The Zero Lower Bound

Eric Sims
University of Notre Dame

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1 Introduction

In the standard New Keynesian model, monetary policy is often described by an interest rate rule (e.g. a Taylor rule) that moves the interest rate in response to deviations of inflation and some measure of economic activity from target. Nominal interest rates are bound from below by 0 – since money is storable, one would never accept a negative nominal return.[1] How does the behavior of the NK model change when interest rates hit zero and cannot freely adjust in response to changing economic conditions?

To answer this question, we consider the implications of an interest rate peg in the model. This isn't literally what happens at the zero lower bound, but what matters in the model is not that the interest rate is zero per se, but rather that it becomes unresponsive to economic conditions. In the experiments I consider, the nominal interest rate is pegged at a fixed value for a finite (and deterministic) period of time. After the peg, monetary policy obeys a simple Taylor rule. It turns out to be relatively straightforward to modify a Dynare code to take this into account. I include a government spending shock in the model so that we can analyze the effects of government spending shocks at the zero lower bound, which has been a topic of much recent interest.

The interest rate peg ends up exacerbating the effects of price stickiness. In particular, output responds even less to a positive supply shock (productivity) and more to "demand" shocks (government spending) than under a standard Taylor rule. This operates through an inflation channel and the Fisher relationship: positive supply shocks lower inflation, which raises real interest rates if nominal rates are unresponsive, with the reverse holding for a demand shock like government spending.

2 Analytics in a Simple Model

I will consider two different ways to implement an interest rate peg analytically in the context of the textbook linearized model. The first is what I'll call a stochastic interest rate peg – the period

[1] Central banks around the world are currently testing this proposition, with many central banks targeting slightly negative interest rates. If there are costs to holding cash, people may be willing to accept slight negative nominal rates of return in return for convenience. What really matters for the analysis below is not that the interest rate gets stuck at zero, but rather that it becomes non-responsive to shocks.

t interest rate is fixed, and then in subsequent periods there is a fixed probability of exiting the interest rate peg. The expected duration is of the peg is not known – it could be only one period, or it could be many. In the deterministic peg case, the interest rate is fixed for a known number of periods (say period t to period $t+H$. For understanding the effects of an interest rate peg, it is important to articulate what policy will look like *after* the peg is over. I'll consider two versions: for the stochastic peg case, I shall assume that the central bank implements a strict inflation target after the peg (which means that, as an equilibrium outcome, the nominal rate equals the natural rate of interest); for the deterministic peg case, I will consider a Taylor rule as characterizing monetary policy after the peg duration.

2.1 Stochastic Interest Rate Peg

Consider the textbook New Keynesian model (assuming log utility over consumption). The main equations can be written:

$$\widetilde{X}_t = E_t \widetilde{X}_{t+1} - \widetilde{i}_t + E_t \pi_{t+1} + \widetilde{r}_t^f \tag{1}$$

$$\pi_t = \gamma \widetilde{X}_t + \beta E_t \pi_{t+1} \tag{2}$$

$$\widetilde{r}_t^f = \rho \widetilde{r}_{t-1}^f + \varepsilon_t \tag{3}$$

I assume that the natural rate of interest obeys some exogenous process (which could be motivated via an exogenous productivity shock, for example).

Monetary policy is characterized as follows. In period t, the nominal interest rate is fixed, which means $\widetilde{i}_t = 0$ (note that this does not necessarily mean it is zero, it just means it is fixed at steady state; in the linear model, fixing it at something other than steady state would not affect dynamics). In period $t+1$, there is a $1-p$ probability that the central bank can "exit" the peg, at which point it implements a strict inflation target, so that $E_t \pi_{t+1} = 0$ which requires $E_t \widetilde{i}_{t+1} = E_t \widetilde{r}_{t+1}^f$. With probability p, the nominal interest rate remains stuck at $\widetilde{i}_{t+1}$. If the peg is not exited in period $t+1$, then there is again a $1-p$ probability of exit in $t+2$, and so on.

The assuming that an inflation target is followed after the peg greatly simplifies analysis, because since the Divine Coincidence holds in the model as written down, we know that inflation and the output gap will both be zero once the peg is complete. This means that the policy functions *after* the peg are simply $E_t \widetilde{\pi}_{t+j} = E_t \widetilde{X}_{t+j} = 0$. Let us guess that the policy functions *during* the peg period are given by $E_t \widetilde{\pi}_{t+j} = \theta_1 E_t \widetilde{r}_{t+j}^f$ and $E_t \widetilde{X}_{t+j} = \theta_2 E_t \widetilde{r}_{t+j}^f$. Let us plug these guesses into the IS equation and Phillips Curve:

$$\theta_1 \widetilde{r}_t^f = \theta_2 \widetilde{r}_t^f + \beta(1-p) \times 0 + \beta p \theta_1 E_t \widetilde{r}_{t+1}^f \tag{4}$$

$$\theta_2 \widetilde{r}_t^f = (1-p) \times 0 + p\theta_2 E_t \widetilde{r}_{t+1}^f - 0 + (1-p) \times 0 + p \times \theta_1 E_t \widetilde{r}_{t+1}^f + \widetilde{r}_t^f \tag{5}$$

