Preference-Based Explanations for the Term Premium and Swanson and Rudebusch (2012, *AEJ: Macro*)

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1 Overview

In standard macro models, we assume that all debt is either one-period or intra-period. This is obviously unrealistic – in the real world, many debt contracts (e.g. mortgage loans, corporate bonds) are long-term, often ten or more years. An important long-term bond is the 10-year Treasury, for example.

A significant component of the *unconventional* monetary policy actions of the last decade-plus has been large scale asset purchases (LSAPs), more commonly referred to as quantitative easing (QE). In its most basic form, QE involves the Fed buying long term bonds (financed via the creation of bank reserves), with the intent of pushing their prices up and yields down. Since many private debt instruments are priced off of long-term Treasuries, such as mortgage bonds, the idea is that if the Fed can push down long-term bond rates, it can nevertheless stimulate interest-sensitive components of expenditure even when short-term rates (like the Fed Funds Rate) are constrained by the zero lower bound (ZLB).

A yield curve is a plot of interest rates (on debt instruments of the same credit risk) against time to maturity. We most commonly look at yield curves for Treasuries. In the data, yield curves are on average upward-sloping – it appears as though long-term bonds are riskier and investors demand compensation to hold them relative to short term bonds in the form of higher average yields. We can fairly easily incorporate long-term bonds into standard macro models but these models will miss the boat. In a linearized macro model, the pure expectations hypothesis will hold, which says that a long bond rate equals the average of the sequence of expected short rates over the life of the long bond. Since, on average, short rates are neither trending up nor down, the pure expectations hypothesis will predict that yield curves are flat on average. This is counterfactual.

The *term premium* is defined as the difference between the yield on a long bond and the hypothetical yield that would be implied by the expectations hypothesis. This is related to the "slope" of a yield curve but is not the same thing – the yield curve slope can change all the time due to changes in the expected path of short-term rates even under the expectations hypothesis. In this note, I will introduce a useful trick to incorporate a long-term bond into an otherwise standard macro model in a tractable way. We can then price long-term bonds and uncover their yields to maturity, and can compare those yields to what would be predicted by the pure expectations hypothesis. In a first order approximation, the pure expectations hypothesis holds, so there is no term premium at all in these models.

To get a term premium, we either need to (i) use a higher order approximation in the solution of the model, which will allow for risk premia to show up, or (ii) introduce some kind of other friction which will generate term premia even to first order. This note is going to focus on (i); we will then next study (ii). Although the model I consider is slightly different, this note is heavily influenced by Rudebusch and Swanson (2012, *AEJ: Macro*). I will construct a sticky price model with short- and long-term bonds. Then I will solve the model via a third order approximation. While the term premium won't be zero in this setup, it turns out to be close to zero and average and nearly constant with "standard" preferences. As suggested by Rudebusch and Swanson (2012), we can alternatively specify preferences according to Epstein and Zin (1989, *Econometrica*), E-Z for short. These preferences separate out risk aversion from intertemporal substitution, which are inextricably linked using standard preferences. With E-Z preferences, we can assume lots of risk aversion and improve upon the asset pricing performance of standard models without much effect on the implications for business cycle variables. The term premium is one such area. As Rudebusch and Swanson (2012) show and as I will confirm, using EZ preferences significantly improves the fit of the model in terms of the average term premium and its volatility. It is not clear, however, how relevant this is for monetary policy, because in this setup the term premium is not something that can be affected much by things like QE. Furthermore, the improved performance with respect to the term premium with E-Z preferences depends on the exact shocks buffeting the economy.

2 Long-Term Bonds

In standard DSGE models, we assume bonds are all one-period. In reality, many debt instruments are multi-period. From a modeling perspective, this can get overwhelming in that introducing multi-period bonds significantly increases the number of variables one has to keep track of in a model.

A useful "trick" is to introduce a *perpetual* bond with a *declining* coupon payment. One can pick the decay parameter to mimic the duration of, say, a 10-year bond in the data. This follows Woodford (2001).

In particular, suppose that issues one dollar of this long-term in bond in period t. Let this new issuances be denoted by CB_t. This obligates the issuer to a coupon payment of one dollar in $t+1$, κ dollars in $t+2$, κ^2 dollars in $t+3$, and so on, with $\kappa \in [0,1]$ (where $\kappa = 0$ is nests the standard one-period bond, whereas $\kappa = 1$ is a true consol/perpetuity). The total coupon liability due in period t is based on *past* issuances of the long-term bond. Let the total coupon liability due in t be equal to B_{t-1} (dated $t-1$ because it is predetermined based on actions taken prior to t). It satisfies:

$$B_{t-1} = CB_{t-1} + \kappa CB_{t-2} + \kappa^2 CB_{t-3} + \kappa^3 CB_{t-4} + \dots \tag{1}$$

Iterate this forward one period for the total coupon liability due in $t+1$:

$$B_t = CB_t + \kappa CB_{t-1} + \kappa^2 CB_{t-2} + \kappa^3 CB_{t-3} \dots \tag{2}$$

If you combine (2) and (1), you see:

$$CB_t = B_t - \kappa B_{t-1} \tag{3}$$

One issues these new long-term bonds in period t at market price Q_t (note, the notation I'm going to use is that the market price, Q_t, is real, whereas the quantity of issuance is dollar-denominated). Because of the decaying coupon structure, bonds issued in period $t-j$ will trade at $\kappa^j Q_t$ for $j \geq 0$. This ends up being super nice, because we don't need to keep track of the price of all previously issued long-term bonds, just the current price. In particular, the value of a long bond portfolio on takes from t to $t+1$ is given by:

$$Q_t B_t = Q_t CB_t + \kappa Q_t CB_{t-1} + \kappa^2 Q_t CB_{t-2} + \dots \tag{4}$$

The upshot of all this is the following. We can include long bonds in the model by only keeping track of one state variable, B_{t-1} (the total outstanding coupon liability), and one price, Q_t, with the household getting to choose how much of the future state variable to issue/hold, B_t. This is exactly how we would do a one-period bond, but it allows us to compare to the richness of the term structure of interest rates that we observe in the data.

