Growth or the Gap? Which Measure of Economic Activity Should be Targeted in Interest Rate Rules?*

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Abstract

What measure of economic activity, if any, should be targeted in simple interest rate rules? This paper analyzes the welfare consequences of responding to the growth rate of output and the output gap in a DSGE model with both nominal and real frictions. In spite of the fact that it shows up directly in the approximate welfare criterion of the simplest version of the model, strong responses to the theoretical output gap can significantly reduce welfare under the following circumstances (or combinations thereof): (i) shocks to price markups are moderately important, (ii) wages are sticky and there is positive trend productivity growth with incomplete real wage indexation, or (iii) there is positive trend inflation and incomplete nominal price and wage indexation. In contrast, responding to the growth rate of output, which is putatively easier to observe than the gap, is often welfare-improving, with the gains from growth-targeting larger under the same sets of circumstances under which gap-targeting is costly. These conclusions reinforce existing results about the relative merits of targeting growth or the gap from the perspective of equilibrium determinacy.

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1 Introduction

Interest rate rules, often called "Taylor Rules" after Taylor (1993), have become a ubiquitous feature of mainstream macroeconomic models. These rules call for central banks to adjust interest rates in response to changes in observable macroeconomic conditions. Though not fully optimal in the Ramsey sense, interest rate rules tend to have good normative properties and yield intuitive and well-understood restrictions to guarantee equilibrium determinacy.

In spite of their widespread application in dynamic stochastic general equilibrium (DSGE) models, there is no set or widely agreed upon specification of Taylor-type rules. What is common to most specifications is for a strong reaction of interest rates to deviations of inflation from target. There is considerably less agreement on which measure of economic activity, if any, should enter into the policy rule. Taylor's original specification featured interest rates rising in response to increases in output above a statistical trend. Many papers instead assume that interest rates are set as a function of the output gap, the deviation between the actual level of output and its "natural rate" – the level of output that would obtain in equilibrium in the absence of nominal rigidities. This specification is usually justified on the grounds that it is the output gap that matters for welfare, not output. Still other specifications of policy rules feature a positive response of interest rates to output growth, either in place of, or addition to, the gap. This specification is often justified on the grounds that output growth is putatively easier to observe in real time than the output gap. Some authors argue against paying attention to any measure of economic activity at all, and instead suggest that policy-makers pursue a policy of strict inflation targeting.

The purpose of this paper is to provide some insight into the simple but apparently unsettled question: what measure of economic activity, if any, should appear in interest rate rules? This question is not merely of academic interest for deciding what kind of Taylor rule specification to include in a DSGE model of the economy. It is also of critical importance for thinking about policy in the current zero lower bound environment. Figure 1 plots the actual behavior of the Federal Funds rate over the period 1985-2012 (black line) along with the implied target level of the funds rate for different policy rule specifications: one in which rates react to the output gap (blue line) and one in which rates instead react to output growth (green line).[1] The actual funds rate has been at or near zero since the end of 2008. Under the gap specification, the implied target rate has been significantly negative for the last four years. Implicitly or explicitly, many who have called for additional monetary stimulus appeal to a picture like this, arguing that nominal rates ought to be negative in the absence of the lower bound, and therefore support non-standard policies like quantitative easing and "forward-guidance." If the Fed were following the growth rate rule, in

[1] To generate the implied target I assume a rule of the form: $i_t = (1-\rho_i)\bar{i} + \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi(\pi_t - \bar{\pi}) + \phi_x(x_t - \bar{x}) + \phi_{\Delta_y}(y_t - y_{t-1}))$. π_t is quarter-over-quarter inflation as defined by the GDP price deflator; x_t is the CBO measure of the log output gap, and y_t is log real GDP. I use parameter values $\rho_i = 0.7$, and $\phi_\pi = 1.5$. In the gap specification $\phi_x = 0.5$ and $\phi_{\Delta_y} = 0$. In the growth rate specification $\phi_x = 0$ and $\phi_{\Delta_y} = 0.5$. Variables with a "bar" denote sample averages over the period 1985-2006. The coefficient values are not from any estimation or optimization, but rather are just common values used in the literature and seem "reasonable" and are simply meant to be instructive. A similar difference between the growth rate and gap specification emerges with different policy rule coefficients within a considerable range. See, for example, Carlstrom and Fuerst (2012).

contrast, the implied target interest rate never actually goes negative, and only gets particularly close to zero in the beginning of 2009. Under that rule, the target interest rate has been around 1-2 percent for the last three years, well above the actual funds rate.

In Section 2, I begin by focusing on the textbook, linearized three equation New Keynesian model with sticky prices. The non-policy block consists of an equation representing the demand side of the model (sometimes called the "IS" curve) and a Phillips Curve describing the relationship between inflation and the output gap. The model is closed with a partial adjustment policy rule in which the interest rate is set as a convex combination of the lagged and target interest rates, where the target rate is a function of the deviations from steady state of inflation, the output gap, and the period-over-period growth rate of output. A second order approximation to household welfare gives rise to a loss function in the variances of inflation and the output gap. This loss function expresses the welfare losses arising due to price rigidity, and can therefore be used to evaluate different monetary policy rules. There are three exogenous shocks in the basic model: a preference shock to the IS curve, a productivity shock to the flexible price (or "natural rate") of output, and a "price-markup," or "cost-push" shock to the Phillips Curve.

