Graduate Macro Theory II: "Business Cycle Accounting" and Wedges

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1 Introduction

Most modern dynamic macro models have at their core a prototypical real business cycle model. Different frictions, adjustment costs, and shocks represent deviations from the basic RBC framework – we have already discussed many of these possible frictions, adjustment costs, and shocks. In an important paper, Chari, Kehoe, and McGrattan (2007, *Econometrica*, hereafter simply "CKM"), develop a methodology which they call "business cycle accounting." They start with a basic one sector real business cycle model, and introduce four exogenous stochastic variables which they call wedges. These wedges are purely meant as reduced form accounting devices – the wedges could emerge because of exogenous shocks, or because of some friction or adjustment cost which means that the basic RBC model is mis-specified. Through the lens of the basic RBC model, they then empirically measure the four different wedges. They then use this exercise to talk about: (i) which exogenous shocks are the most promising candidate drivers of the business cycle and (ii) which kind of frictions and adjustment costs researchers ought to build into their models.

The four wedges on which CKM focus are called the efficiency wedge, the labor wedge, the investment wedge, and the government consumption wedge. The efficiency wedge is isomorphic to a productivity shock – it is simply measures TFP. The government consumption wedge is the residual output component not explained by consumption or investment, and is therefore equivalent to government spending in a simple model. It could also include net exports in an open economy model. The labor and investment wedges are isomorphic to distortionary tax rates on labor income and investment, respectively.

In what follows, I will describe the basic model and where the wedges appear. I will then measure the wedges in the data, essentially as residuals of the first order conditions from the model. Then we will feed the observed wedges back into the prototypical model and do a variance decomposition of output and other variables. Then I shall conclude with a discussion about the use (and potential abuse) of business cycle accounting more generally.

2 The Model

Consider what appears to be a standard RBC model. Consider first the household problem. For simplicity, assume that there are no bonds available to the household, and the only means through which it can transfer resources intertemporally is by accumulating capital. The household faces the flow budget constraint:

$$C_t + (1+\tau_t^I)I_t \leq (1-\tau_t^N)w_t N_t + R_t K_t + \Pi_t - T_t \tag{1}$$

Where τ_t^I is like a tax on investment (it alters the relative price between consumption and investment), while τ_t^N is a tax on labor income. We will refer to $1-\tau_t^N$ as the *labor wedge* and $1+\tau_t^I$ as the *investment wedge*. The household takes Π_t as given, and T_t is a lump sum tax. Capital accumulates according to:

$$K_{t+1} = I_t + (1-\delta)K_t \tag{2}$$

Preferences are standard:

$$u(C_t, N_t) = \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} \tag{3}$$

The household discounts future utility flows by $0 < \beta < 1$. A Lagrangian for the household is:

$$\mathbb{L} = E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln C_{t+j} - \theta \frac{N_{t+j}^{1+\chi}}{1+\chi} + \lambda_{t+j} \left[(1-\tau_{t+j}^N) w_{t+j} N_{t+j} + R_{t+j} K_{t+j} + \Pi_{t+j} - T_{t+j} - C_{t+j} - (1+\tau_{t+j}^I) I_{t+j} \right. \right.$$

$$\left. \cdots + \mu_{t+j} \left[I_{t+j} + (1-\delta) K_{t+j} - K_{t+j+1} \right] \right\}$$

The first order conditions are:

$$\frac{\partial \mathbb{L}}{\partial C_t} = 0 \Leftrightarrow \frac{1}{C_t} = \lambda_t \tag{4}$$

$$\frac{\partial \mathbb{L}}{\partial N_t} = 0 \Leftrightarrow \theta N_t^{\chi} = \lambda_t(1-\tau_t^N)w_t \tag{5}$$

$$\frac{\partial \mathbb{L}}{\partial I_t} = 0 \Leftrightarrow (1+\tau_t^I)\lambda_t = \mu_t \tag{6}$$

$$\frac{\partial \mathbb{L}}{\partial K_{t+1}} = 0 \Leftrightarrow \mu_t = \beta E_t\left[\lambda_{t+1}R_{t+1} + \mu_{t+1}(1-\delta)\right] \tag{7}$$

Re-arranging so as to eliminate multipliers yields:

$$\theta N_t^{\chi} = \frac{1}{C_t}(1-\tau_t^N)w_t \tag{8}$$

$$(1+\tau_t^I)\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}\left[R_{t+1} + (1+\tau_{t+1}^I)(1-\delta)\right] \tag{9}$$

The firm problem is standard. Output and the capital and labor demand curves are:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{10}$$

$$w_t = (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha} \tag{11}$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{12}$$