3 Model

3.1 Household

There is a representative household with standard preferences over consumption and labor. The household owns and accumulates the capital stock. It earns income from supplying labor and leasing capital to firms. It can save via a standard one period nominal bond, D_t, or through the long-term bond discussed about, B_t. P_t is the price of goods. Its budget constraint is:

$$P_t C_t + P_t I_t + D_t + Q_t(B_t - \kappa B_{t-1}) \leq W_t L_t + R_t K_t + R^d_{t-1} D_{t-1} + B_{t-1} + DIV_t \tag{5}$$

On the expenditure side, the household consumes; purchases new capital; saves via one period bonds, denoted via D_t; and purchases new long-term bonds, which is Q_t times the issuance, which as noted above can be written $B_t - \kappa B_{t-1}$. On the income side, the household earns labor and rental income, interest plus principal on its short-term bonds, the total coupon payment from its holdings of long-term bonds, B_{t-1}, and a nominal dividend payout, DIV_t, from its ownership stake in production firms.

Capital accumulates according to a law of motion with Christiano, Eichenbaum, and Evans (2005) style "I-dot" adjustment costs:

$$K_{t+1} = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)K_t \tag{6}$$

The function $S(\cdot)$ satisfies: $S(1) = S'(1) = 0$, while $S''(1) = \phi_i \geq 0$.

Formally, the household problem is:

$$\max_{C_t, L_t, D_t, B_t, I_t, K_{t+1}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} \right\}$$

$$\text{s.t.}$$

$$P_t C_t + P_t I_t + D_t + Q_t(B_t - \kappa B_{t-1}) \leq W_t L_t + R_t K_t + R^d_{t-1} D_{t-1} + B_{t-1} + DIV_t$$

$$K_{t+1} = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)K_t$$

A downside of these preferences is that they are not consistent with balanced growth unless $\sigma = 1$, in which case flow utility from consumption reduces to the natural log. Let's ignore that – I'm not going to write down the model with permanent shocks anyway. Let λ_t be the multiplier on the budget constraint, and μ_t the multiplier on the capital accumulation equation. Form a Lagrangian:

$$\begin{aligned}\mathbb{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} + \mu_t \left[\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)K_t - K_{t+1} \right] \\ & + \lambda_t \left[W_t L_t + R_t K_t + R^d_{t-1} D_{t-1} + B_{t-1} + DIV_t - P_t C_t - P_t I_t - D_t - Q_t(B_t - \kappa B_{t-1}) \right] \Bigg\}\end{aligned}$$

The FOC are:

$$\frac{\partial \mathbb{L}}{\partial C_t} = C_t^{-\sigma} - \lambda_t P_t$$

$$\frac{\partial \mathbb{L}}{\partial L_t} = -\psi L_t^{\chi} + \lambda_t W_t$$

$$\frac{\partial \mathbb{L}}{\partial I_t} = \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] - \lambda_t P_t + \beta\, \mathbb{E}_t\, \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

$$\frac{\partial \mathbb{L}}{\partial D_t} = -\lambda_t + \beta\, \mathbb{E}_t\, \lambda_{t+1} R_t$$

$$\frac{\partial \mathbb{L}}{\partial B_t} = -\lambda_t Q_t + \beta\, \mathbb{E}_t\, \lambda_{t+1}\left(1 + \kappa Q_{t+1}\right)$$

$$\frac{\partial \mathbb{L}}{\partial K_{t+1}} = -\mu_t + \beta\,\mathbb{E}_t\,\lambda_{t+1} R_{t+1} + \beta\,\mathbb{E}_t\,\mu_{t+1}(1-\delta)$$

Set these equal to zero and eliminate λ_t. For the labor supply condition, we get:

$$\psi L_t^{\chi} = w_t C_t^{-\sigma} \tag{7}$$

For the short-term bond condition, we get:

$$\frac{1}{P_t C_t^{\sigma}} = \beta\,\mathbb{E}_t\,\frac{1}{P_{t+1} C_{t+1}^{\sigma}} R_t$$

Which can be written:

$$1 = \mathbb{E}_t\,\Lambda_{t,t+1} R_t \Pi_{t+1}^{-1} \tag{8}$$

Where I have introduced $\Lambda_{t,t+1}$ as the household's real stochastic discount factor:

$$\Lambda_{t-1,t} = \beta \left(\frac{C_{t-1}}{C_t}\right)^{\sigma} \tag{9}$$

And have defined $\Pi_t = P_t/P_{t-1}$ as the gross inflation rate.
For the long-term bond FOC, we have:

$$\frac{Q_t}{P_t C_t^{\sigma}} = \beta\,\mathbb{E}_t\,\frac{1}{P_{t+1} C_{t+1}^{\sigma}}(1+\kappa Q_{t+1})$$

Which may similarly be written using the SDF:

$$Q_t = \mathbb{E}_t\,\Lambda_{t,t+1} \Pi_{t+1}^{-1}(1+\kappa Q_{t+1}) \tag{10}$$

The FOC for capital may be written:

$$\mu_t = \beta\,\mathbb{E}_t\left[\lambda_{t+1} R_{t+1} + \mu_{t+1}(1-\delta)\right]$$

Subbing out λ_{t+1}, and defining $r_t = R_t/P_t$ as the real rental rate, we have:

$$\mu_t = \beta\,\mathbb{E}_t\left[\frac{1}{C_{t+1}^{\sigma}} r_{t+1} + \mu_{t+1}(1-\delta)\right]$$

Now, define $q_t = \mu_t C_t^{\sigma}$. This is the ratio of the Lagrange multiplier on the accumulation equation to the marginal utility of consumption. The Lagrange multiplier says how many utils you get from having more capital (i.e. by relaxing the constraint); dividing by the marginal utility of consumption (with these preferences multiplying by consumption) tells us how many additional units of consumption is equivalent to having more capital. Multiply both sides of the above by C_t:

$$\mu_t C_t^{\sigma} = \beta\,\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} r_{t+1} + C_t^{\sigma} \mu_{t+1}(1-\delta)\right]$$

Now, multiply and divide by C_{t+1}^{σ} inside the brackets:

$$\mu_t C_t^{\sigma} = \beta \, \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} r_{t+1} + \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} C_{t+1}^{\sigma} \mu_{t+1} (1-\delta) \right]$$

But then using this new definition, we have:

$$q_t = \mathbb{E}_t \, \Lambda_{t,t+1} \left[r_{t+1} + (1-\delta) q_{t+1} \right] \tag{11}$$

(11) is just a standard asset pricing condition – the value of capital today is the discounted value of its flow benefit (the rental rate) plus its continuation value (adjusted for depreciation).

