The Term Premium and Rudebusch and Swanson (2012, *AEJ: Macro*)

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Term Premium

The term premium is the difference between the yield on a long maturity bond and the hypothetical yield predicted by the expectations hypothesis

In the data, it seems to be positive (about 100 basis points when looking at 10 year Treasuries) and reasonably volatile

Unconventional monetary policy actions, such as large scale asset purchases / quantitative easing (LSAP/QE) implicitly were targeted at lowering term premia

But how?

Term Premium (cont)

Term premium can be interpreted just as a risk premium for interest rate risk (long bond prices fall when short term interest rates rise)

To first order, how we solve many macro models, there are no risk premia, so no term premia

Two basic approaches to get a term premium

1. Higher order solution plus play with preferences: can get term premium but not clear what relevance there is for policy
2. Segmentation frictions: will get term premium to first order and can be affected by policy

A Simple Model

Representative household preferences:

$$U = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j})$$

Can save via one (B) or two (Z) period pure discount bonds (prices P and Z) with face value 1:

$$C_t + P_t B_t + Q_{t,t,t+2} Z_{t,t,t+2} + Q_{t,t-1,t+1} Z_{t,t-1,t+1} \leq \\ Y_t + B_{t-1} + Q_{t,t-1,t+1} Z_{t-1,t-1,t+1} + Z_{t-1,t-2,t}$$

Subscript notation:

- First subscript: date of choice
- Second subscript: date of issuance
- Third subscript: date of maturity

FOC

One period bond, B_t:

$$P_t = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \right]$$

Two period bond, new issue, $Z_{t,t,t+2}$:

$$Q_{t,t,t+2} = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} Q_{t+1,t,t+2} \right]$$

Two period bond, previous issue, $Z_{t,t-1,t+1}$:

$$Q_{t,t-1,t+1} = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} \right] = P_t$$

Note: price of bond doesn't depend on date of issuance, just time to maturity: $Q_{t,t-1,t+2} = P_t$

Combining

Combine these FOC together to get the price of the two period bond in period t, and drop middle subscript since date of issuance doesn't matter for price:

$$Q_{t,t+2} = \mathbb{E}_t \left[\frac{\beta u'(C_{t+1})}{u'(C_t)} P_{t+1} \right]$$

One is **tempted** to distribute the expectations operator, which would then give us:

$$Q^{EH}_{t,t+2} = P_t \, \mathbb{E}_t \, P_{t+1}$$

This is the **expectations hypothesis** – the price of a long bond is the product of the prices of the sequence of current and expected short bond prices

Yields and Prices

Yield to maturity (yield) equates PDV of cash flows from holding until maturity to bond price. So:

$$P_t = \frac{1}{1 + r_{1,t}}$$

$$Q_{t,t+2} = \frac{1}{(1 + r_{2,t})^2}$$

Expectations hypothesis then says:

$$\frac{1}{(1 + r_{2,t})^2} = \frac{1}{1 + r_{1,t}} \mathbb{E}_t \frac{1}{1 + r_{1,t+1}}$$

Or:

$$r_{2,t} \approx \frac{1}{2} \left[r_{1,t} + \mathbb{E}_t \, r_{1,t+1} \right]$$

Expectations Hypothesis Intuition

Intuition for Expectations Hypothesis is simple:

- Bonds of different maturities are **perfect substitutes**
- They are just different means of transferring resources intertemporally
- This imposes tight restrictions on prices/yields of bonds with different maturities

Empirical Issues

Yield curves (plots of yields against time to maturity for given credit risk, e.g. Treasuries) are almost always upward-sloping

This is difficult to reconcile with expectations hypothesis – the typical yield curve should be flat

Term premium: difference between actual yield to maturity and yield to maturity implied by expectations hypothesis

For general time to maturity:

$$tp_t = r_{h,t} - r_{h,t}^{EH}$$

Not the same thing as slope of yield curve except in steady state

Quantitative Easing

QE (or Large Scale Asset Purchases): essentially buying large quantity of long-term bonds, with intent to push up price and push down yield

Antidote to the problem of ZLB and inability to adjust short-term rates to provide further stimulus

But according to Expectations Hypothesis, this won't work

- Ben Bernanke: "The problem with QE is that it works in practice but not in theory."