CKM refer to A_t as the efficiency wedge – it is just the standard RBC model productivity variable. Assume that the government chooses spending, G_t. Assume that the lump sum tax is chosen to balance the government's budget, so that $T_t = G_t - \tau_t^I I_t - \tau_t^N w_t N_t$ (effectively, then, distortionary tax revenue is remitted back to the household lump sum). Then the aggregate resource constraint is:

$$Y_t = C_t + I_t + G_t \tag{13}$$

CKM refer to G_t as the government consumption wedge. Not counting the four exogenous wedges, the equilibrium of the model economy is then summarize by:

$$\theta N_t^{\chi} = \frac{1}{C_t}(1-\tau_t^N)w_t \tag{14}$$

$$(1+\tau_t^I)\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}\left[R_{t+1} + (1+\tau_{t+1}^I)(1-\delta)\right] \tag{15}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{16}$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{17}$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{18}$$

$$Y_t = C_t + I_t + G_t \tag{19}$$

$$K_{t+1} = I_t + (1-\delta) K_t \tag{20}$$

2.1 Log-Linearization

Let's log linearize the model about some non-stochastic steady state. For the purposes of this linearization, define the labor wedge as $\psi_t^N = 1 - \tau_t^N$ and the investment wedge as $\psi_t^I = (1+\tau_t^I)$. The linearized labor supply curve is:

$$\chi \widetilde{N}_t = -\widetilde{C}_t + \widetilde{\psi}_t^N + \widetilde{w}_t \tag{21}$$

The linearized labor demand condition can be written:

$$\widetilde{w}_t = \widetilde{Y}_t - \widetilde{N}_t \tag{22}$$

I can get this by noting that the marginal product of labor can be written as proportional to the average product of labor. Combining these, I get:

$$\chi \widetilde{N}_t = -\widetilde{C}_t + \widetilde{\psi}_t^N + \widetilde{Y}_t - \widetilde{N}_t \tag{23}$$

Or, simplifying further:

$$\widetilde{\psi}_t^N = (1+\chi)\widetilde{N}_t + \widetilde{C}_t - \widetilde{Y}_t \tag{24}$$

In other words, given data on consumption, GDP, and labor input, as well as a value of χ, I can in essence measure $\widetilde{\psi}_t^N$ as a the residual. It measures, in a sense, how much the static labor FOC does not hold in the data.

The linearized production function is straightforward and can be re-arranged to yield:

$$\widetilde{A}_t = \widetilde{Y}_t - \alpha \widetilde{K}_t - (1-\alpha)\widetilde{N}_t \tag{25}$$

In other words, the efficiency wedge can be measured as TFP.

The linearized resource constraint can be written:

$$\widetilde{G}_t = \frac{Y}{G}\widetilde{Y}_t - \frac{C}{G}\widetilde{C}_t - \frac{I}{G}\widetilde{I}_t \tag{26}$$

In other words, the government consumption wedge can be measured as the residual of output not accounted for by consumption and investment.

Now let's linearize the Euler equation for capital. This one is a bit more involved so I'll show steps. Start by taking logs (ignoring the expectation operator):

$$\ln \psi_t^I - \ln C_t = \ln \beta - \ln C_{t+1} + \ln \left[R_{t+1} + \psi_{t+1}^I(1-\delta)\right]$$

Totally differentiate:

$$\widetilde{\psi}_t^I - \widetilde{C}_t = -\widetilde{C}_{t+1} + \frac{\beta}{\psi^*}\left[dR_{t+1} + (1-\delta)d\psi_{t+1}\right]$$

This follows because $R^* + (1-\delta)\psi^* = \frac{\psi^*}{\beta}$. Write this in tilde notation:

$$\widetilde{\psi}_t^I - \widetilde{C}_t = -\widetilde{C}_{t+1} + \frac{\beta R^*}{\psi^*}\widetilde{R}_{t+1} + (1-\delta)\beta\widetilde{\psi}_{t+1}^I$$

Note that we can write:

$$R^* = \psi^*\left(\frac{1}{\beta} - (1-\delta)\right)$$

This means we can write:

$$\widetilde{\psi}_t^I - \widetilde{C}_t = -\widetilde{C}_{t+1} + (1-\beta(1-\delta))\,\widetilde{R}_{t+1} + \beta(1-\delta)\widetilde{\psi}_{t+1}^I$$