For a conventional calibration of the textbook model, I numerically find policy rule parameters which minimize the expected welfare loss from price rigidity (equivalently, maximize expected welfare in the economy with sticky prices). The optimal interest rate rule features a large response to inflation, a moderate amount of interest smoothing, no response to the output gap, and a mild, positive response to output growth. Strong positive responses to the output gap can result in significant welfare losses. That it is welfare-reducing to react to the gap, and welfare-improving to target growth, seems counterintuitive at first. This is because the variance of the gap shows up in the welfare loss function, whereas output growth does not. Targeting the gap ought to reduce the variance of the gap, which would seemingly increase welfare. In contrast, because of the "natural rate" property of the model, output growth tends to be high when output is below potential. Raising interest rates when output is below potential runs counter to conventional stabilization logic.

The relative merits of gap and growth targeting hinge on two features of the Phillips Curve in the model: the presence of the price-markup shock and the role of expected inflation. The price-markup shock introduces a tension between gap and inflation stabilization: conditional on this shock, stabilizing inflation or the gap comes at the expense of more volatility in the other. Given that the relative weight on the volatility of the output gap in the loss function is small for reasonable parameterizations, the gains from lowering gap volatility are outweighed by a higher variance of inflation, making responding to the output gap welfare-reducing conditional on these shocks. Conditional on preference or productivity shocks, responding positively to the gap is welfare-improving. The overall desirability of gap-targeting thus depends on the relative importance of price-markup shocks. In the baseline calibration, these shocks account for a relatively small component of output fluctuations, explaining about 15 percent of the unconditional variance of output growth. I show that if these shocks account for anything more than about 5 percent of the volatility of output growth, then it is welfare-reducing to respond to the gap.

The presence of expected inflation in the Phillips Curve makes inflation forward-looking. The less expected inflation reacts to a shock, the better is the available "menu" of current inflation and the gap available to a central bank. This means that policies which better "anchor" expected inflation are likely to result in welfare gains. Responding to output growth does exactly this. By tying current policy to the past, a rule which reacts to growth sends better signals about future policies, working to better anchor expected inflation. For example, suppose the economy is hit by a shock which pushes output below potential. By responding positively to output growth, a central bank can cut rates immediately (when output growth declines), with an implicit promise to raise rates in the future when output growth turns positive as output heads back toward potential. The implicit promise of a future anti-inflationary stance better anchors expected inflation, allowing for a more desirable menu of inflation and the gap in the present.

Section 3 extends the analysis to a more realistic "medium-scale" version of the New Keynesian model. In addition to endogenous capital accumulation, the model also features sticky wages and allows for a number of real frictions and a multitude of shocks. The model makes two other departures from the baseline model in allowing for trend productivity growth and positive trend inflation. Most of the basic conclusions from the basic model carry over. The optimal policy rule features a strong response to inflation, no response to the output gap, and a positive response to output growth. Strong positive responses to the output gap can again result in significant welfare losses. The non-desirability of targeting the gap hinges on three features of the model: the presence of price-markup shocks, trend productivity growth, and trend inflation. Any combination of these three features being present makes gap targeting welfare-reducing. In contrast, the welfare gains from growth targeting in the policy rule are greatest when these features are present.

The role of price-markup shocks on the welfare effects of gap targeting is essentially the same as in the simpler model without capital. Responding to the gap conditional on these shocks induces excess inflation volatility, which lowers welfare on net. The roles of trend productivity growth and trend inflation are new. Combined with nominal wage rigidity, either of these features (in isolation or together) make wage dispersion a first order phenomenon. Wage dispersion drives a wedge between labor supply and labor actually employed, and can be very costly from a welfare perspective, much more so than price dispersion. Positive steady state wage dispersion significantly increases the welfare costs of inflation variability relative to gap volatility. Even if there are no price-markup shocks, there still exists a tradeoff between nominal and real stabilization if both prices and wages are sticky. With the heightened relative cost of inflation volatility when trend productivity growth or inflation are positive, gap targeting becomes highly non-desirable from a welfare perspective. In contrast, because both of these features effectively make price- and wage-setting more forward-looking, the welfare gains from targeting output growth in the policy rule are larger when trend productivity growth or inflation are positive, due to the anchoring effect of growth-targeting on expected inflation.

This paper is closely related to several different papers in the literature on monetary policy design within the New Keynesian framework. Clarida, Gali, and Gertler (1999) provide a comprehensive survey in the context of the basic three equation linearized model. Woodford (2001) and Svennson (2003) examine how basic Taylor type rules perform from the perspective of Ramsey optimal policies. Woodford (1999), Woodford (2003), Carlstrom and Fuerst (2008), and Giannoni (2012) discuss the advantages of inertia in policy rules. Papers that study the empirical fit of Taylor type interest rate rules include Judd and Rudebusch (1998); Clarida, Gali, and Gertler (2000); Orphanides (2001); Rudebusch (2006); Pappell, Molodtsova, and Nikolsko-Rzhevskyy (2008); and Coibion and Gorodnichenko (2012).

Similarly to the exercises conducted in this paper, Schmitt-Grohe and Uribe (2006, 2007) study the properties of "simple and implementable" rules in which interest rates are constrained to react to only handful of easily observable variables. A central result in their papers is that these rules should not react to the level of output. They do not consider the output gap as a potential target variable in the policy rule, putatively because of the difficulty in observing it. They also do not consider shocks to the price and/or wage Phillips Curves in their model, which are of central importance to the desirability (or non-desirability) of gap targeting. They also do not consider trend productivity growth. Walsh (2003) studies Ramsey optimal monetary policy under commitment and discretion. He shows that a myopic central bank that acts under discretion will implement the socially optimal policy under commitment if it is presented with a loss function that seeks to minimize variation in output gap changes as opposed to the level. The intuition for how this result arises is similar to that discussed above: by making current policy contingent on the past, focusing on growth rates as opposed to levels better anchors inflation expectations. He does not consider including output growth either in place of or in addition to the output gap in a simple interest rate reaction function, however. Faia and Monacelli (2007) look at optimal policy rule coefficients in a model with credit frictions. Levin, Wieland, and Williams (1999) undertake a similar exercise to Schmitt-Grohe and Uribe (2006, 2007) in several different empirically motivated monetary models. They do not consider policy rules which react to the output growth rate.