Long-bond price can't be affected absent a change in the path of short-term rates according to Expectations Hypothesis

To make sense of QE at all, need some sort of failure of the expectations hypothesis

How to Get Non-Zero and Variable Term Premia

To say anything about policies like QE, need term premium to be **non-zero** and **variable**

In a standard macro model solved to first-order, there will be **no term premium**

Why? To first order, you **can** distribute the expectations operator as I did above

Need one of two things (or potentially both)

1. Solve model via higher-order approximation: this allows for risk premia more generally
2. Introduce some kind of friction that makes bonds of different maturities imperfect substitutes (segmentation)

A Macro Model with Real and Nominal Risk

I'm going to focus on the higher-order approximation approach in this lecture

Write down a model with **nominal** bonds and sticky prices, standard preferences

This means there is both real and nominal risk for bond prices

Solve the model via higher order approximation

This will generate a term premium but it is trivially small

Rudebusch and Swanson (2012)

Rudebusch and Swanson (2012, *AEJ: Macro*) introduce Epstein-Zin (1989) preferences

- Separates risk aversion from intertemporal substitution
- Can get lots of risk aversion without assuming too little intertemporal substitution
- Improves asset pricing performance of model more generally (i.e. equity premium puzzle) without substantially affecting macro dynamics
- But still basically irrelevant for QE-type policies

Modeling Long-Term Bonds

Having a bunch of bonds with different maturities gets clunky very quickly (see above)

Useful trick based on Woodford (2001): include a **perpetual bond** with a **decaying** coupon payment

The decay parameter, $\kappa \in [0, 1]$, nests the traditional one period bond ($\kappa = 0$) or a pure perpetuity ($\kappa = 1$)

Can pick κ to match a desired duration (e.g. 10 year maturity)

Generalized Consol

Suppose you issue CB_t in nominal perpetual bonds (consols) in period t

This obligates issuer to one dollar in coupon payments in $t+1$, κ dollars in $t+2$, and κ^2 dollars in $t+3$, and so on

Total coupon liability due in t:

$$B_{t-1} = CB_{t-1} + \kappa CB_{t-2} + \kappa^2 CB_{t-3} + \dots$$

Leading forward one period:

$$B_t = CB_t + \kappa CB_{t-1} + \kappa^2 CB_{t-2} + \dots$$

Hence:

$$CB_t = B_t - \kappa B_{t-1}$$

Bond Prices

The neat thing: you just need to keep track of coupon liability in t, B_{t-1}, and in $t+1$, B_t, not the entire sequence of issuances

New issues trade at price Q_t in period t

Past issues trade at proportional price, $\kappa^j Q_t$, since cash flows are just scaled versions of one another

Total value of bonds issued is therefore just proportional to coupon liability:

$$Q_t CB_t + \kappa Q_t CB_{t-1} + \kappa^2 CB_{t-2} + \cdots = Q_t B_t$$

Bottom line: just need to keep track of B_t and Q_t (like we would do for a one period bond)

A NK Model with Long Bonds

Households

Preferences:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} \right\}$$

Capital accumulation:

$$K_{t+1} = \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t + (1-\delta) K_t$$

Flow budget constraint (nominal). D_t: one period bonds, B_t: generalized consol

$$P_t C_t + P_t I_t + D_t + Q_t (B_t - \kappa B_{t-1}) \leq W_t L_t + R_t K_t + R_{t-1}^d D_{t-1} + B_{t-1} + DIV_t$$

FOC

Let $\Lambda_{t-1,t}$ be the *real* stochastic discount factor:

$$\Lambda_{t-1,t} = \beta \left(\frac{C_{t-1}}{C_t} \right)^{\sigma}$$

FOC for one period bonds is standard:

$$1 = \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1} R_t^d$$

As is labor supply condition:

$$\psi L_t^{\chi} = w_t C_t^{-\sigma}$$

Capital and Investment

These FOC are also standard

Capital:

$$q_t = \mathbb{E}_t \, \Lambda_{t,t+1} \left[r_{t+1} + (1-\delta) q_{t+1} \right]$$