Now, let's re-arrange this slightly to isolate the ψ_t terms on one side. Doing so yields:

$$\beta(1-\delta)E_t\widetilde{\psi}_{t+1}^I - \widetilde{\psi}_t^I = E_t\widetilde{C}_{t+1} - \widetilde{C}_t - (1-\beta(1-\delta))\,E_t\widetilde{R}_{t+1}$$

Now, let's assume that $\widetilde{\psi}_{t+1}^I$ follows an AR(1) process, so that $E_t\widetilde{\psi}_{t+1}^I = \rho_I\widetilde{\psi}_t^I$. Then the left hand side becomes:

$$(\beta(1-\delta)\rho_I - 1)\,\widetilde{\psi}_t^I = E_t\widetilde{C}_{t+1} - \widetilde{C}_t - (1-\beta(1-\delta))\,E_t\widetilde{R}_{t+1}$$

Now, where does this get us? From the firm's FOC, I can get $E_t\widetilde{R}_{t+1}$ as:

$$E_t\widetilde{R}_{t+1} = \widetilde{Y}_{t+1} - \widetilde{K}_{t+1}$$

Define the transformed investment wedge as $\widetilde{\psi}^I_{2,t} = (\beta(1-\delta)\rho_I - 1)\,\widetilde{psi}^I_t$. Then I can write:

$$(\beta(1-\delta)\rho_I - 1)\,\widetilde{\psi}^I_{2,t} = E_t\widetilde{C}_{t+1} - \widetilde{C}_t - (1-\beta(1-\delta))\left(E_t\widetilde{Y}_{t+1} - \widetilde{K}_{t+1}\right) \tag{27}$$

In other words, I can measure a (scaled) investment wedge essentially as the residual from expected consumption growth and the expected marginal product of capital. This allows me to get the investment wedge.

In other words, if I have data on output, capital, consumption, investment, and labor input, and am willing to take a stand on a few parameter values, I can back out the wedges essentially as residuals from the first order conditions in (24), (25), (26), and 27).

3 Measuring the Wedges in the Data

I collect data on output, the capital stock, consumption, investment, and labor hours. The data on output, capital, and labor hours are from John Fernald's supplement producing a quarterly TFP measure. Consumption is the real personal consumption expenditure series (which is not ideal in the sense that it includes durable goods purchases, which are better thought of as investment) and real private fixed investment is the investment series. After logging each series, I remove a linear time trend (so as to be consistent with the idea of deviations from the steady state in the equations above). The sample period is 1947q1 - 2016q4.

I then measure the wedges using the log-linearized equations above and these data. I assume that $\beta = 0.99$, $\delta = 0.02$, $\alpha = 1/3$, and $\chi = 1$. Figure 1 plots the time series of the four wedges. Shaded gray regions are recessions as defined by the NBER. One thing that sticks out is that both the efficiency and labor wedges are quite persistent and very procyclical – they tend to be low in periods identified by the NBER as recessions. The investment wedge seems to have quite a bit of high frequency volatility but is not very persistent and seems to be, if anything, slightly countercyclical (i.e. it is high in the Great Recession). The government consumption wedge moves around quite a bit, with a very big spike in the early 1950s, which is associated with the Korean War and the start of the Cold War. It's not obviously pro or countercyclical from eyeballing the series.