The FOC for investment may be written:

$$\lambda_t P_t = \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta \, \mathbb{E}_t \, \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

Which is:

$$1 = \mu_t C_t^{\sigma} \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta C_t^{\sigma} \, \mathbb{E}_t \, \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

Multiply and divide by C_{t+1} in the final term:

$$1 = \mu_t C_t^{\sigma} \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta C_t^{\sigma} \, \mathbb{E}_t \, \frac{1}{C_{t+1}^{\sigma}} C_{t+1}^{\sigma} \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

But then in terms of q_t:

$$1 = q_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \, \Lambda_{t,t+1} q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \tag{12}$$

If there were no adjustment cost, then this would just tell us $q_t = 1$.

3.2 Wholesale Firm

There is a wholesale firm, denoted with a w subscript. It produces output using:

$$Y_{w,t} = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{13}$$

The wholesale firm hires labor and leases capital from the household. It sells its output to retail firms (discussed below) at $P_{w,t}$. Its problem is:

$$\max_{L_t, K_t} \; P_{w,t} A_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - R_t K_t$$

The FOC are:

$$W_t = (1-\alpha) P_{w,t} A_t K_t^{\alpha} L_t^{-\alpha}$$

$$R_t = \alpha P_{w,t} A_t K_t^{\alpha-1} L_t^{1-\alpha}$$

Define $mc_t = P_{w,t}/P_t$ (this will be real marginal cost for the retailers). These FOC in terms of real prices are then:

$$w_t = (1-\alpha) mc_t A_t K_t^{\alpha} L_t^{-\alpha} \tag{14}$$

$$r_t = \alpha mc_t A_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{15}$$

The wholesale firm earns zero real profit in equilibrium:

$$mc_t A_t K_t^{\alpha} L_t^{1-\alpha} - (1-\alpha) mc_t A_t K_t^{\alpha} L_t^{1-\alpha} - \alpha mc_t A_t K_t^{\alpha} L_t^{1-\alpha} = 0$$

3.3 Final Goods Firm

There are a continuum of retailers indexed by $z \in [0,1]$. They costlessly transform wholesale output, $Y_{w,t}$, purchased at P_t^w, into retail output, $Y_t(z)$. They then sell this retail output to a competitive final goods firm at $P_t(z)$. The competitive final goods firm produces final output:

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

Demand for each retail good is:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t \tag{17}$$

And the price index is:

$$P_t^{1-\epsilon} = \int_0^1 P_t(z)^{1-\epsilon} dz \tag{18}$$

3.4 Retailers

Retailers simply repackage wholesale output, so their production function is $Y_t(z) = Y_{w,t}(z)$. Their nominal profit is:

$$DIV_t = P_t(z) Y_t(z) - P_{w,t} Y_{w,t}(z) = P_t(z) Y_t(z) - P_{w,t} Y_t(z)$$

Plug in the demand function, (17):

$$DIV_t = P_t(z)^{1-\epsilon} P_t^{\epsilon} Y_t - P_{w,t} P_t(z)^{-\epsilon} P_t^{\epsilon} Y_t$$

Written in real terms:

$$div_t = P_t(z)^{1-\epsilon} P_t^{\epsilon-1} Y_t - mc_t P_t(z)^{-\epsilon} P_t^{\epsilon} Y_t$$

Retailers can only update their price in a given period with probability $1-\theta$; this is also the fraction of retailers who can update in any period. A price chosen in period t will be in effect k periods into the future with probability θ^k.

The problem of an updating retailer is to pick $P_t(z)$ to maximize the PDV of real dividends, discounting by the household's SDF and the probability that a price chosen today will still be in effect in the future:

$$\max_{P_t(z)} \; \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \Big\{ P_t(z)^{1-\epsilon} P_{t+s}^{\epsilon-1} Y_{t+s} - mc_{t+s} P_t(z)^{-\epsilon} P_{t+s}^{\epsilon} Y_{t+s} \Big\}$$

The FOC is:

$$(1-\epsilon) P_t(z)^{-\epsilon}\, \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} P_{t+s}^{\epsilon-1} Y_{t+s} + \epsilon P_t(z)^{-\epsilon-1}\, \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} mc_{t+s} P_{t+s}^{\epsilon} Y_{t+s}$$

Setting equal to zero, and noting that $P_t(z) = P_t^*$ which does not vary across retailers:

$$P_t^* = \frac{\epsilon}{\epsilon-1} \frac{\mathbb{E}_t \displaystyle\sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} mc_{t+s} P_{t+s}^{\epsilon} Y_{t+s}}{\mathbb{E}_t \displaystyle\sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} P_{t+s}^{\epsilon-1} Y_{t+s}}$$

Define $\Pi_t^* = P_t^*/P_t$ as the relative reset price, and introduce two auxiliary variables as recursive representations of the infinite sums. We get:

$$\Pi_t^* = \frac{\epsilon}{\epsilon-1} \frac{z_{1,t}}{z_{2,t}} \tag{19}$$

$$z_{1,t} = mc_t Y_t + \theta\, \mathbb{E}_t\, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} z_{1,t+1} \tag{20}$$

$$z_{2,t} = Y_t + \theta\, \mathbb{E}_t\, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} z_{2,t+1} \tag{21}$$

3.5 Monetary Policy

The gross nominal interest rate is set according to a Taylor rule:

$$R_t = R^{1-\rho_R} R_{t-1}^{\rho_R} \left[\Pi_t^{\phi_\pi} \left(Y_t / Y \right)^{\phi_Y} \right]^{1-\rho_R} \exp(s_R \varepsilon_{R,t}) \tag{22}$$