The "$\times 0$" terms enter because for period $t+1$ values of inflation and the output gap, there is a $1-p$ probability that the central bank exists the peg and that these are then zero. We can also make use of the fact that $E_t\widetilde{r}^f_{t+1} = \rho\widetilde{r}^f_t$. From the Phillips Curve, we get:

$$\theta_1\widetilde{r}^f_t = \theta_2\widetilde{r}^f_t + \beta p\rho\theta_1\widetilde{r}^f_t \tag{6}$$

This expression holding requires:

$$\theta_1 = \frac{\gamma}{1-\beta p\rho}\theta_2 \tag{7}$$

From the IS equation, we get:

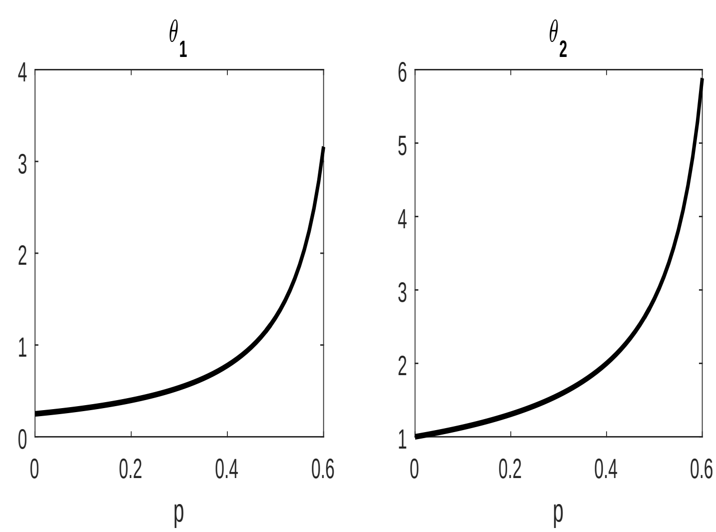
$$\theta_2(1-p\rho) = p\rho\theta_1 + 1 \tag{8}$$

Combining these, we get:

$$\theta_1 = \frac{\gamma}{(1-\beta p\rho)(1-p\rho) - p\rho\gamma} \tag{9}$$

$$\theta_2 = \frac{1-\beta p\rho}{(1-\beta p\rho)(1-p\rho) - p\rho\gamma} \tag{10}$$

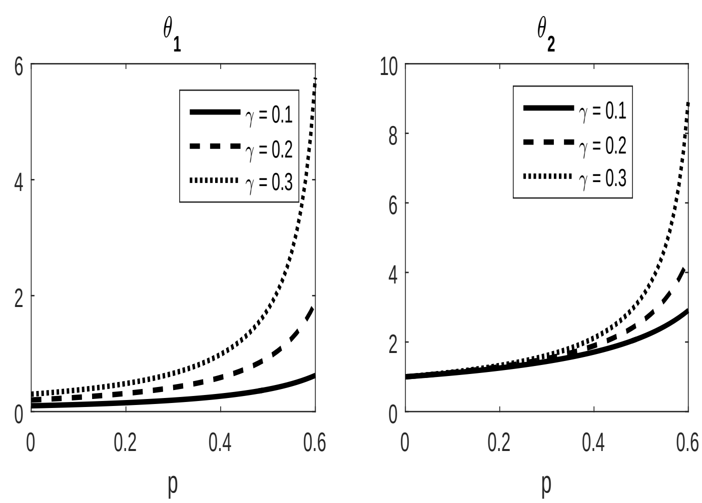
The figure below plots these coefficients as a function of p (the bigger is p, the longer is the expected duration of the interest rate peg period). In generating this figure, I assume that $\gamma = 0.25$, $\beta = 0.99$, and $\rho = 0.90$. I show the plot for values of p ranging from 0 to 0.6. We can see that both coefficients are increasing in p. This suggests that an interest rate peg means that inflation and the output gap react more to a shock to the natural rate than they otherwise would (under an inflation targeting regime they do not react at all). Furthermore, they react more to the natural rate shock the longer is the expected duration of the interest rate peg period.



There is a paradoxical feature at work here, however. In particular, it is straightforward to see that there exists a p where these coefficients tend toward infinity (or more precisely are not defined). Beyond that p, these coefficients will both be negative rather than positive. For my parameterization, this occurs at about $p = 0.67$. These "sign flips" are documented in Carlstrom, Fuerst, and Paustian (2014, 2015). They do not occur under a deterministic interest rate peg (considered in the subsection below).