Figure 1: Time Series of the Wedges

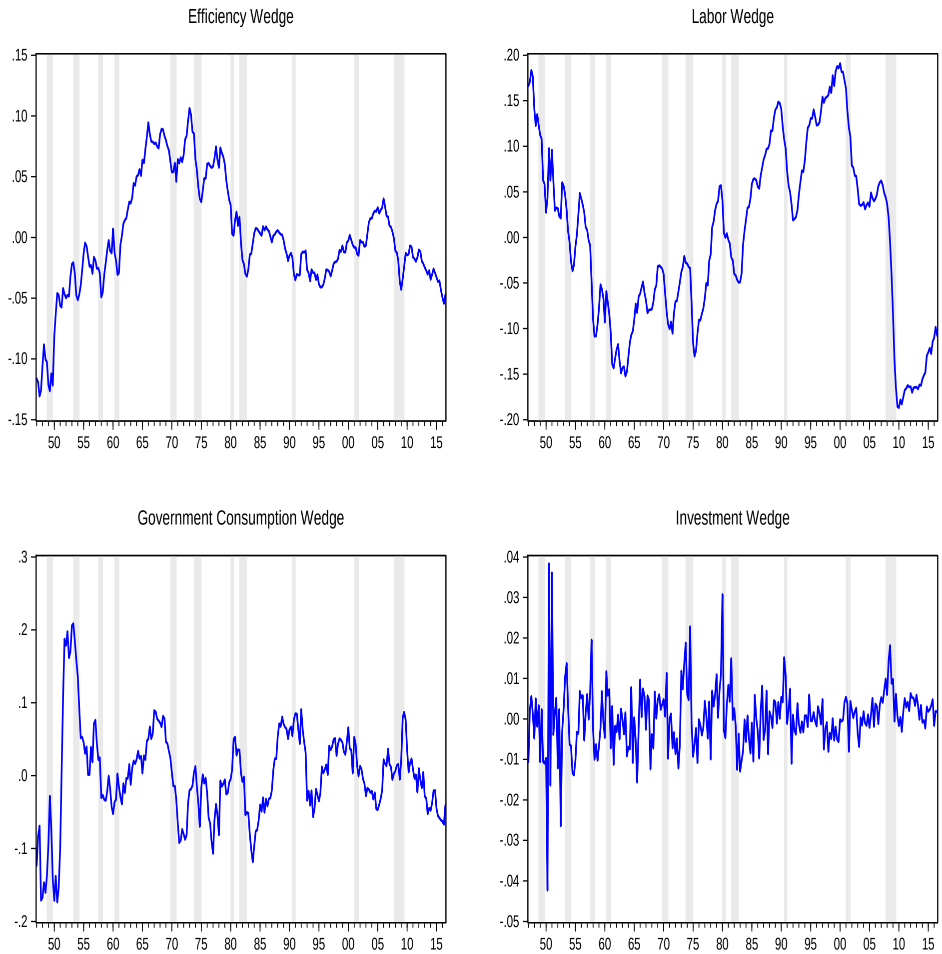

Table 1 shows some simple summary statistics for the wedges. The labor wedge is the most volatile and the investment wedge is least volatile. All are positively correlated with output (note these are correlations are based on linearly detrended data, not HP filtered data, and so may not line up with what you might see elsewhere).

Table 1: Moments of the Wedges

Wedge	Standard Deviation	Correlation w/ Output
Efficiency	0.0462	0.6100
Labor	0.0992	0.5178
Government	0.0645	0.2398
Investment	0.0080	0.1087

That the efficiency wedge is procyclical should not be surprising – the efficiency wedge is just measured TFP. What is interesting is that the labor wedge is both (i) quite volatile and (ii) very procyclical. Recall that the labor wedge above is defined as $\psi_t^N = 1 - \tau_t^N$. In other words, the labor wedge so defined being low when output is low (such as was the case during the Great Recession) means that it is as though there is a tax rate of labor income that is *high*. In other words, the procyclical labor wedge implies that the labor market seems to be countercyclically distorted (i.e. it is as though there is a tax on labor income that is high in recessions and low in expansions).

Simply looking at unconditional moments of the wedges doesn't really tell us much about how important they are or are not for understanding fluctuations. To do that, we need to take the model and feed these wedges into it and look at how much of the volatility of output (and other variables) can be explained by these wedges. To do this, we need to parameterize stochastic processes for the wedges. I'm going to consider a fairly simple exercise – I simply estimate AR(1) processes on each wedge, and report the AR coefficient and the standard error of the innovation. That is, I assume that the wedge processes can be written:

$$\widetilde{A}_t = \rho_A \ln \widetilde{A}_{t-1} + s_A \epsilon_{A,t} \tag{28}$$

$$\widetilde{G}_t = \rho_G \widetilde{G}_{t-1} + s_G \epsilon_{G,t} \tag{29}$$

$$\widetilde{\psi}_t^N = \rho_N \widetilde{\psi}_{t-1}^N + s_N \epsilon_{N,t} \tag{30}$$

$$\widetilde{\psi}_t^I = \rho_I \widetilde{\psi}_{t-1}^I + s_N \epsilon_{I,t} \tag{31}$$

Table 2 gives the estimated parameters of these processes:

Table 2: AR(1) Coefficients from Wedge Regressions

Wedge	AR Parameter	SD of Innovation
Efficiency	0.9739	0.0083
Labor	0.9854	0.0151
Government	0.9193	0.0245
Investment	0.0640	0.0079

There are a couple of things to note about these regressions. First, there is no constant (technically, I did estimate them with constants) because the wedges are based on linearly detrended data, and hence are mean zero. Second, there is a potential issue with what I did. In particular, the wedges are correlated with one another in the data. Writing down independent AR(1) processes models these wedges as uncorrelated processes. CKM actually write down a VAR process for the wedges and estimate that, which allows for the innovations to the wedges to be correlated as well as for lagged values of one wedge to impact other wedges. What I'm doing is simpler, but does not affect the main message of the exercise to follow. Consonant with visual inspection of the series, the efficiency, labor, and government wedges are quite persistent, while the investment wedge is not.