3.6 Aggregation

Aggregate production and price-setting conditions are:

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta)(\Pi_t^*)^{1-\epsilon} \tag{23}$$

$$A_t K_t^{\alpha} L_t^{1-\alpha} = Y_t v_t^p \tag{24}$$

v_t^p is a measure of price dispersion, which can be written:

$$v_t^p = (1-\theta)(\Pi_t^*)^{-\epsilon} + \theta \Pi_t^{\epsilon} v_{t-1}^p \tag{25}$$

A_t obeys an AR(1) process that is mean-zero in the log:

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t} \tag{26}$$

We do not need to specify anything about the quantities of long- or short-term debt to solve the model. These quantities are actually irrelevant the way in which we have written down the model. The aggregate resource constraint is standard:

$$Y_t = C_t + I_t \tag{27}$$

3.7 Bond Returns and the Term Premium

We can define the holding period return on the long bond, $R_{B,t}$, as:

$$R_{B,t} = \frac{1 + \kappa Q_t}{Q_{t-1}} \tag{28}$$

In the numerator, we have the coupon payment from buying a bond in $t-1$, 1, plus the continuation value of a bond issued in $t-1$ in period t, which is κQ_t. In the denominator we have the purchase price. So (28) is the gross holding period return.

The yield to maturity on the long bond is the (gross) discount rate that equates the price of the bond to the PDV of cash flows from holding it forever. It therefore satisfies:

$$Q_t = \frac{1}{R_{y,t}} + \frac{\kappa}{R_{y,t}^2} + \frac{\kappa^2}{R_{y,t}^3} + \dots$$

This can be written:

$$Q_t = \frac{1}{R_{y,t}} \left[1 + \frac{\kappa}{R_{y,t}} + \frac{\kappa^2}{R_{y,t}^2} + \dots \right] = \frac{1}{R_{y,t}} \frac{R_{y,t}}{R_{y,t} - \kappa} = \frac{1}{R_{y,t} - \kappa}$$

We therefore have:

$$R_{y,t} = Q_t^{-1} + \kappa \tag{29}$$

Note that (29) is only the same as (28) in the steady state.

The *term premium* is defined as the difference between the yield on the long-bond and a hypothetical "expectations hypothesis" bond. The price of the hypothetical expectations hypothesis bond satisfies:

$$Q_{EH,t} = \frac{1 + \kappa \, \mathbb{E}_t \, Q_{EH,t+1}}{R_t^d} \tag{30}$$

In other words, the expectations hypothesis bond price, $Q_{EH,t}$, is implicitly defined by discounting the stream of cash flows by the safe, short-term gross interest rate, R_t^d. The yield to maturity on the hypothetical expectations hypothesis bond takes the same form as (29):

$$R_{EH,t} = Q_{EH,t}^{-1} + \kappa \tag{31}$$

The gross term premium, TP_t, is then the ratio of the yield on the long-term bond and the hypothetical expectations hypothesis yield:

$$TP_t = \frac{R_{y,t}}{R_{EH,t}} \tag{32}$$

3.8 Full Set of Equilibrium Conditions

- Household

$$\psi L_t^{\chi} = w_t C_t^{-\sigma} \tag{33}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1} R_t^d \Pi_{t+1}^{-1} \tag{34}$$

$$\Lambda_{t-1,t} = \beta \left(\frac{C_{t-1}}{C_t} \right)^{\sigma} \tag{35}$$

$$Q_t = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} (1 + \kappa Q_{t+1}) \tag{36}$$

$$q_t = \mathbb{E}_t \Lambda_{t,t+1} \left[r_{t+1} + (1-\delta) q_{t+1} \right] \tag{37}$$

$$1 = q_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \Lambda_{t,t+1} q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \tag{38}$$

- Wholesale Firm:

$$w_t = (1-\alpha) mc_t A_t K_t^{\alpha} L_t^{-\alpha} \tag{39}$$

$$r_t = \alpha mc_t A_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{40}$$

- Retail Firm:

$$\Pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{z_{1,t}}{z_{2,t}} \tag{41}$$

$$z_{1,t} = mc_t Y_t + \theta \, \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} z_{1,t+1} \tag{42}$$

$$z_{2,t} = Y_t + \theta \, \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} z_{2,t+1} \tag{43}$$

- Monetary Policy:

$$R_t^d = (R^d)^{1-\rho_R}(R_{t-1}^d)^{\rho_R}\left[\Pi_t^{\phi_\pi}(Y_t/Y)^{\phi_Y}\right]^{1-\rho_R}\exp(s_R\varepsilon_{R,t}) \tag{44}$$

- Aggregate Conditions:

$$1 = \theta\Pi_t^{\epsilon-1} + (1-\theta)(\Pi_t^*)^{1-\epsilon} \tag{45}$$

$$A_t K_t^\alpha L_t^{1-\alpha} = Y_t v_t^p \tag{46}$$

$$v_t^p = (1-\theta)(\Pi_t^*)^{-\epsilon} + \theta\Pi_t^\epsilon v_{t-1}^p \tag{47}$$

$$\ln A_t = \rho_A \ln A_{t-1} + s_A\varepsilon_{A,t} \tag{48}$$

$$K_{t+1} = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)K_t \tag{49}$$

$$Y_t = C_t + I_t \tag{50}$$

- Bond returns and yields:

$$R_{B,t} = \frac{1+\kappa Q_t}{Q_{t-1}} \tag{51}$$

$$R_{y,t} = Q_t^{-1} + \kappa \tag{52}$$

$$Q_{EH,t} = \frac{1+\kappa\,\mathbb{E}_t\,Q_{EH,t+1}}{R_t^d} \tag{53}$$

$$R_{EH,t} = Q_{EH,t}^{-1} + \kappa \tag{54}$$

$$TP_t = \frac{R_{y,t}}{R_{EH,t}} \tag{55}$$

This is $\left\{Y_t, C_t, L_t, K_t, I_t, A_t, \Lambda_{t-1,t}, w_t, r_t, q_t, mc_t, R_t^d, \Pi_t, \Pi_t^*, z_{1,t}, z_{2,t}, v_t^p, Q_t, R_{B,t}, R_{y,t}, Q_{EH,t}, R_{EH,t}, TP_t\right\}$ 23 equations and 23 variables. The only two exogenous shocks are to productivity and monetary policy – i.e. a "supply" and a "demand" shock.