In the baseline model, positive supply shocks (e.g. a transitory increase in productivity) tend to increase $\widetilde{Y}_t^f$ but decrease $\widetilde{r}_t^f$. Since θ_1 and θ_2 are both positive in the "non-pathological" region of p, this means that both inflation and the output gap will decline after a positive supply shock, and will decline more the longer is the expected duration of the peg (again, ruling out the pathological values of p). The output gap can decline so much that output, $\widetilde{Y}_t$, actually falls when there is a positive supply shock. Eggertsson (2010) has called this the "paradox of toil."

Another quizzical feature of the model is that θ_1 and θ_2 are *bigger* the bigger is γ (i.e. the more flexible are prices). This can be seen in the figure below:



The essential intuition for what is it play here is that an interest rate peg operates through an "expected inflation channel." Solving the IS equation forward, we get:

$$X_t = -E_t \sum_{j=0}^{\infty} \left(\widetilde{i}_{t+j} - \widetilde{\pi}_{t+j+1} - \widetilde{r}_{t+j}^f \right) \tag{11}$$

The current output gap is negatively proportional to the sum of current and future real interest rate gaps. When the nominal interest rate is fixed, the current real interest rate is the negative of the expected rate of inflation. Positive supply shocks lower $\widetilde{r}_t^f$ and $E_t \widetilde{\pi}_{t+j}$ under a peg. This means that the current real interest rate goes up, rather than down as the natural rate of interest does, resulting in a positive real interest rate gap (and a negative output gap). The longer is the expected duration of the peg, the more expected inflation falls; therefore, the real interest rate rises by more, the real interest rate gap goes up by more, and the output gap falls by more. The more flexible are prices, the more expected inflation reacts to shocks to the natural rate, which results in bigger real interest rate gap movements and hence bigger movements in the output gap.

2.2 A Deterministic Interest Rate Peg

Suppose we have NK model linearized about a zero inflation steady state. I'm going to write it in terms of the output gap and the natural rate of interest. The two principal equations are:

$$\widetilde{X}_t = E_t\widetilde{X}_{t+1} - \widetilde{i}_t + E_t\pi_{t+1} + \widetilde{r}_t^f \tag{12}$$

$$\pi_t = \gamma\widetilde{X}_t + \beta E_t\pi_{t+1} \tag{13}$$

Let's assume that policy obeys a simple Taylor rule, where $\phi_\pi > 1$:

$$\widetilde{i}_t = \phi_\pi\pi_t \tag{14}$$

Here I have assumed that $\sigma = 1$. Instead of following an AR(1), suppose instead that the natural rate of interest follows a process wherein in expectation it is equal to the current period shock for the current and subsequent H periods, after which time in expectation it is equal to its steady state:

$$E_t\widetilde{r}_{t+s}^f = \begin{cases} \varepsilon_t & \text{if } s \leq H \\ 0 & \text{for } s > H \end{cases} \tag{15}$$

With this specification, the natural rate of interest will go up in response to a positive shock ε_t and remain up for the current and subsequent H periods, after which time it goes back to steady state. What is nice about this setup is that there are no endogenous variables. With this setup for the shock (as opposed to our conventional AR(1) specification), the economy will return to steady state after period $t+H$. This means that we can solve for things by hand backwards by using the condition that all variables will equal 0 after $t+H$.

Suppose that $H = 1$. This means that the natural rate of interest jumps up for two periods, after which time it returns to steady state. This means that $E_tX_{t+2} = E_t\pi_{t+2} = 0$. Use this to solve for the $t+1$ values of these variables, in conjunction with the Taylor rule specification:

$$
\begin{aligned}
E_t\widetilde{X}_{t+1} &= 0 - \phi_\pi E_t\pi_{t+1} + \varepsilon_t \\
E_t\pi_{t+1} &= \gamma E_t\widetilde{X}_{t+1} \\
&\Rightarrow \\
E_t\widetilde{X}_{t+1} &= \frac{1}{1+\phi_\pi\gamma}\varepsilon_t \\
E_t\pi_{t+1} &= \frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t
\end{aligned}
$$

Now that we have $E_t\widetilde{X}_{t+1}$ and $E_t\pi_{t+1}$, we can solve for π_t and $\widetilde{X}_t$:

$$
\begin{aligned}
\widetilde{X}_t &= \frac{1}{1+\phi_\pi\gamma}\varepsilon_t - \phi_\pi\pi_t + \frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t + \varepsilon_t \\
\pi_t &= \gamma\widetilde{X}_t + \beta\frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t \\
&\Rightarrow \\
\widetilde{X}_t &= \frac{1}{1+\phi_\pi\gamma}\varepsilon_t - \phi_\pi\gamma\widetilde{X}_t - \beta\phi_\pi\frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t + \frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t + \varepsilon_t \\
\widetilde{X}_t &= \left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right]\varepsilon_t \\
\pi_t &= \left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right] + \frac{\beta\gamma}{1+\phi_\pi\gamma}\right)\varepsilon_t
\end{aligned}
$$