In the most basic three equation New Keynesian model, the "Divine Coincidence" (Blanchard and Gali, 2007) holds, and, absent markup shocks, there is no tradeoff between output gap and inflation stabilization. Erceg, Henderson, and Levin (2000) show that nominal wage stickiness can generate a non-trivial tradeoff even without resorting to markup shocks. In fact, they show that pure inflation targeting regimes (e.g. interest rate rules which only react to inflation) do poorly relative to rules which more strongly target the gap. There result abstracts from the three things which make gap targeting most costly: trend growth, trend inflation, and markup shocks. When incorporating any of these features into a model with both price and wage stickiness, gap targeting again does poorly, while growth targeting is beneficial.

This paper also connects to a different strand of the literature which focuses on monetary policy and equilibrium determinacy. Coibion and Gorodnichenko (2011b) show that, once trend inflation is taken into account, responding strongly to the output gap in the policy rule can be destabilizing, while reacting to output growth can help induce equilibrium determinacy. This result dovetails nicely with the conclusions in the present paper: shifting from a focus on the gap to output growth is likely to be welfare-improving conditional on being in a determinate equilibrium, but is also likely to help ensure that a determinate equilibrium exists in the first place. Empirically, there is some evidence to support the notion that, over the "Great Moderation" period (since the early 1980s), the Fed has indeed shifted from a focus on the gap to growth. Smets and Wouters (2007), when estimating a medium-scale DSGE model via Bayesian methods on both the pre- and post-Volcker era, find a significantly smaller coefficient on the gap over the last thirty years. Coibion and Gorodnichenko (2011b), employing a limited information estimation strategy, estimate that the post-Volcker period has been characterized by a both smaller response coefficient on the output gap and a significantly stronger reaction to output growth.

The remainder of the paper is organized as follows. Section 2 presents the basic New Keynesian model, analyzes the welfare consequences of different kinds of policy rules, computes optimized policy rules, and considers a number of extensions. Section 3 describes a medium scale version of the model including capital, wage stickiness, and a number of other real frictions. Section 4 conducts quantitative analysis on the medium scale model. The final section concludes.

2 The Basic New Keynesian Model

This section considers the welfare effects of interest rate rules in the context of the textbook three equation, linearized New Keynesian model. For a full derivation see Woodford (2003), Gali (2008), or Walsh (2010).

The non-monetary side of the economy is characterized by two equations – an equation describing aggregate demand and an aggregate supply relation.[2] These are:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) + \frac{1}{\sigma}(1-\rho_\nu)\nu_t \tag{1}$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t^p \tag{2}$$

Equation (1) is derived from log-linearizing a household's consumption Euler equation and imposing the aggregate resource constraint that all output must be consumed. y_t is the log deviation of output from its non-stochastic steady state, i_t is the nominal interest rate relative to steady state, and ν_t is a shock to the marginal utility of consumption. σ is the coefficient of relative risk aversion from an iso-elastic and additively separable utility specification in consumption and labor, and ρ_ν is the AR(1) parameter in the stochastic process for ν_t.

Equation (2) is the Phillips Curve. π_t is inflation and x_t is the gap between the actual and the efficient levels of output, e.g. $y_t - y_t^e$.[3] κ is a parameter reflecting the degree of price-stickiness. Under Calvo (1983) pricing, it is given by $\kappa = \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}(\sigma + \eta)$, where $\theta_p \in (0,1)$ is the probability a firm cannot adjust its price in a period, β is a discount factor, σ is the coefficient of relative risk aversion, and η is the inverse Frisch labor supply elasticity. u_t^p is a variable which represents exogenous variation in the gap between the efficient and flexible price levels of output,

[2] As is common in much of this literature, I am abstracting from money altogether, so implicitly referring to a "monetary" side of the economy is a bit of a misnomer.

[3] It is more common to see the output gap defined in terms of the difference between actual output and what it would be if prices were flexible. The reason I write the gap this way is that it allows the "price markup" shock, which affects the flexible price, but not the efficient, level of output, to show up as a residual in the Phillips Curve.

e.g. $y_t^e - y_t^f$. One interpretation, pursued further in Ireland (2004) and modeled more formally in the medium scale model in the next section, is that it represents time series variation in desired markups of price over marginal cost.[4] For this reason I will refer to it is a "price markup" shock. It is sometimes also called a "cost-push" shock.