3.9 Steady State

Let variables without time subscripts denote non-stochastic steady state values. I assume zero trend inflation (so $\Pi = 1$ in gross terms) A couple are easy and just drop out:

$$\Lambda = \beta \tag{56}$$

$$R^d = \beta^{-1} \tag{57}$$

$$\Pi^* = 1 \tag{58}$$

$$v^p = 1 \tag{59}$$

$$mc = \frac{\epsilon}{\epsilon - 1} \tag{60}$$

$$q = 1 \tag{61}$$

We can solve for the steady state long bond price as:

$$Q = \left(\frac{1}{\beta} - \kappa\right)^{-1} \tag{62}$$

We can immediately see that the steady state expectations hypothesis bond price is the same thing:

$$Q_{EH} = \left(\frac{1}{\beta} - \kappa\right)^{-1} \tag{63}$$

But since the price of the long-bond and the hypothetical expectations hypothesis bond price are the same, their yields will also be the same:

$$R_y = R_{EH} = \frac{1}{\beta} \tag{64}$$

Which then implies that there is no term premium in the steady state:

$$TP = 1 \tag{65}$$

This is an important point that we shall return to below – in a frictionless model (frictionless from the perspective of bond-pricing; there are other frictions like sticky prices and monopolistic competition in this model) there is no term premium in the steady state. The term premium can only arise in terms of stochastic means based on covariances between bond returns and the stochastic discount factor. Since the steady state is non-stochastic, there will be no term premium in the steady state. And since in a first-order approximation, the stochastic mean of a variable equals the non-stochastic steady state, there will be no mean term premium (and hence no slope of the yield curve) without going to a higher order approximation.

For the rest of the model, let us normalize $L = 1$ by appropriately picking ψ. $A = 1$ in the steady state. Combine the expression for the rental rate on capital with the household's FOC for capital knowing that $q = 1$ (which comes from the FOC for investment given assumptions on $S(\cdot)$). We get:

$$1 = \beta \left[\alpha mc K^{\alpha-1} + (1-\delta)\right]$$

But then we can solve for K:

$$K = \left(\frac{\alpha mc}{\frac{1}{\beta} - (1-\delta)}\right)^{\frac{1}{1-\alpha}} \tag{66}$$

But once we know steady state K, we know steady state I. And given $L = 1$, we know steady state Y, w, and r. And hence we also know steady state C:

$$Y = K^{\alpha} \tag{67}$$

$$I = \delta K \tag{68}$$

$$C = K^{\alpha} - \delta K \tag{69}$$

$$w = (1-\alpha) mc K^{\alpha} \tag{70}$$

$$r = \alpha mc K^{\alpha-1} \tag{71}$$

But then we can solve for the requisite ψ to be consistent with our normalization of $L = 1$:

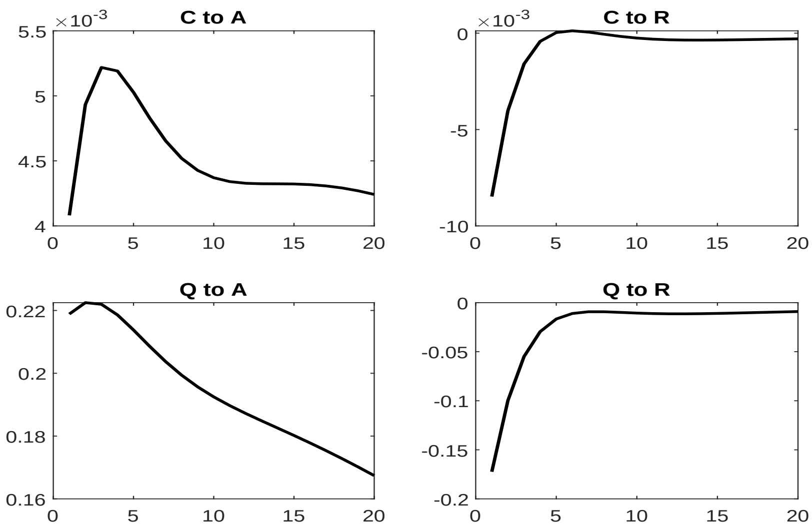
$$\psi = \frac{(1-\alpha) mc K^{\alpha}}{(K^{\alpha} - \delta K)^{\sigma}} \tag{72}$$

4 Parameterization, Moments, and Higher Order Approximations

I use a standard parameterization: $\beta = 0.99$, $\alpha = 1/3$, $\delta = 0.025$, $\sigma = 2$, $\epsilon = 11$, $\chi = 1$, and $\theta = 0.75$. I specify the Taylor rule with $\phi_\pi = 1.5$, $\phi_y = 0.25/4$. I calibrate the productivity shock to have $\rho_A = 0.95$ and $s_A = 0.005$, while the standard deviation of the monetary shock is $s_R = 0.003$. For the long bond, I set $\kappa = 1 - 40^{-1}$, which implies roughly a duration of 10 years (40 quarters). The investment adjustment cost function takes the form $S(\cdot) = \frac{\psi_i}{2}\left(I_t/I_{t-1} - 1\right)^2$, and I set $\psi_i = 2$.

If we solve the model via a first order approximation, we get impulse responses of consumption and the long bond price of:

Figure 1: IRFs to Shocks, First Order Solution



Why do I plot these? Focus on a productivity shock (first column). This causes long bonds to do well – their price goes up. Essentially what is going on is that the shock is deflationary, which causes the Fed to lower short term rates. Lower short rates are good for long-term bond prices. But while long term bond prices go up, consumption is also rising. This means, conditional on this shock, that long-term bonds are *risky* in the sense that they do well (high prices) when you don't value them doing well that much (consumption high, so marginal utility low). Flipping things around, when consumption is low (marginal utility is high), you'd really value an asset that has high payouts in those states. But the long bond does poorly in such states.