Now, instead of a Taylor rule, suppose that monetary policy follows an interest rate peg, wherein the interest rate is held fixed at its steady state value for the duration of the period in which $\widetilde{r}_t^f$ is different from steady state:

$$
E_t\widetilde{i}_{t+s} = \begin{cases} 0 & \text{if } s \leq H \\ \phi_\pi E_t\pi_{t+s} & \text{for } s > H \end{cases} \tag{16}
$$

That is, the nominal interest is fixed for the duration of the natural rate shock, but reverts to the Taylor rule after that. Reversion to the Taylor rule with $\phi_\pi > 1$ is sufficient for determinacy, but because there are no state variables the nominal interest rate will in expectation always be fixed after a natural rate shock.

We can use this setup to solve backwards for the t and $t+1$ values of the output gap and inflation:

$$E_t\widetilde{X}_{t+1} = \varepsilon_t$$
$$E_t\pi_{t+1} = \gamma\varepsilon_t$$
$$\widetilde{X}_t = \varepsilon_t + \gamma\varepsilon_t + \varepsilon_t = (2+\gamma)\varepsilon_t$$
$$\pi_t = (\gamma(2+\gamma) + \beta\gamma)\,\varepsilon_t = \gamma\,(2+\gamma+\beta)\,\varepsilon_t$$

It is not immediately obvious from looking at the equations, but what we get here is the π_t and X_t respond more to ε_t when the interest rate is pegged. Suppose that I use values of $\gamma = 0.2$, $\beta = 0.99$, and $\phi_\pi = 1.5$. For the peg case, I get:

$$\widetilde{X}_t = 2.2\varepsilon_t$$
$$\pi_t = 0.638\varepsilon_t$$
$$E_t\widetilde{X}_{t+1} = \varepsilon_t$$
$$E_t\pi_{t+1} = 0.2\varepsilon_t$$

Now, compare these numbers to the Taylor rule case. For that case, I get:

$$\widetilde{X}_t = 1.306\varepsilon_t$$
$$\pi_t = 0.413\varepsilon_t$$
$$E_t\widetilde{X}_{t+1} = 0.7692\varepsilon_t$$
$$E_t\pi_{t+1} = 0.1538\varepsilon_t$$

So, what we see here is that $\widetilde{X}_t$ and π_t respond *more* to changes in $\widetilde{r}_t^f$ than they do under a Taylor rule.

For the sake of completeness, let's now also work through a case of $H = 2$. This means that the natural rate shock will last for three periods (the current period plus two more). Consider first the Taylor rule case. Solve backwards imposing the terminal condition:

$$E_t\widetilde{X}_{t+2} = 0 - \phi_\pi E_t\pi_{t+2} + \varepsilon_t$$
$$E_t\pi_{t+2} = \gamma E_t\widetilde{X}_{t+2}$$
$$\Rightarrow E_t\widetilde{X}_{t+2} = \frac{1}{1+\phi_\pi\gamma}\varepsilon_t$$
$$E_t\pi_{t+2} = \frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t$$

These are exactly the same expressions we had in the $H = 1$ case for the $t+1$ values of the variables. Now that we have the $t+2$ values, we can solve for the $t+1$ values:

$$E_t\widetilde{X}_{t+1} = \frac{1}{1+\phi_\pi\gamma}\varepsilon_t - \phi_\pi E_t\pi_{t+1} + \frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t + \varepsilon_t$$
$$E_t\pi_{t+1} = \gamma E_t\widetilde{X}_{t+1} + \frac{\beta\gamma}{1+\phi_\pi\gamma}\varepsilon_t$$
$$\Rightarrow$$
$$E_t\widetilde{X}_{t+1} = \frac{1}{1+\phi_\pi\gamma}\varepsilon_t - \phi_\pi\gamma E_t\widetilde{X}_{t+1} - \frac{\phi_\pi\beta\gamma}{1+\phi_\pi\gamma}\varepsilon_t + \frac{\gamma}{1+\phi_\pi\gamma}\varepsilon_t + \varepsilon_t$$
$$\Rightarrow$$
$$E_t\widetilde{X}_{t+1} = \left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right]\varepsilon_t$$
$$E_t\pi_{t+1} = \left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right] + \frac{\beta\gamma}{1+\phi_\pi\gamma}\right)\varepsilon_t$$