There are three exogenous shocks: the preference shock, ν_t; the price-markup shock, u_t^p; and a productivity shock, a_t. Each follow stationary AR(1) processes in the log, with the shocks drawn from standard normal distributions and s_j, $j = a, \nu, up$ the innovation standard deviations:

$$a_t = \rho_a a_{t-1} + s_a e_{a,t} \tag{3}$$

$$\nu_t = \rho_\nu \nu_{t-1} + s_\nu e_{\nu,t} \tag{4}$$

$$u_t^p = \rho_{up} u_{t-1}^p + s_{up} e_{up,t} \tag{5}$$

The efficient level of output can be solved analytically, depending only on the productivity and preference shocks:

$$y_t^e = \left(\frac{1+\eta}{\sigma+\eta}\right) a_t + \left(\frac{1}{\sigma+\eta}\right) \nu_t \tag{6}$$

The model is closed with a description of monetary policy in the form of an interest rate feedback rule. With some abuse of terminology, I will often refer to interest rate rules as "Taylor rules" after Taylor (1993). I consider the following generalized specification of a Taylor type interest rate rule:

$$\begin{aligned} i_t &= \rho_i i_{t-1} + (1-\rho_i) i_t^T, \;\; 0 \leq \rho_i < 1 \\ i_t^T &= \phi_\pi \pi_t + \phi_x x_t + \phi_{\Delta_y} \Delta y_t \end{aligned} \tag{7}$$

The actual interest rate is set as a convex combination of the previous period's rate and the target rate, i_t^T. All variables are either deviations or log deviations from trend, and are hence mean zero. The parameter ρ_i measures the degree of interest smoothing and the target rate, i_t^T, is expressed as a linear function of inflation, the output gap, and output growth.[5] One sometimes also sees policy rules which feature a response to the level of output. It turns out that responding to the level of output is always welfare-reducing for any parameterization of the model that I consider. For this reason, I do not consider the level of output as a potential target variable in the policy rule.[6] Though written in terms of current period values of the target variables, this policy rule can easily be amended to accommodate forward- or backward-looking terms. I require that $\phi_\pi > 1$ and that all other parameters be non-negative.[7]

[4]There are several alternative interpretations of this variable, such as variation in distortionary labor income taxes. Gilchrist and Leahy (2002) and Carlstrom, Fuerst, and Paustian (2010) show that net worth shocks are another potential source of the this kind of shock. Adam and Woodford (2013) show that shocks to housing demand and housing productivity could also generate what looks to be a price-markup shock.

[5]I later also consider "difference" and "inertial" rules in which $\rho_i \geq 1$.

[6]In an earlier version of the paper I also allowed a response to the growth rate of the output gap. This turns out to have similar effects as responding to output growth and is hence omitted.

[7]The so-called "Taylor principle" calls for the central bank to raise nominal interest rates more than one-for-one

Equations (1)-(7) characterize an equilibrium in the variables y_t, π_t, i_t, y_t^e, a_t, u_t^p, and ν_t. Assuming that a Pigouvian subsidy offsets the steady state distortion from monopoly power, one can express the welfare cost of price rigidity according to a simple quadratic loss function in inflation and the output gap:

$$\mathbb{W} = -\Omega E_t \sum_{j=0}^{\infty} \beta^j L_{t+j} \tag{8}$$

$$L_{t+j} = \pi_{t+j}^2 + \lambda x_{t+j}^2 \tag{9}$$

This loss function measures the average welfare loss due to price rigidity. Its units are constructed such that it measures the fraction of steady state output that one would need to give up in the flexible price, efficient economy to have the same welfare as in the sticky price economy. In terms of the underlying structural parameters of the model, the coefficients of the loss function are given by $\Omega = \frac{\theta_p(\sigma+\eta)}{2\kappa}$ and $\lambda = \frac{\kappa}{\epsilon_p}$, where ϵ_p is the elasticity of substitution among intermediate goods, with $\epsilon_p > 1$.

Interest rate rules are a kind of instrument rule since they describe how a central bank's main instrument ought to be set as a function of macroeconomic conditions. There is also a substantial literature that studies optimal monetary policy in the form of targeting rules. Targeting rules are the solution to a Ramsey problem – a central picks a time path of the nominal interest rate to minimize the loss function described above. The implementation of optimal targeting rules places potentially large informational burdens on central banks, where it is necessary to know the underlying structure of the economy and the specific shocks hitting it. Relatively simple interest rate rules only require central banks to adjust interest rates in response to a handful of more easily observable endogenous variables. In addition, the conditions under which interest rate rules give rise to a determinate rational expectations equilibrium are well understood.

2.1 Optimal Policy Rules: Quantitative Analysis

Solving the model and analyzing welfare requires picking values for the parameters. Table 1 shows the benchmark parameter values for the model. The unit of time is taken to be a quarter, so $\beta = 0.99$. $\sigma = 1$, so that utility from consumption is log, and $\eta = 1$, so that the Frisch labor supply elasticity is one. The Calvo parameter, θ_p, is set to 0.75, implying an average duration between price changes of four quarters. The elasticity of substitution is set to $\epsilon_p = 10$, which implies desired markups in steady state of about 10 percent. This parameterization leads to a slope of the Phillips curve of $\kappa = 0.17$. The relative weight on the output gap in the loss function is $\lambda = 0.02$.

The persistence parameters in the three exogenous processes are all set to 0.95. The shock standard deviations are $s_a = 0.0075$, $s_\nu = 0.015$, ad $s_{up} = 0.001$. Under a "standard" parameterization of the policy rule in which $\rho_i = 0.7$, $\phi_\pi = 1.5$, and $\phi_x = \phi_{\Delta_y} = 0$, this parameterization implies that the volatility of output growth is 1.2 percent and that the interest rate and inflation are mildly

with movements in inflation. In the model as written down, $\phi_\pi > 1$ is a slightly stronger restriction than is necessary to achieve determinacy, as determinacy also depends on the other response coefficients. See Woodford (2003).

countercyclical. These are consistent with US data. Under this parameterization, preference shocks account for the bulk of the variance of output growth at 57 percent. Productivity shocks account for a quarter of output fluctuations. The price-markup shock accounts for about 15 percent of the variance of output growth. These numbers are loosely in line with Ireland (2004).