We see the same pattern (albeit reversed) conditional on the monetary policy shock. A contractionary policy shock raises short-term rates. This causes long-term bond prices to fall and consumption to fall. So long-term bonds are doing poorly precisely when you'd like them to do well (i.e. when consumption is low, so marginal utility of consumption is high).

Without getting into the mathematics, what this means is that we ought to expect to see long bonds trade at a *discount* relative to what the hypothetical expectations hypothesis bond would, where, as noted above, discounting is by the short-term rate rather than the stochastic discount factor. Because yields are inversely related to prices, we would therefore expect the yield on the long bond to reflect a premium for bearing this risk.

To investigate whether in fact we get a premium, we have to solve the model beyond first-order. To first-order, the model is certainty equivalent and there are no risk premia. If we solve the model via a second-order approximation, we will got non-zero risk premia but these will be constant. So let's go out to a third-order approximation. When I solve the same model via a third-order approximation, I do in fact get a positive term premium. But it is extremely small and close to constant – in my solution, the (annualized percentage) term premium is merely 0.03, while its standard deviation is 0.0006. These results are consistent with Rudebusch and Swanson (2012) – in the data, the average term premium on a 10 year bond is about 100 basis points (so 1 instead of 0.017) and its volatility is about 0.5. With expected utility preferences (see Table 2 in their paper), I am able to get a slightly positive average term premium, but it is very close to zero and close to constant.

The conclusion here is that, using the "standard preferences" we like to use in macro models, you really can't get much of a term premium at all. This result belongs to a long-list of asset-pricing failures in macro models with standard preferences.

5 Preference-Based Explanations for the Term Premium

Where Swanson and Rudebusch (2012) come in is they change preferences. In particular, they use Epstein-Zin preferences. These are not super easy to work with, but the basic gist is that they allow one to separate the coefficient of relative risk aversion from the intertemporal elasticity of substitution. With the standard preferences we like to work with (like the ones above)

Let the lifetime utility function be:

$$V_t = u(C_t, L_t) + \beta \left(\mathbb{E}_t \, V_{t+1}^{1-\zeta} \right)^{\frac{1}{1-\zeta}} \tag{73}$$

When $\zeta = 0$, this just gives us back our standard specification, where I have assumed $u(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi}$. Note that one would be tempted to "distribute and cancel" the $1-\zeta$ terms. But you cannot do this – the exponent $1-\zeta$ is *inside* an expectations operator while the $\frac{1}{1-\zeta}$ is applying to the entire expectation. Suppose, for example, that there are two states of nature in $t+1$, with probability p and $1-p$. Call these states (1) and (2). Then this last term that gets discounted would be:

$$\left(\mathbb{E}_t \, V_{t+1}^{1-\zeta} \right)^{\frac{1}{1-\zeta}} = \left(p V_{t+1}(1)^{1-\zeta} + (1-p) V_{t+1}(2)^{1-\zeta} \right)^{\frac{1}{1-\zeta}}$$

Hence, you cannot distribute and hence eliminate the terms involving ζ, unless, of course, $\zeta = 0$, in which case we'd be back in the usual case. Swanson and Rudebusch make this all sort of clear by explicitly using state notation. I'm going to in part do that in what follows.

Let's form a Lagrangian for the household problem. This is going to look a bit different because we're going to include (73) as a constraint. The Lagrangian is:

$$\mathbb{L} = V_0 - \mathbb{E}_0 \sum_{t=0}^{\infty} \omega_t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} + \beta \left(\sum_{s_{t+1}} \pi(s_{t+1}) V_{t+1}(s_{t+1})^{1-\zeta} \right)^{\frac{1}{1-\zeta}} - V_t \right\} +$$

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left[\left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t + (1-\delta) K_t - K_{t+1} \right]$$

$$+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[W_t L_t + R_t K_t + R_{t-1}^d D_{t-1} + B_{t-1} + DIV_t - P_t C_t - P_t I_t - D_t - Q_t (B_t - \kappa B_{t-1}) \right]$$

$\pi(s_{t+1})$ denotes the probability of each possible state of nature, s_{t+1}, materializing (implicitly this is conditional on the current state, s_t). I only use this notation for the part explicitly referencing the value function, though it is implicitly picked up by the expectations operators for other parts of the Lagrangian. $V_{t+1}(s_{t+1})$ is the value function in a particular state. The summation operator sums across possible states. It's important to be clear about this. We will be taking a derivative wrt to $V_t(s_t)$ above, where the optimization occurs in period $t-1$. You are effectively picking the value function in each state ahead of time.

The derivatives are:

$$\frac{\partial \mathbb{L}}{\partial C_t} = \omega_t C_t^{-\sigma} - \beta^t \lambda_t P_t$$

$$\frac{\partial \mathbb{L}}{\partial L_t} = -\omega_t \psi L_t^{\chi} + \beta^t \lambda_t W_t$$

$$\frac{\partial \mathbb{L}}{\partial D_t} = -\lambda_t + \beta \, \mathbb{E}_t \, \lambda_{t+1} R_t$$

$$\frac{\partial \mathbb{L}}{\partial I_t} = \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] - \lambda_t P_t + \beta \, \mathbb{E}_t \, \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

$$\frac{\partial \mathbb{L}}{\partial K_{t+1}} = -\mu_t + \beta \, \mathbb{E}_t \, \lambda_{t+1} R_{t+1} + \beta \, \mathbb{E}_t \, \mu_{t+1} (1-\delta)$$

$$\frac{\partial \mathbb{L}}{\partial V_t} = \omega_{t-1} \frac{\beta}{1-\zeta} \left(\sum_{s_t} \pi(s_t) V_t(s_t)^{1-\zeta} \right)^{\frac{1}{1-\zeta} - 1} (1-\zeta) V_t(s_t)^{-\zeta} - \mathbb{E}_{t-1} \, \omega_t$$