Note again, these are exactly the same expressions we had for the t variables in the $H=1$ case. Now, let's use these to solve for the period t variables in the $H=2$ case:

$$\widetilde{X}_t = \left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right]\varepsilon_t - \phi_\pi\pi_t + \left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right] + \frac{\beta\gamma}{1+\phi_\pi\gamma}\right)\varepsilon_t + \varepsilon_t$$
$$\pi_t = \gamma\widetilde{X}_t + \beta\left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right] + \frac{\beta\gamma}{1+\phi_\pi\gamma}\right)\varepsilon_t$$
$$\Rightarrow$$
$$(1+\phi_\pi\gamma)\widetilde{X}_t = \varepsilon_t + \left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right]\varepsilon_t + (1-\beta\phi_\pi)\left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right] + \frac{\beta\gamma}{1+\phi_\pi\gamma}\right)\varepsilon_t$$
$$\Rightarrow$$
$$\widetilde{X}_t = \left\{\frac{1}{1+\phi_\pi\gamma} + \left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^3}\right] + (1-\beta\phi_\pi)\left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^3}\right] + \frac{\beta\gamma}{(1+\phi_\pi\gamma)^2}\right)\right\}\varepsilon_t$$

$$\pi_t = \left[\gamma\left\{\frac{1}{1+\phi_\pi\gamma} + \left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^3}\right] + (1-\beta\phi_\pi)\left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^3}\right] + \frac{\beta\gamma}{(1+\phi_\pi\gamma)^2}\right)\right\}\right.$$
$$\left. +\beta\left(\gamma\left[\frac{2+\gamma(1+\phi_\pi(1-\beta))}{(1+\phi_\pi\gamma)^2}\right] + \frac{\beta\gamma}{1+\phi_\pi\gamma}\right)\right]\varepsilon_t$$

Now let's solve for these expression in the case of an interest rate peg of the same duration as the shock. Working backwards, we get:

$$E_t\widetilde{X}_{t+2} = \varepsilon_t$$
$$E_t\pi_{t+2} = \gamma\varepsilon_t$$

Now go to the $t+1$ expressions:

$$E_t\widetilde{X}_{t+1} = \varepsilon_t + \gamma\varepsilon_t + \varepsilon_t = (2+\gamma)\varepsilon_t$$
$$E_t\pi_{t+1} = (\gamma(2+\gamma) + \beta\gamma)\,\varepsilon_t = \gamma\,(2+\gamma+\beta)\,\varepsilon_t$$

These are both the same things we had in the $H = 1$ case, just led forward one period. Now solve for the period t variables:

$$\widetilde{X}_t = (2+\gamma)\varepsilon_t + \gamma\,(2+\gamma+\beta)\,\varepsilon_t + \varepsilon_t = [3+\gamma+\gamma\,(2+\gamma+\beta)]\,\varepsilon_t$$
$$\pi_t = (\gamma\,[3+\gamma+\gamma\,(2+\gamma+\beta)] + \beta\gamma\,(2+\gamma+\beta))\,\varepsilon_t$$

Let's do what we did above, using the same numbers to quantify these things. First, start with the Taylor rule. We have:

$$\widetilde{X}_t = 1.6179\varepsilon_t$$
$$\pi_t = 0.7325\varepsilon_t$$
$$E_t\widetilde{X}_{t+1} = 1.306\varepsilon_t$$
$$E_t\pi_{t+1} = 0.413\varepsilon_t$$
$$E_t\widetilde{X}_{t+2} = 0.7692\varepsilon_t$$
$$E_t\pi_{t+2} = 0.1538\varepsilon_t$$

Now, do the comparable exercise for the interest rate peg case:

$$\widetilde{X}_t = 3.8380\varepsilon_t$$
$$\pi_t = 1.3992\varepsilon_t$$
$$E_t\widetilde{X}_{t+1} = 2.2000\varepsilon_t$$
$$E_t\pi_{t+1} = 0.6380\varepsilon_t$$
$$E_t\widetilde{X}_{t+2} = \varepsilon_t$$
$$E_t\pi_{t+2} = 0.2000\varepsilon_t$$

Now, let's compare the responses of $\widetilde{X}_t$ and π_t on impact in the $H = 2$ under the peg relative to the Taylor rule. For $\widetilde{X}_t$, we get a ratio of 2.37, and for π_t we get a ratio of 1.9102. Let's look at how these compare to the same ratios for the $H = 1$ case. There we had a ratio of 1.6845 for $\widetilde{X}_t$ and 1.5448 for π_t. From this, we can draw the following conclusions:

1. Both $\widetilde{X}_t$ and π_t respond more to a shock to the natural rate when the interest rate is pegged than when it follows a Taylor rule. In other words, an interest rate peg exacerbates the effects of price rigidity (since $\widetilde{X}_t$ wouldn't react to a shock to the natural rate with flexible prices).

2. Both $\widetilde{X}_t$ and π_t respond more under a peg relative to the Taylor rule the longer is the interest rate peg, H.

In the model, we often think of "demand shocks" (e.g. a government spending shock) as raising $\widetilde{r}_t^f$ and Y_t, while "supply shocks" (e.g. a productivity shock) lower $\widetilde{r}_t^f$ but raise Y_t. An interest rate peg is going to exacerbate the effects on $Y_t - Y_t$ will react more to a demand shock when the interest rate is pegged, but will react less (since $\widetilde{r}_t^f$ falls) to a supply shock.