The first row of Table 3, labeled "Baseline," shows optimized policy rule parameters for the basic Taylor rule. To find these parameters, I numerically search over $\phi_\pi \in (1.01, 3)$, $\rho_i \in (0, 0.99)$, and ϕ_x, $\phi_{\Delta_y} \in (0, 3)$ to find the combination that minimizes $\mathbb{W}$. The table reports the minimized value of the period loss function and the optimal parameters. The interpretation of the units of the minimized loss function is as the fraction of total consumption that would need to be given to the household each period in the sticky price economy to have the same welfare as in the same economy with flexible prices. The optimized policy rule features a moderate amount of smoothing ($\rho_i = 0.58$), a strong response to inflation ($\phi_\pi = 3$), and a modest response to output growth, with $\phi_{\Delta_y} = 0.11$. The optimal rule features a zero response to the output gap. The optimized value of the loss function implies that price rigidity only amounts to a welfare loss of about 0.04 percent of consumption (which would translate into roughly 4 percent of a single period's consumption). This welfare loss is low but in line with most of the work on New Keynesian models with price rigidity.[8]

Subsequent rows in the first panel of Table 3 show optimal policy parameters and the optimized value of the objective function for different restricted versions of the rule. The first row fixes the response to the output gap at a popular value of $\phi_x = 0.5$. This results in a substantial welfare loss, nearly quadrupling the consumption equivalent difference in welfare between the flexible and sticky price economies (0.04 versus 0.15). Other than a larger coefficient on the lagged interest rate, the optimized values of the other policy rule parameters are roughly the same as in the baseline. Requiring no interest smoothing has virtually no welfare cost, and the optimized rule compensates with a slightly larger coefficient on output growth. Forcing the coefficient on output growth to be zero has little welfare effect either, and the rule compensates with more smoothing. Forcing a smaller coefficient on inflation results in a near doubling of the welfare loss, with a larger coefficient on the lagged interest rate and a smaller response to the gap. Strict inflation targeting (just a large response to inflation and nothing else) does almost as well as the optimal rule.

The exercises carried out in Table 3 suggest that reacting positively to the output gap is welfare-reducing, while a mild positive response to output growth is beneficial. This conclusion may seem odd given the welfare loss function above, which depends on the variances of inflation and the gap. A stronger response to the gap ought to reduce gap volatility, and therefore seems like it should be welfare-improving. In contrast, output growth tends to be high when the output gap is low, given the "natural rate" property of the model. Positively responding to growth therefore seems to run counter to conventional stabilization logic.

The intuition for the relative merits of gap and growth targeting in the policy rule arise from

[8] Note that the welfare loss is expressed relative to the efficient, flexible price economy, not the optimal Ramsey targeting rule (as in, for example, Schmitt-Grohe and Uribe, 2007). The welfare loss under the optimal Ramsey policy under commitment is 0.034 and under discretion (solving the problem period-by-period with no commitment to future paths of the interest rate) is 0.051.

two features of the Phillips Curve: the price-markup shock, u_t^p, and the presence of expected future inflation, $E_t\pi_{t+1}$. If price-markup shocks are absent, then the "Divine Coincidence" (Blanchard and Gali, 2007) holds, and it is possible to simultaneously stabilize both the gap and inflation. This is easy to see: if $x_t = 0$ for all periods, then iterating the Phillips Curve forward, it is straightforward to see that $\pi_t = 0$ as well. This could be accomplished through a large value of ϕ_x in the policy rule. If price markup shocks are present, in contrast, then there is a tradeoff at play: stabilizing the gap comes at the expense of more inflation, which may or may not be welfare-improving.

The presence of expected future inflation in the Phillips Curve means that a central bank can achieve a better "menu" of current inflation and the gap the more stable is expected inflation. Positive response coefficients to output growth make policy history dependent (implicitly through lagged output), which has the effect of better "anchoring" expected future inflation. For example, suppose a shock hits which drives output below potential. By responding positively to growth, the central bank can cut interest rates in the present (when output growth declines), while implicitly promising to raise rates in the future (when output growth turns positive as output heads back to potential). The implicit promise to raise rates in the future keeps expected inflation from reacting too much, yielding a better inflation-gap menu in the present.

Figure 2 graphically depicts how the variances of inflation and the output gap vary with the coefficients on the output gap (left panel) and output growth (right panel). These variances are computed relative to a "baseline" rule with $\rho_i = 0.7$, $\phi_\pi = 1.5$, and $\phi_x = \phi_{\Delta_y} = 0$. Stronger responses to the output gap do indeed reduce the variance of the output gap, but at the expense of an everywhere increasing variance of inflation. Given the low weight on the gap in the objective function ($\lambda = 0.02$), this evidently works to reduce welfare on net. Positive responses to output growth also work to reduce the variance of the output gap, though the gains are smaller here than when responding directly to the gap. Quite differently than the coefficient on the gap, however, positive responses to output growth actually work to also reduce the variance of inflation (at least over a range). Evidently, a mild response to growth allows a central bank to achieve lower gap and inflation volatility, resulting in greater welfare.