The last condition can more compactly be written using conventional expectation operator notation as:

$$\omega_{t-1} \beta (\mathbb{E}_{t-1} \, V_t^{1-\zeta})^{\frac{\zeta}{1-\zeta}} V_t^{-\zeta} = \mathbb{E}_{t-1} \, \omega_t$$

Setting the first equal to zero, we have:

$$\lambda_t = \frac{\omega_t}{\beta^t P_t C_t^{\sigma}}$$

Plugging this into the second, for example, the λ^t and ω_t cancel, leaving a standard FOC for labor:

$$\psi L_t^{\chi} = w_t C_t^{-\sigma} \tag{74}$$

Now, let's start to think about the stochastic discount factor. Go to the FOC for bonds. We get:

$$1 = \beta \, \mathbb{E}_t \, \frac{\lambda_{t+1}}{\lambda_t} R_t^d$$

Plugging in for λ_t, we'd have:

$$1 = \beta \, \mathbb{E}_t \, \frac{\omega_{t+1}}{\beta^{t+1} P_{t+1} C_{t+1}^{\sigma}} \frac{\beta^t P_t C_t^{\sigma}}{\omega_t} R_t^d$$

Which would be:

$$1 = \mathbb{E}_t \, \frac{\omega_{t+1}}{\omega_t} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{P_t}{P_{t+1}} R_t^d$$

Where is this getting us? Now to the FOC for V_t, setting it equal to zero:

$$\mathbb{E}_{t-1} \, \omega_t = \beta \omega_{t-1} \left(\mathbb{E}_{t-1} \, V_t^{1-\zeta} \right)^{\frac{-\zeta}{1-\zeta}} \mathbb{E}_{t-1} \, V_t^{-\zeta} \tag{75}$$

Now, suppose that $\zeta = 0$. This should correspond to the standard expected utility case. This would then imply that:

$$\mathbb{E}_{t-1} \, \omega_t = \beta \omega_{t-1}$$

Iterating forward one period, we'd have:

$$E_t \omega_{t+1} = \beta \omega_t$$

But plugging this into the bond Euler equation, we'd have:

$$1 = \beta \, \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} R_t^d \Pi_{t+1}^{-1}$$

But this would be exactly standard! So the SDF would be identical to the usual case when $\zeta = 0$. But when $\zeta > 0$, it's more complicated. Let's consider the more general case. Go back to (75). Iterate it forward one period. We get:

$$\mathbb{E}_t \, \omega_{t+1} = \beta \omega_t \left(\mathbb{E}_t \, V_{t+1}^{1-\zeta} \right)^{\frac{-\zeta}{1-\zeta}} V_{t+1}^{-\zeta} \tag{76}$$

We have:

$$\frac{\mathbb{E}_t \, \omega_{t+1}}{\omega_t} = \beta \left(\mathbb{E}_t \, V_{t+1}^{1-\zeta} \right)^{\frac{-\zeta}{1-\zeta}} V_{t+1}^{-\zeta} \tag{77}$$

Plugging that in for the Euler equation above, for example, we have:

$$1 = \mathbb{E}_t \, \beta \left(\mathbb{E}_t \, V_{t+1}^{1-\zeta} \right)^{\frac{\zeta}{1-\zeta}} V_{t+1}^{-\zeta} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} R_t^d \Pi_{t+1}^{-1}$$

We can thus define the real stochastic discount factor as:

$$\Lambda_{t,t+1} = \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{V_{t+1}}{\left(\mathbb{E}_t \, V_{t+1}^{1-\zeta} \right)^{\frac{1}{1-\zeta}}} \right)^{-\zeta}$$

With this, the multipliers drop out and give us the standard-looking intratemporal labor supply condition. Furthermore, the rest of the equilibrium conditions in the model look identical. All that is different is the stochastic discount factor, with $\zeta = 0$ reverting to the case I used above.

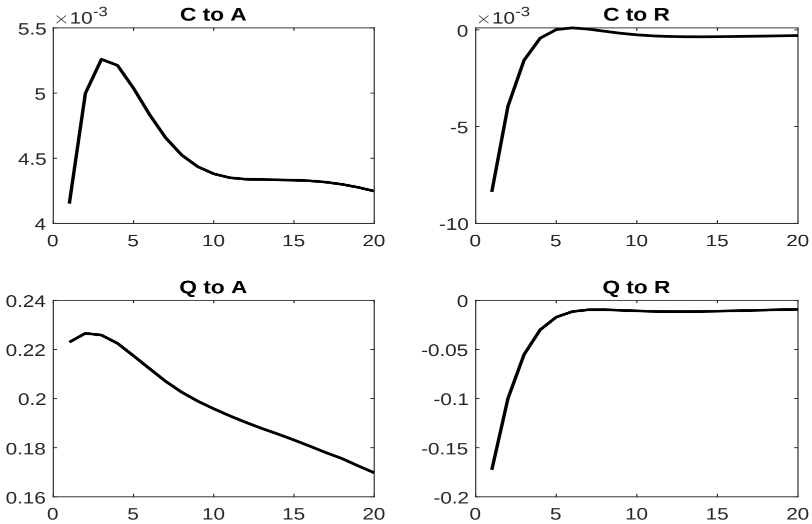
5.1 Quantitative Analysis

I'm going to use the same parameterization as above, but am going to (roughly) follow Rudebusch and Swanson and set $\zeta = -150$ (note the corresponding parameter in their notation is α, not ζ, where I have used α for the production function). Note that there is a subtle issue about the sign of ζ. See the sentences in Rudebusch and Swanson under equation (4): "When $u \geq 0$ everywhere, higher values of α correspond to greater degrees of risk aversion. When $u \leq 0$ everywhere, the opposite is true: higher values of α correspond to lesser degrees of risk aversion." With $\sigma > 1$, the sign of the flow utility function is in fact negative, so we need $\zeta < 0$ to correspond to more risk aversion. But if we changed things to scale where flow utility was positive (or, say, we assumed $\sigma < 1$), we would instead need positive values of ζ.