3 Quantitative Model

The analytic stuff we did above is useful, if not a bit laborious. It gets very tricky when you start including endogenous state variables into the model, because then the economy will not be at steady state at the end of the period of the shock. Furthermore, the setup we used above required a hokey shock structure in which the natural rate of interest jumps up and stays up for $H + 1$ periods, but then goes back to steady state. It doesn't work as well for an AR(1) process.

There is an intuitively simple (though quantitatively somewhat burdensome) way to implement an interest rate peg in the model. What is nice about this is that the model can be as complicated as you like, and you can also solve the model with perturbations above first order. It involves augmenting the Taylor rule with "news shocks." Given some other shock, you solve for the value of the news shocks which keeps the nominal rate pegged for a desired length of time. So effectively, we can think about the effects of a shock under an interest rate peg as being something like the sum of the direct effect of the shock, plus the effects of current and anticipated monetary policy shocks so as to keep the nominal interest rate pegged.

The model is a standard New Keynesian model with price stickiness. To make things a little more interesting, as well as to tie into some of the recent literature, I included government spending in the model.

I abstract from money altogether. Monetary policy is characterized by an interest rate rule. The money supply is determined passively in the background so as to equate demand and supply of real balances at the central bank's target interest rate. The full set of equilibrium conditions are given by:

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma}(1 + i_t)(1 + \pi_{t+1})^{-1} \tag{17}$$

$$\psi N_t^{\eta} = C_t^{-\sigma} w_t \tag{18}$$

$$mc_t = \frac{w_t}{A_t} \tag{19}$$

$$Y_t = C_t + G_t \tag{20}$$

$$Y_t = \frac{A_t N_t}{v_t^p} \tag{21}$$

$$v_t^p = (1-\phi)(1+\pi_t^{\#})^{-\epsilon}(1+\pi_t)^{\epsilon} + (1+\pi_t)^{\epsilon}\phi v_{t-1}^p \tag{22}$$

$$(1+\pi_t)^{1-\epsilon} = (1-\phi)(1+\pi_t^{\#})^{1-\epsilon} + \phi \tag{23}$$

$$1+\pi_t^{\#} = \frac{\epsilon}{\epsilon-1}(1+\pi_t)\frac{x_{1,t}}{x_{2,t}} \tag{24}$$

$$x_{1,t} = C_t^{-\sigma} mc_t Y_t + \phi\beta E_t(1+\pi_{t+1})^{\epsilon} x_{1,t+1} \tag{25}$$

$$x_{2,t} = C_t^{-\sigma} Y_t + \phi\beta E_t(1+\pi_{t+1})^{\epsilon-1} x_{2,t+1} \tag{26}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t} \tag{27}$$

$$\ln G_t = (1-\rho_g)\ln G^* + \rho_g \ln G_{t-1} + \varepsilon_{g,t} \tag{28}$$

$$i_t = i^* + \phi_\pi(\pi_t - \pi^*) \tag{29}$$

G^* is an exogenously chosen steady state value of government spending, and $i^* = \frac{1}{\beta}(1+\pi^*) - 1$ is the steady state nominal interest rate. This is a standard interest rate rule and model. I use the following parameter values in quantitative solutions of the model: $\sigma = 1$, $\psi = 1$, $\eta = 0.5$, $\epsilon = 10$, $\phi = 0.75$, $\beta = 0.99$, $\rho_a = 0.90$, $\rho_g = 0.9$, $\pi^* = 0$, $\phi_\pi = 1.5$, and $G^* = 0.2$. This parameterization ends up implying that government spending is about 20 percent of steady state output. I set the standard deviations of both the government spending and productivity shocks to 0.01. I abstract from a smoothing term in the interest rate rule and do not model an explicit monetary policy shock.

3.1 An Interest Rate Peg

The zero lower bound refers to the fact that nominal interest rates cannot be negative (whereas real rates can). The argument for why this is the case is fairly straightforward, though without money explicitly in the model it is not terribly transparent. The nominal interest rate tells you the dollar return on foregoing one dollar's worth of current consumption, whereas the real interest rate tells you the consumption return on forgoing one unit of current consumption. If consumption goods are not storable, then you may be willing to accept a negative real return – giving up 1 unit of fruit today for 0.9 fruits tomorrow isn't a great deal, but if your outside option is zero fruit tomorrow, you may be willing to take this. But since money is a store of value, one would never take a negative return on money – you could simply hold your wealth in money between periods and have as many dollars tomorrow as you saved today. Hence, no one would ever take a negative nominal interest rate.

What is relevant in this model is not whether the nominal interest rate is zero or positive per se, but rather whether the nominal interest rate is responsive to changing economic conditions. Suppose that a central bank desired a negative nominal interest rate of 2 percent, given conditions. Then, given reasonably sized movements in inflation, the central bank would keep the nominal interest rate fixed at zero, until such a time as its desired interest rate (given its policy rule) is positive, after which time it would go back to its standard rule.