Where q_t is the ratio of Lagrange multipliers on accumulation equation relative to the budget constraint – gives the consumption value of additional capital

Investment:

$$1 = q_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \, \Lambda_{t,t+1} q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2$$

FOC for Long Bonds

The FOC is:

$$Q_t = \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1} (1 + \kappa Q_{t+1})$$

In words, price today is expectation of product of nominal SDF ($\Lambda_{t,t+1} \Pi_{t+1}^{-1}$) with future payouts – coupon payment of 1 and continuation value of κQ_{t+1}

Define $R_{B,t}$ as return on long bond:

$$R_{B,t} = \frac{1 + \kappa Q_t}{Q_{t-1}}$$

Can equivalently write FOC:

$$1 = \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1} R_{B,t+1}$$

Comparing FOC for Long and Short Bonds

FOC for short bonds (deposits), which offer a *predetermined* nominal return:

$$1 = R_t^d \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1}$$

FOC for long bonds, where nominal return is unknown at t:

$$1 = \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{-1} R_{B,t+1}$$

If there were no uncertainty, we could ignore expectations operators and get $\mathbb{E}_t \, R_{B,t+1} = R_t^d$

There would be no upward-sloping yield curve, no term premium

To first order approximation, this is in fact what we would get

Production Sector and Monetary Policy

Three part production sector

1. Wholesale firm: producing using capital and labor, sells output at $P_{w,t}$ to retailers
2. Continuum of retailers: repackage wholesale output. Subject to Calvo price stickiness (θ)
3. Final good: CES aggregate of retailer output

Generates standard-looking "supply-side" equations

Monetary policy: Taylor rule

Yield to Maturity

Yield to maturity is (gross) discount rate equate pricing of bond to cash flows:

$$Q_t = \frac{1}{R_{y,t}} + \frac{\kappa}{R_{y,t}^2} + \frac{\kappa^2}{R_{y,t}^3}$$

This satisfies:

$$R_{y,t} = Q_t^{-1} + \kappa$$

Term Premium

To say something about the term premium, we need to postulate a hypothetical expectations hypothesis (EH) bond

Priced according to safe short-run rate, not the SDF:

$$Q_{EH,t} = \frac{1 + \kappa\, \mathbb{E}_t\, Q_{EH,t+1}}{R_t^d}$$

Yield to maturity satisfies:

$$R_{EH,t} = Q_{EH,t}^{-1} + \kappa$$

Term premium (gross) is ratio of the two:

$$TP_t = \frac{R_{y,t}}{R_{EH,t}}$$

Calibration

Standard values: $\beta = 0.99$, $\alpha = 1/3$, $\delta = 0.025$, $\sigma = 2$, $\epsilon = 11$, $\chi = 1$, $\theta = 0.75$. ψ chosen to normalize $L = 1$

Pick $\kappa = 1 - 40^{-1}$ (ten year duration)

Investment adjustment cost parameter: $S(\cdot) = \frac{\psi_i}{2}\left(I_t / I_{t-1} - 1\right)^2$, $\psi_i = 2$.

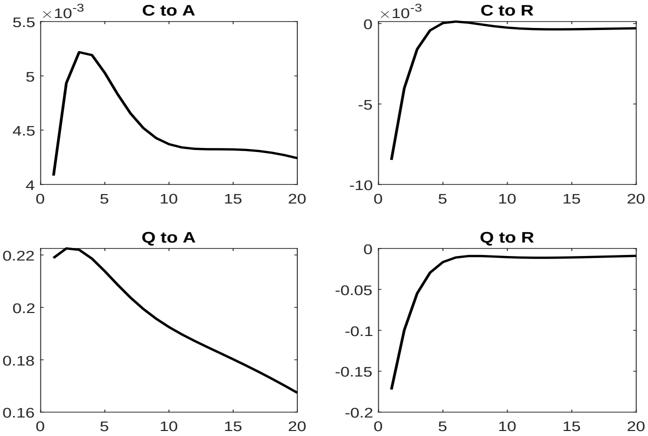
Note: term premium is **zero** in the non-stochastic steady state

And solved to first order, it is constant

Term premium relies on **higher order** covariance terms

Only two exogenous shocks: productivity and monetary policy

Impulse Responses



Term Premium?