What I do next is to solve the model using these specified AR(1) processes for the wedges. In so doing, I assume that the efficiency, labor, and investment wedges are all in fact mean zero. But I need to specify a non-stochastic steady state value for G. I assume that government spending (taken literally in the model it is government spending, but more generally it is output not accounted for by consumption and investment, which in reality includes both government spending and net exports) is 20 percent of steady state output.

Table 3 shows the unconditional variance decomposition of output, hours, investment, and consumption to the four wedge shocks in the model. The results in the table below are pretty stark and are consistent with what CKM claim. In particular, the efficiency and labor wedges explain virtually all of the variance of these variables. The government spending and investment wedges are essentially completely irrelevant. Loosely speaking, the efficiency wedge accounts for about 60 percent of the variance of output, and the labor wedge the other 40 percent.

Table 3: Unconditional Variance Decomposition

	Percent of Unconditional Variance Due to			
Variable	Efficiency	Labor	Government	Investment
Output	61.7	37.8	0.2	0.3
Labor hours	3.5	92.9	1.9	1.7
Investment	62.8	26.8	3.2	7.3
Consumption	56.5	40.7	1.8	1.0

4 Interpreting the Wedges

Two of the four wedges – the efficiency and government consumption wedges – can be interpreted as exogenous stochastic shocks (to productivity and government spending). The labor and investment wedges do not have a clear interpretation as exogenous shocks. These could reflect exogenous shocks, or they could represent some mis-specification of the model along some dimension.

What I have laid out above is simply an accounting exercise (hence the title of the CKM paper, "Business Cycle Accounting"). Through the lens of a RBC model, we examine where the model needs wedges to explain the data. Going from these wedges to structural economic shocks, with the potential exception of the efficiency and government consumption wedges, is a bit more dicey.

One key message that comes out of the CKM paper is that we need to come up with a way to account for the labor wedge. The static first order condition for labor supply fails very badly in the data, suggesting that there is some important mis-specification or some important missing shock. Since the publication of their paper, there has been a substantial amount of research aimed at explaining the labor wedge.

There are several potential alternative interpretations of what gives rise to the labor wedge. In the model as laid out above, one possibility is literally a time-varying tax on labor income. Statutory tax rates do not vary that much in the data to take this potential explanation seriously (though one could argue that effective tax burdens vary in a countercyclical way over the business cycle). An alternative explanation of the labor wedge is simply a shock to the disutility from labor:

$$\nu_t \theta N_t^{\chi} = \frac{1}{C_t} w_t \tag{32}$$

Here, after log-linearization ν_t would be isomorphic to a time-varying tax on labor income. Precisely because the model-implied labor wedge moves around so much in the data, papers which seek to formally estimate the stochastic properties of different structural shocks very often find that labor supply shocks, like ν_t as modeled above, are very important drivers of the business cycle. Smets and Wouters (2007, *AER*) estimate a structural New Keynesian DSGE model and argue that a "wage markup shock" is one of the key drivers of the business cycle. To a first order approximation, the wage markup shock looks identical to a labor supply preference shock. In a separate paper, Chari, Kehoe, and McGrattan (2009, *AEJ Macro*) make this point, and hence argue that the Smets-Wouters wage markup shock cannot be considered structural. There are very different policy implications if something is causing the labor market to be more distorted in recessions (the wage markup interpretation) than if people simply dislike working more in recessions (the labor supply interpretation). Time-varying monopoly power in product markets could also help account for the labor wedge, either because of explicit time-variation in monopoly power or because of price stickiness resulting in countercyclical price markups (as we will see when we study New Keynesian models).

It is well-accepted that the static first order condition for labor supply from the baseline model is not consistent with the data. In that sense, CKM's paper makes an important (though not entirely novel, as Hall (1997, *Journal of Labor Economics*) and Gali, Gertler, and Lopez-Salid (2007, *ReStat*) previously made a very similar argument) point that macroeconomists need to better understand the labor wedge and why the first order condition for labor supply fails relative to the data. The more provocative claim in CKM's paper is that the lack of importance of the investment wedge means that research focusing on financial shocks and frictions is not likely to be a fruitful avenue for future research. Many feel that this claim is too strong, and it is not difficult to write down a model with a type of financial constraint that manifests directly as a labor wedge (as we shall see). One could also envision a model in which a financial shock leads to a poor allocation of factors of production across firms in a way that would manifest as an efficiency wedge.