A simple way for us to model this behavior that requires only minor modification of our model and codes is to not worry about the zero lower bound per se, but rather to model an interest rate peg. A peg means that the interest rate is fixed for a known duration of time, after which time it follows the standard Taylor rule. This will simulate the situation described above of the Fed desiring a very negative nominal interest rate, and therefore keeping the actual nominal rate fixed for a period of time.

We can model an interest rate peg by including "news shocks" to the policy rule as follows:

$$i_t = i^* + \phi_\pi(\pi_t - \pi^*) + \sum_{j=0}^{H-1} e_{i,t-j} \tag{30}$$

In this specification, the $e_{i,t-j}$ are policy rate shocks. For $j > 0$, these are shocks known to agents in advance of them actually impacting the policy rule. We can impose an interest rate peg as follows. First, hit the economy with some shock. Then, solve for the $e_{i,t-j}$, $j = 0, \dots, H-1$, which will keep $i_{t+s} = i_{t-1}$ for $s = 0, \dots, H-1$. Effectively, this solves for the shocks which keep the interest rate unresponsive to a shock for the current and subsequent $H-1$ periods, for a total of H periods. I will refer to H as the peg period. It is important to write these as anticipated shocks in the policy rule – this mean that, at the time of another shock, agents will anticipate that the interest rate will be unresponsive for H total periods.

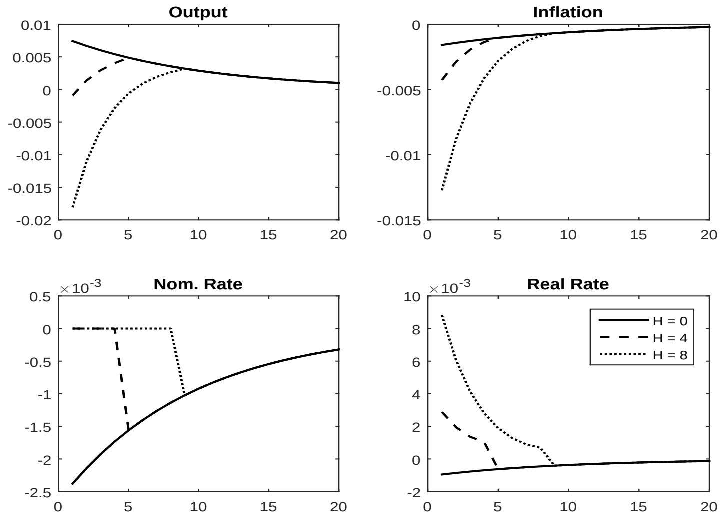
This is easy to do in Dynare. For, say, a four period peg, just write your policy rule like this:

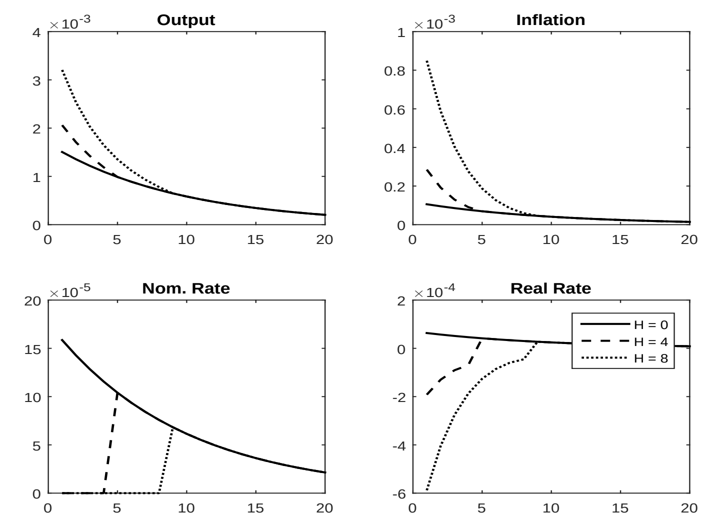
$$i_t = i^* + \phi_\pi(\pi_t - \pi^*) + e_t + e_{1,t}(-1) + e_{2,t}(-2) + e_{3,t}(-3) \tag{31}$$

Then you solve the model as you normally would. If you don't do anything else, Dynare will produce normal impulse responses to other shocks (e.g. productivity and government spending). But, with a little work on your own, you can construct IRFs on your own, and in the process you solve for the values of e_t, $e_{1,t}$, $e_{2,t}$, and $e_{3,t}$ such that the impulse response of i_t to the shock will be zero for horizons zero through three.

4 Analysis

I solve the model in Dynare using a first order approximation under three different policy scenarios: the standard Taylor rule, a $H = 4$ period interest rate peg, and a $H = 8$ period interest rate peg. Below I plot impulse responses first to a productivity shock and then to the government spending shock under the different policy rules. The solid lines show the responses under a standard Taylor rule, the dashed lines under a four period peg, and the dotted lines under an eight period peg (the periods can be interpreted as quarters).