The second panel of Table 3 shows optimized policy rule coefficients conditional on each of the three shocks one at a time. In all cases, it is optimal to have large positive response to inflation. Consistent with the intuition discussed above, conditional on preference or productivity shocks, it is optimal to have a strong response to the output gap, while it is best to not respond to the gap at all conditional on price markup shocks. Even though price markup shocks account for relatively little variation in output growth, they account for the bulk of movements in inflation and the output gap. Hence, the non-desirability of gap targeting conditional on this shock outweighs the benefits of gap targeting conditional on the other two shocks, making a positive response to the gap welfare-reducing on net. It is optimal to have strong positive responses to output growth conditional on either productivity or price-markup shocks, but responding to growth is welfare-reducing conditional on preference shocks. The intuition for this is that output growth is approximately proportional to lagged real interest rates, which can be seen in (1). Positive responses to output growth work to stabilize real rates, which is not optimal conditional on preference shocks, because these shocks induce large movements in the "natural," or Wicksellian, rate of interest. Nevertheless, the benefits of growth targeting conditional on the other two shocks outweigh the costs of doing so conditional on preference shocks, making it desirable to respond to growth on net. This obtains in spite of the fact that preference shocks are the dominant source of output movements under the baseline parameterization.

How important must price-markup shocks be for it to be non-optimal to respond to the gap in the policy rule? This is an important question for two reasons. First, responding to the gap can evidently be quite costly from a welfare perspective if these shocks are important. Second, there is a good bit of debate in the literature on just how important these shocks are (e.g. Justiniano, Primiceri, and Tambalotti, 2013). In the model, the "importance" of price-markup shocks depends on both the persistence, ρ_{up}, and innovation standard deviation, s_{up}, of the shock process. The left panel of Figure 3 plots the set of (s_{up}, ρ_{up}) pairs for which it is not welfare-reducing to move from a zero to small positive (0.05) response to the output gap, relative to the baseline rule with $\rho_i = 0.7$ and $\phi_\pi = 1.5$. This plot has the flavor of an indifference map – at points above the curve, it is welfare-reducing to respond to the gap, and vice-versa for points below the curve. The right panel plots the fraction of the unconditional variance of output growth for which the price-markup shock would account at the points in the left panel. For plausible measures of persistence (e.g. $\rho_{up} > 0.75$), if the price-markup shock accounts for more than 5 percent of variation in output growth, then it is welfare-reducing to respond to the output gap.

2.2 Robustness

This subsection considers several robustness exercises. The results are summarized in Table 4 and Figure 4.

I first consider alterations on the structure of the policy rule itself. The first two rows of Table 4 drop the "partial adjustment" specification embodied in (7) by allowing $\rho_i \geq 1$. Mechanically, this necessitates dropping the $1 - \rho_i$ term multiplying the target variables on the right hand side of the policy rule. The optimal policy parameters when ρ_i can be greater than unity are shown in the first row, labeled "super-inertial." The optimal $\rho_i = 1.07$, there is a large positive response to inflation, essentially no response to the output gap, and a small, but positive, response to output growth. The row labeled "difference" constraints $\rho_i = 1$ and features similar coefficients and achieves about the same level of welfare. Either of these specifications achieve a slightly higher level of welfare than the partial adjustment specification. Nevertheless, the basic conclusions about gap versus growth targeting still obtain.

The next two rows of the table, labeled "Backward-looking" and "Forward-looking," replace the target variables on the right hand side with their realizations lagged or led one period, respectively. The backward-looking specification achieves a slightly better average welfare outcome than the baseline "contemporaneous" version of the policy rule, while the forward-looking specification does a little worse. These differences are nevertheless small. In both cases the optimal coefficient on inflation is large, there is no response to the output gap, there is moderate interest smoothing, and a mild positive response to output growth. The optimal response to growth is larger in the forward-looking specification than in the backward-looking specification.

The next part of Table 4 shows optimal policy rule parameters under different versions of the welfare loss function. In the baseline parameterization, inflation volatility is about 50 times more important than gap volatility ($\lambda = 0.02$). Though this comes out of an approximation to household utility and is based on a standard parameterization of the model, these relative weights seem out of line with how actual central bankers seem to operate. I therefore consider arbitrary welfare weights on the gap. I do not present optimized values of the loss function, because with arbitrary values of λ there is no natural interpretation of the units of the loss function and these numbers thus cannot be compared to earlier analyses. I also fix the smoothing parameter to be $\rho_i = 0$; this better facilitates comparisons of the coefficients on the gap and growth across different values of λ.[9] For it to be optimal to have a non-negative response coefficient on the gap, λ must be greater than about 0.1, which is 5 times its value based on the approximation to household welfare. Quite naturally, as λ increases from there, the desired response coefficient on the gap also increases. Interestingly, the desired response to output growth also increases with λ. For a value of $\lambda = 0.25$, for example, the desired response to output growth is about 4 times larger than the optimal response to the gap. Even for $\lambda = 1$, the optimal response to output growth is larger than the parameter on the output gap.

I also consider robustness to different parameter values, though I only discuss these qualitatively. The only circumstance in which it is optimal to respond positively to the output gap in the policy rule is when the price-markup shock generates very little output volatility (see Figure 3). In all parameterizations it is optimal to respond strongly to inflation. The desired response coefficient on output growth is increasing the amount of price rigidity, θ_p; increasing in risk aversion, σ; decreasing in the elasticity of substitution, ϵ_p (which affects the optimal weight on the output gap in the loss function); increasing in the Frisch labor supply elasticity, η^{-1}; increasing in the discount factor, β; increasing in the quantitative importance of price-markup and productivity shocks; and decreasing in the importance of preference shocks. The conclusions concerning shock magnitudes are consistent with the analysis in Table 3. Because responding to growth has the effect of better anchoring expected inflation, the desired response to growth is larger under parameterizations where price-setting is more forward-looking, such as when θ_p or β are larger.

