary shocks are very similar to standard NK model

QE and credit shocks are the same up to scale: mechanically, they allow $C_{b,t}$ to go up, which increases aggregate demand

Child consumption does not react to productivity and monetary shock

- As noted above, this is a consequence of full bailout assumption, which facilitates system reduction when linearizing
- Does not significantly alter responses – see detailed notes
- Another way to get endogenous child consumption would be to have the child supply labor and earn some labor income
  - Which would also help with co-movement issues

Optimal Monetary Policy

Return to Linearized Model

After a decent amount of work (see notes), one can get the IS and Phillips curves as presented above:

$$x_t = \mathbb{E}_t\, x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \mathbb{E}_t\,\pi_{t+1} - r_t^f\right) - z\left(\frac{b^{FI}}{b}\left(\mathbb{E}_t\,\theta_{t+1} - \theta_t\right) + \frac{b^{cb}}{b}\left(\mathbb{E}_t\,\widehat{qe}_{t+1} - \widehat{qe}_t\right)\right)$$

$$\pi_t = \gamma\zeta x_t - \frac{\sigma\gamma z}{1-z}\left(\frac{b^{FI}}{b}\theta_t + \frac{b^{cb}}{b}\widehat{qe}_t\right) + \beta\,\mathbb{E}_t\,\pi_{t+1}$$

Instead of assuming processes for r_t^s and $\widehat{qe}_t$, think about optimal choices – these are the policy instruments

Optimal Policy Under Discretion

Let objective of central bank be:

$$\mathbb{L} = \mu x_t^2 + \pi_t^2$$

Wants to pick r_t^s and qe_t to minimize this, subject to the Euler equation and Phillips Curves

Discretion is easier – don't worry about future instruments and take the future as given (you will re-optimize each period)

FOC: policy rate

FOC for r_t^s is:

$$2\mu x_t \frac{dx_t}{dr_t^s} + 2\pi_t \frac{d\pi_t}{dr_t^s} = 0$$

Where:

$$\frac{dx_t}{dr_t^s} = -\frac{1-z}{\sigma}$$

$$\frac{d\pi_t}{dr_t^s} = \gamma\zeta \frac{dx_t}{dr_t^s}$$

So:

$$-\frac{\mu(1-z)}{\sigma} x_t - \frac{\gamma\zeta(1-z)}{\sigma}\pi_t = 0$$

Standard "lean against the wind" condition:

$$\pi_t = -\frac{\mu}{\gamma\zeta} x_t$$

FOC: QE

FOC for qe_t is:

$$2\mu x_t \frac{dx_t}{dqe_t} + 2\pi_t \frac{d\pi_t}{dqe_t} = 0$$

Where:

$$\frac{dx_t}{dqe_t} = z\frac{b^{cb}}{b}$$

$$\frac{d\pi_t}{dqe_t} = \gamma\zeta\frac{dx_t}{dqe_t} - \frac{\sigma\gamma z}{1-z}\frac{b^{cb}}{b}$$

So:

$$\mu z\frac{b^{cb}}{b}x_t + \left(\gamma\zeta z\frac{b^{cb}}{b} - \frac{\sigma\gamma z}{1-z}\frac{b^{cb}}{b}\right)\pi_t = 0$$

Another "lean against the wind" condition:

$$\pi_t = -\frac{\mu(1-z)}{\gamma\zeta(1-z) - \sigma\gamma}x_t$$

FOC: QE

With **both** instruments available, both FOC holding requires $\pi_t = x_t = 0$

This implies in equilibrium:

$$r_t^s = r_t^*$$

$$qe_t = -\frac{b^{FI}}{b^{cb}}\theta_t$$

- Move policy rate one-for-one with natural rate
- Move balance sheet opposite credit shocks

The ZLB

At the ZLB, the FOC for policy rate can't hold

But the FOC for QE does

Stochastic interest rate peg: $r_t^s = 0$ today, lifts with probability $1 - \alpha$ each subsequent period

Solve for paths of variables in response to a natural rate shock

Note: not being able to use the policy rate is not a problem for credit shocks; can combat with just balance sheet policies

Some Results

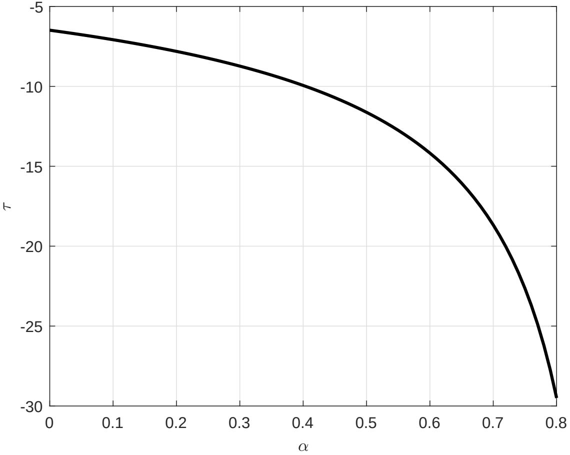
QE reacts more to a natural rate shock the longer is the ZLB