We see something pretty interesting here. In particular, the interest rate peg *exacerbates* the effects of price stickiness. For the case of the productivity shock, under a standard Taylor rule with sticky prices output rises by less than it would if prices were flexible. The longer the interest rate is pegged, the less output rises. For the case of the government spending shock, output would rise more with sticky prices under a standard Taylor rule than if prices were flexible. The longer the interest rate is pegged, the more output reacts. For both the productivity and government spending shocks, inflation also reacts more the longer is the peg – it falls by more in the case of the productivity shock, and rises by more in the case of the government spending shock. We clearly see how the nominal interest rate is fixed for the specified number of periods. After the peg is over, the IRFs lie on top of one another – this occurs because there is no endogenous state variable in the model, so once the peg is over it is irrelevant that the peg was ever in place in the first place.

What's going on here is fairly simple and operates through the real interest rate in conjunction with the Fisher relationship. The positive productivity shock is a "supply shock" that results in lower inflation. When the central bank follows a Taylor rule, it reacts to lower inflation by lowering the nominal interest rate, which prevents the real interest rate from rising by much (indeed, in the baseline scenario here under a Taylor rule it falls). But when the nominal interest rate is fixed, falling inflation means a higher real interest rate. This higher real interest rate works to "choke off" demand. The longer the interest rate is pegged, the more inflation initially falls. And with a fixed nominal rate, this translates into a bigger increase in the real interest rate and an even more contractionary effect on output.

We see the reverse pattern in the case of a government spending shock. This is a kind of "demand" shock that leads to an increase in inflation. Under a standard Taylor rule, higher inflation means a higher nominal interest rate, which translates into a higher real interest rate. But if the interest rate is fixed, higher inflation has the opposite effect on the real interest rate: higher inflation means a lower real interest rate, which is stimulative. The longer the peg, the more inflation increases, which means the real interest rate falls by more, translating into an even bigger effect on output.

We can quantify the effects of the government spending shock via the "multiplier": $\frac{dY_t}{dG_t}$. To do this, simply calculate the ratio of the impact impulse responses of output and government spending, and multiply by the steady state ratio of output to government spending to put it in "dollar form" (this is because the impulse responses are in percentage terms, and the multiplier is usually expressed in levels, not log deviations). Under a standard Taylor rule, the multiplier under my parameterization is 0.75. In other words, output rises by less than government spending, meaning that consumption gets "crowded out." The crowding out of consumption results because of the increase in the real interest rate. Under the interest rate peg, in contrast, the multiplier under a four period peg is 1.03, and under an 8 period peg is 1.60. The reason why the multiplier is bigger than one under the peg is because the government spending shock results in a decrease in the real interest rate under the peg, rather than an increase, which "crowds in" consumption. It is based on simple logic like this that many commentators have argued for more fiscal stimulus in our current low interest rate environment.

5 Policy Implications

To the extent to which a central bank doesn't like output gaps (output being different from its natural, or flexible price, level) or inflation, it's pretty clear that not being able to adjust the interest rate to shocks is a bad thing. The interest rate being pegged exacerbates the role of price stickiness: the economy responds even less to a supply shock, and even more to a demand shock, than if prices were flexible, the more so the longer is the peg. Inflation also reacts by more. Clearly, a central bank operating with an interest rate rule would like to avoid the situation in which it is unable to adjust the nominal interest rate.

Hence, central banks would clearly like to avoid the incidence of nominal interest rates hitting the zero lower bound. The most straightforward way to do this is to have a higher inflation target. The steady state nominal interest rate is $1 + i^* = \frac{1}{\beta}(1 + \pi^*)$. The central bank can't control β, but it can raise π^*. Raising π^* raises i^*, and if nominal interest rates fluctuate about a higher steady state level, it is natural that the incidence of hitting zero will be lower. So should central banks raise inflation targets? It's not so obvious, and there are tradeoffs. First, raising inflation targets implies larger and larger deviations from the Friedman Rule, which would have deflation along trend and zero interest rates. To the extent to which the Friedman rule is optimal in a flexible price model, then a positive inflation target implies a larger steady state distortion. Secondly, in the sticky price model, positive trend inflation induces positive steady state price dispersion, which is strictly welfare reducing. Positive trend inflation also makes price dispersion a first order (as opposed to second order) phenomenon, so not only will positive trend inflation lead to a steady state distortion, it will result in an added distortion in terms of the dynamics of price dispersion about the steady state.

Coibion, Gorodnichenko, and Wieland (2012, *Review of Economic Studies*) study the question of the optimal inflation target in light of the zero lower bound in a standard New Keynesian model. In spite of the fact that hitting the zero lower bound is very costly, it occurs rarely enough in these models that it is not worth the cost of high positive trend inflation. They argue that the optimal level of trend inflation is typically 2 percent per year or less, which is remarkably close to recent US experience.