An important issue from the perspective of current policy is the zero lower bound on nominal interest rates, which my welfare calculations ignore. As shown in Coibion, Gorodnichenko, and Wieland (2012), hitting the zero lower bound can be quite costly from a welfare perspective. When the interest rate cannot react to shocks, the response of inflation is typically much larger than under a standard policy rule, which leads to large and inefficient movements in price dispersion. Figure 4 plots how the incidence of hitting the zero lower bound varies as a function of the policy rule coefficients on the output gap and inflation for three different levels of trend inflation.[10] In the

[9] In particular, as λ increases, the optimal ρ_i increases. As discussed above, there is something of a tradeoff between ρ_i and the coefficient on output growth. ρ_i is fixed at 0 to make comparisons across λ easier. In any event, the differences in the optimized value of the loss function from fixing $\rho_i = 0$ are small.

[10] To generate this figure, I linearize the basic model allowing for non-zero trend inflation (the basic equations

baseline paramterization with zero trend inflation the incidence is quite low (around one percent annually); naturally, as the level of trend inflation increases, the probability of hitting the zero lower bound decreases for all values of the policy rule parameters. In the left panel, we see that the probability of hitting the zero lower bound is everywhere increasing in the coefficient on the output gap. Moderate responses to the gap can lead to rather large probabilities of hitting the zero bound. In contrast, there is not much effect of the coefficient on output growth on the incidence of the nominal interest rate hitting zero – though it is difficult to see, the probability of hitting zero is actually mildly decreasing in the coefficient on growth for low values of this coefficient. These results about the zero lower bound reinforce the welfare analysis ignoring the floor on interest rates: responding to the gap can be costly while there are some gains to be had from targeting growth.[11]

3 A Medium Scale DSGE Model

This section extends the analysis of the previous section to a medium scale version of the New Keynesian model. The model features both price and wage stickiness, productive physical capital, a number of real rigidities, and several shocks. It also allows for trend growth and trend inflation. The model is similar to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). The model is composed of the following actors: an employment agency, households, a final good firm, and intermediate goods firms. Below I describe the problem facing each of these actors and define an equilibrium. The Appendix lists the full set of equilibrium conditions.

3.1 Employment Agency

There is a competitive employment agency that combines differentiated labor input from a continuum of households indexed by $h \in (0,1)$. Labor input available to lease to firms, $N_{d,t}$, is:

$$N_{d,t} = \left(\int_0^1 N_t(h)^{\frac{\epsilon_{w,t}-1}{\epsilon_{w,t}}} dh \right)^{\frac{\epsilon_{w,t}}{\epsilon_{w,t}-1}}$$

The variable $\epsilon_{w,t}$ is a potentially time-varying elasticity of substitution between different types of labor. Its mean value is assumed to be greater than 1. Fluctuations in $\epsilon_{w,t}$ can be interpreted as time-variation in the desired wage-markup. The agency takes both the real wage on labor it sells to firms, W_t, and the real wage charged by households, $W_t(h)$, as given. Profit maximization gives

shown above in the text, in contrast, assume zero trend inflation). Then I simulate the model for 200,000 periods and calculate the frequency of the nominal interest rate being less than or equal to zero. The frequencies are based on the "baseline" policy rule with $\rho_i = 0.7$ and $\phi_\pi = 1.5$, with $\phi_x = \phi_{\Delta_y} = 0$. Though not shown in the figure, the incidence of hitting the zero lower bound appears everywhere decreasing in ρ_i.

[11]That responding to the gap increases the probability of hitting the zero lower bound rests on the presence of the price-markup shock. In simulations without this shock, the incidence of hitting the zero lower bound is everywhere decreasing in ϕ_x. The intuition for this is straightforward – if there are no price-markup shocks, then responding to the gap reduces the volatility of both the gap and inflation, which in turn reduces the volatility of nominal interest rates. In contrast, if markup shocks are present, then a larger response to the gap reduces gap volatility but increases inflation volatility. Given the larger coefficient on inflation in the policy rule, this leads to more interest rate variability and increases the probability of hitting zero.

rise to a downward-sloping demand curve for each type of labor and an aggregate real wage index:

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\epsilon_{w,t}} N_{d,t} \tag{10}$$

$$W_t = \left(\int_0^1 W_t(h)^{1-\epsilon_{w,t}} dh\right)^{\frac{1}{1-\epsilon_{w,t}}} \tag{11}$$

3.2 Households

Households, $h \in (0,1)$, have identical preferences but supply differentiated labor. They can choose consumption, labor supply, capital utilization, capital accumulation, and holdings of riskless one period bonds. The problem of a typical household is:

$$\max_{C_t(h),N_t(h),u_t(h),I_t(h),B_t(h),K_{t+1}(h)} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\nu_t \ln(C_t(h) - bC_{t-1}(h)) - \psi_t \frac{N_t(h)^{1+\eta}}{1+\eta}\right)$$

s.t.

$$C_t(h) + I_t(h) + \frac{B_t(h)}{P_t} + \leq W_t(h)N_t(h) + R_t u_t(h)K_t(h) - \Gamma(u_t)\frac{K_t(h)}{Z_t} + (1+i_{t-1})\frac{B_{t-1}(h)}{P_t} + \Pi_t(h) + T_t(h)$$

$$K_{t+1}(h) = Z_t\left(1 - S\left(\frac{I_t(h)}{I_{t-1}(h)}\right)\right) I_t(h) + (1-\delta)K_t(h)$$