- In equilibrium, while ZLB binds, we have:

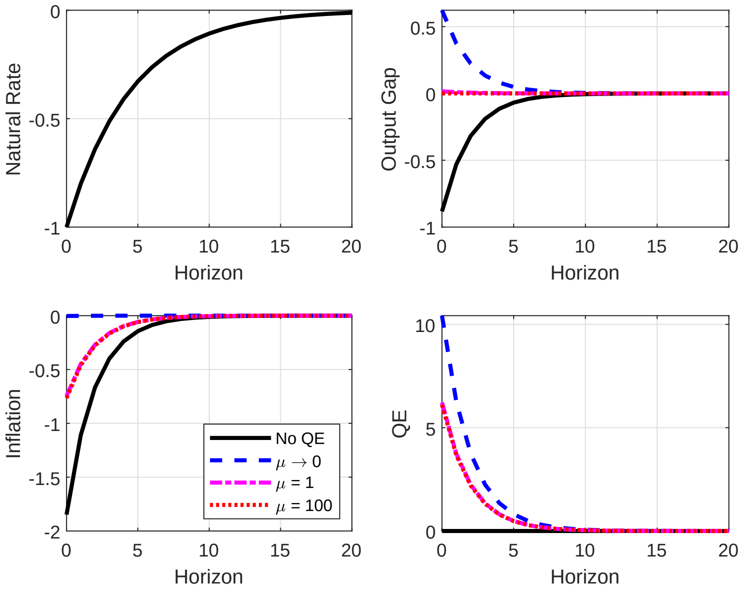
$$qe_t = -\frac{b^{FI}}{b^{cb}}\theta_t + \tau r_t^*$$

Use of QE **significantly** reduces costs of ZLB

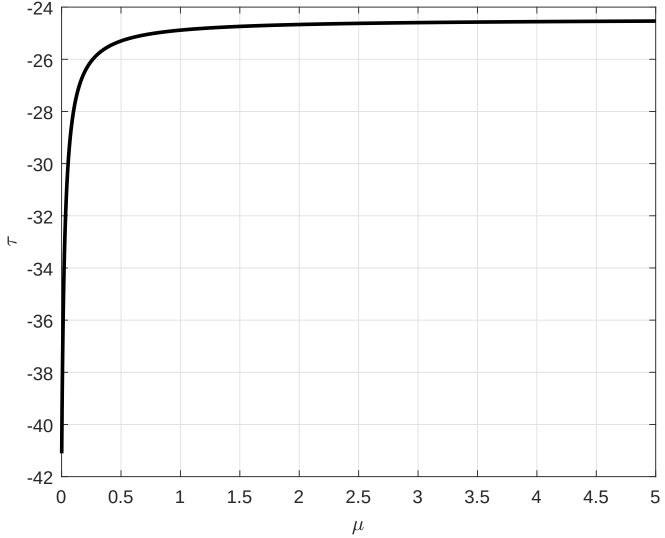
QE Response as Function of Duration of ZLB



Responses to Natural Rate Shock, Different μ



τ as a Function of μ



Interest Rate Policy Without QE

Pre-Great Recession, Fed did not use QE

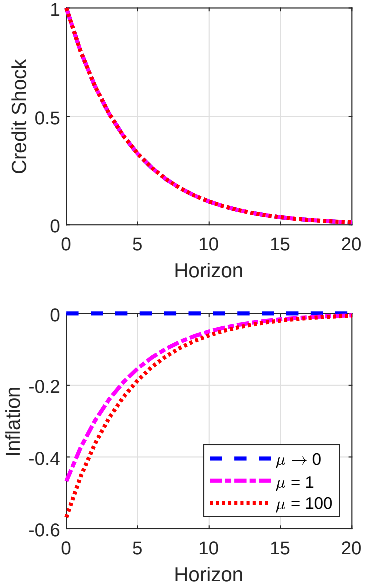
FOC for policy rate holds, does not hold for QE ($qe_t = 0$)

In equilibrium, policy rate will react to credit shocks (unlike with both instruments available) with:

$$r_t^s = r_t^* + \eta\theta_t$$

How policy rate reacts to credit shocks a function of relative weight on gap, μ

Responses to Credit Shock



Policy Rate Responses: η as a function of μ