Payout on long bonds, Q_t, is covaries **positively** with consumption conditional on both shocks

Intuition: productivity shock:

- Causes output and consumption to rise, but inflation to fall
- Taylor rule results in falling short term rate
- Low short rate: good for long bond prices

Intuition: monetary shock

- Short rate is exogenously higher
- This causes long bond prices to fall

Long bond payout covaries **negatively** with marginal utility of consumption – demand a premium in terms of higher yield to hold

Third Order Solution

To get a term premium, need higher order solution

Second order: will get non-zero average risk premia, but they will be constant

- Basically to second order, agents don't like risk, but they don't really do anything about it (e.g. quadratic utility)

So I solve and simulate the model via third order approximation, as in Rudebusch and Swanson (2012)

Results

In data, average term premium is about 100 basis points and relatively volatile (50 basis points)

In solution of the model, I get an average term premium of 0.003 (annualized percentage points) and volatility of 0.0006

Basically: with standard preferences can't generate a high and volatile term premium

In a sense, this is just a manifestation of the equity premium puzzle

Need the SDF to move a lot – need lots of risk aversion

But with standard preferences, this means little intertemporal substitution, which weakens fit of model in terms of macro moments

Epstein-Zin Preferences

Epstein-Zin Preferences

Let value function be:

$$V_t = u(C_t, L_t) + \beta \left(\mathbb{E}_t \, V_{t+1}^{1-\zeta} \right)^{\frac{1}{1-\zeta}}$$

If $\zeta = 0$, we have the standard expected utility case

Loosely, ζ controls risk aversion

- Whether you need that positive or negative depends on sign of flow utility (which could be positive or negative)

Stochastic Discount Factor

FOC for static variables like labor are exactly the same

FOC for dynamic variables look like the same too, written in terms of SDF

But SDF is different:

$$\Lambda_{t,t+1} = \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\zeta} \right)^{\frac{1}{1-\zeta}}} \right)^{-\zeta}$$

$\zeta = 0$ corresponds to base case from earlier

Note also: in a linear approximation, since you can effectively "distribute" expectations operators, the augmentation factor based on the value function drops out

Quantitative Results

I set $\zeta = -150$, basically what Swanson and Rudebusch (2012) do

I get an average term premium of 0.38 annualized percentage points, a *significant* improvement over case with expected utility preferences

Volatility is higher too, though still too low relative to data (and lower than what Swanson and Rudebusch find)

Note also: macro moments and IRFs are basically the same with $\zeta = -150$ and a third order approximation as when $\zeta = 0$

These preferences essentially allow you to do better on matching asset pricing facts without affecting much else

Two Dissatisfying Things

E-Z preferences can **significantly** improve asset pricing performance

But . . .

1. To what end, and of what relevance to policy? You can't really impact the term premium via policy here – it depends on a covariance between SDF and long bond price – and even if you could, it's not really relevant for quantities

2. Improved fit depends on kind of shocks

MEI shock

Include a marginal efficiency of investment (MEI) shock (Justiniano, Primiceri, and Tambalotti 2010, 2011):

$$K_{t+1} = \nu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta) K_t$$

Only affects FOC for investment:

$$1 = q_t \nu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \mathbb{E}_t \, \Lambda_{t,t+1} q_{t+1} \nu_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

Fuerst and Mau (2019)

Fuerst and Mau (2019) note that MEI shocks tend to overturn these results about the average term premium

In particular, with the MEI shock, we get long bonds being a hedge – the payout is high when consumption is low

This results in a *negative* average term premium, not positive, with E-Z preferences

They argue that a segmentation approach, as in for example Carlstrom, Fuerst, and Paustian (2017, *AEJ: Macro*) is a better way to think about the term premium and its relevance for monetary policy