Using exactly the same calibration as above, I am able to get a significantly more positive average term premium. In particular, I get an average term premium of 0.38 and a standard deviation of the term premium of 0.009. These are not exactly the same as what Swanson and Rudebusch (2012) report but are in the same ballpark, particularly for the mean. The model details are somewhat different – I have variable investment and they do not, for example, plus I'm using perpetual bonds with decaying coupons rather than a straight up forty quarter bonds.

It is interesting to note that using E-Z preferences with a large ζ and solving the model via a third-order approximation have essentially *no effect* on the behavior of macro variables. See the impulse responses below and compare them to the IRFs with standard preferences under a first-order solution – they are virtually identical.

Figure 2: IRFs to Shocks, Third Order Solution, $\zeta = -150$



The following is a loose and probably not 100 percent correct characterization. What matters for macro dynamics is the elasticity of substitution, not the amount of risk aversion per se. What matters for asset prices (and only to higher-order) is the coefficient of relative risk aversion. With "standard" preferences these are one in the same – the elasticity of intertemporal substitution is the inverse of the coefficient of relative risk aversion. Generating amplification and persistence in a macro model requires high elasticities of substitution, such as implied by the popular log specification (i.e. $\sigma = 1$). But this is very low risk aversion, resulting in macro models doing very poorly in matching facts about asset prices. This is really nothing more than a restatement of the equity premium puzzle.

In a nutshell, Epstein-Zin preferences allow you to separate risk aversion from the elasticity of substitution. You can assume lots of risk aversion without impacting the elasticity of substitution and hence the behavior of macro variables. But it improves the asset pricing performance of the model – as we can see here, with E-Z preferences we get a much higher average term premium and it is more volatile. But since these preferences aren't really relevant for macro dynamics, they're not really relevant for policy. For example, large scale asset purchases (LSAPs, or QE) are putatively aimed at impacting term premia. But with preference-based explanation for the term premia such as resorting to E-Z preferences, there really is not way for policy to impact the term premium (because it is based on covariances of long bond prices with consumption), and even if it could it wouldn't matter. So this is somewhat dissatisfying.

Another potential problem that arises with this preference based explanation of the term premium is that it will only work conditional on certain types of shocks. If one thinks about recessions as periods when short-term interest rates are low, then long-term bonds should do well – bond prices move opposite interest rates, so if interest rates are low in a recession, long bond prices will be high, which makes them a hedge against low consumption (high marginal utility of consumption) in a recession. So with a preference-based explanation of the term premium, if we think recessions are period where short-term rates are low, we'd expect the term premia to on average be negative, not positive.

So why am I able to get a positive average term premium in the analysis above? It's because the shocks are rather carefully chosen to undue the intuition from the previous paragraph. A negative technology shock causes inflation to rise, which in turn causes the central bank to raise the short-term rate. This would be bad for long-bond prices, which would fall. These prices would be falling precisely when the household would like them to not (i.e. when consumption is low / marginal utility high). Similarly, consider a monetary shock. An exogenous contractionary shock causes short rates to rise and long bond prices to fall at the same time consumption is falling. The household again doesn't like this – it would like an asset where Q is high when consumption is low, and demands compensation in the form of a higher yield to hold it. If you make the household risk averse enough via ζ, you can get the average term premium to be empirically plausible conditional on these shocks.

So let's think about another kind of shock that won't have this feature. A good candidate is a marginal efficiency of investment shock (Justiniano, Primiceri, and Tambalotti 2010, 2011). Such a shock would appear as an exogenous term in the capital accumulation equation:

$$K_{t+1} = \nu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)K_t \tag{78}$$

ν_t governs the efficiency of transforming investment into new physical capital. Several authors, such as Justiniano, Primiceri, and Tambalotti (2010, 2011) have stressed this kind of shock as an important business cycle shock. This is only going to show up in the FOC for investment as follows:

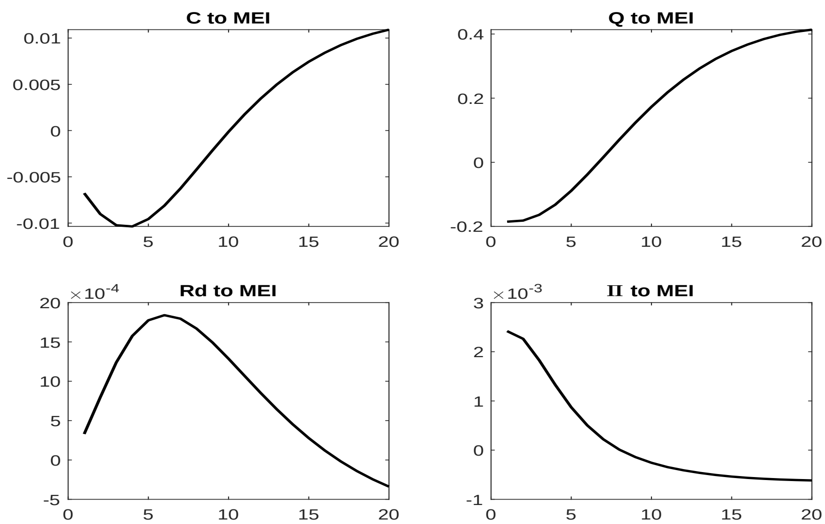
$$1 = q_t\nu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \mathbb{E}_t\,\Lambda_{t,t+1}q_{t+1}\nu_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{79}$$

Assume it follows an AR(1) with non-stochastic mean normalized to unity:

$$\ln\nu_t = \rho_\nu \ln\nu_{t-1} + s_\nu\varepsilon_{\nu,t} \tag{80}$$

Below are the impulse responses to a positive MEI shock assuming $\rho_\nu = 0.90$ and using all the same values for other parameters.

Figure 3: IRFs to MEI Shock



Consumption initially falls (output would rise); this is because ν_t being high makes the household want to substitute away from consumption and into investment. The shock is inflationary, and hence associated with short-term rates rising. The long-bond price actually initially declines, but rises after about six periods. Consumption, in contrast, stays below its starting point for more than ten periods. This makes the long bond a good hedge compared to the productivity and monetary shocks discussed above – the long bond does well when consumption is low. This results in the average term premium being *negative* at a mean value of -0.3 (annualized percentage points).