ν_t is a common shock to the utility from consumption, while ψ_t is a time-varying labor supply shock. $b \in (0,1)$ is a measure of internal habit persistence, and $\eta > 0$ is the inverse Frisch labor supply elasticity. B_{t-1} denotes the holdings of bonds with which a household enters a period. R_t is a common rental rate on leasing capital services, where capital services are the product of utilization, $u_t(h)$, and physical capital, $K_t(h)$. $\Gamma(\cdot)$ is a convex adjustment cost from utilization, and has the properties that $\Gamma(1) = \Gamma'(1) = 0$ and $\Gamma''(\cdot) = \gamma > 0$. Z_t is a shock to the marginal efficiency of investment. $S(\cdot)$ is an investment adjustment cost as in Christiano, Eichenbaum, and Evans (2005). It has properties $S(g_I) = S'(g_I) = 0$ and $S''(g_I) = \kappa \geq 0$, where g_I is the steady state growth rate of investment. Π_t and T_t are distributed profits from ownership in firms and lump sum taxes/transfers, respectively.

Each period, only a randomly chosen fraction of households, $1-\theta_w$, $\theta_w \in (0,1)$, are able to adjust their wage. This has the complication that it induces heterogeneity into household income. As is common in the literature since Erceg, Henderon, and Levin (2000), I implicitly assume the existence of state-contingent bonds which insure households against idiosyncratic wage income risk. Given separability between consumption and leisure, this has the implication that all households will make identical consumption, investment, utilization, and bond-holding choices. Updating households choose their current wage to maximize the expected present discounted value of utility, subject to the restriction of supplying as much labor as is demanded from (11). Households not given the opportunity to update their wage can index nominal wages to lagged inflation at rate $\zeta_w \in (0,1)$.

3.3 Final Good Firm

There is a competitive, representative firm that produces a final good available for consumption, Y_t. It is a composite of a continuum of intermediate goods, $Y_t(j)$, $j \in (0,1)$:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_{p,t}-1}{\epsilon_{p,t}}} dj \right)^{\frac{\epsilon_{p,t}}{\epsilon_{p,t}-1}}$$

The variable $\epsilon_{p,t}$ is a potentially time-varying elasticity of substitution among goods. Its mean value is greater than 1. Fluctuations in ε_t represent time-variation in the desired price-markup. The first order conditions resulting from profit maximization are a demand curve for each intermediate good and an aggregate price index:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_{p,t}} Y_t \tag{12}$$

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_{p,t}} dj \right)^{\frac{1}{1-\epsilon_{p,t}}} \tag{13}$$

3.4 Intermediate Goods Firms

There are a continuum of intermediate goods firms indexed by $j \in (0,1)$. The production function of a typical firm is:

$$Y_t(j) = A_t K_{s,t}(j)^{\alpha} N_{d,t}(j)^{1-\alpha} - g_y F$$

$K_{s,t}(j)$ is the quantity of capital services (the product of physical capital and utilization) which firms rent from households. $N_{d,t}(j)$ is the quantity of labor demanded by firm j at time t. A_t is a productivity disturbance common across firms. Firms behave atomistically and take real factor prices, R_t and W_t, as well as aggregates, P_t and Y_t, as given. A_t is an exogenous productivity shock. F is a fixed cost scaled by the trend growth rate of output. Its magnitude is chosen so as to ensure zero profits along a balanced growth path.

Each period, only a randomly chosen fraction of firms, $1-\theta_p$, are allowed to update their price. Updating firms choose price to maximize the present discounted value of flow profits, discounting by the household's stochastic discount factor. Non-updating firms can partially index price to lagged inflation at $\zeta_p \in (0,1)$. Regardless of whether a firm updates its price in a period, it hires capital and labor to minimize total costs. This has the implication that firms have identical real marginal cost and hire capital and labor in the same ratio.

3.5 Aggregation and Equilibrium

The central bank is assumed to follow the same form of interest rate rule as in the simpler model. A couple of modifications are necessary, however. First, because I do not assume the existence of a Pigouvian subsidy to offset monopoly distortions, the steady state value of the output gap,

x^*, will be positive. Second, I allow for trend growth in output, so that the steady state output growth will not be zero. Finally, I also potentially allow for non-zero trend inflation, π^*. Define $x_t = \ln Y_t - \ln Y_t^e$ and $\Delta y_t = \ln Y_t - \ln Y_{t-1}$. Y_t^e is the level of output that would obtain absent price and wage rigidity and monopoly power.[12] The modified target rule can be written (the partial adjustment part of the rule is the same as earlier):

$$i_t^T = \phi_\pi \left(\pi_t - \pi^*\right) + \phi_x \left(x_t - x^*\right) + \phi_{\Delta_y} \left(\Delta y_t - \ln g_y\right) \tag{14}$$

In terms of exogenous processes, the neutral productivity and marginal efficiency of investment processes each fluctuate about a deterministic trend:

$$\ln A_t = \ln A_t^\tau + \ln \widetilde{A}_t \tag{15}$$

$$\ln A_t^\tau = g_a t \tag{16}$$

$$\ln \widetilde{A}_t = \rho_a \ln \widetilde{A}_{t-1} + s_a e_{a,t} \tag{17}$$

$$\ln Z_t = \ln Z_t^\tau + \ln \widetilde{Z}_t \tag{18}$$

$$\ln Z_t^\tau = g_z t \tag{19}$$

$$\ln \widetilde{Z}_t = \rho_z \ln \widetilde{Z}_{t-1} + s_z e_{z,t} \tag{20}$$

The variables governing wage and price markups follow stationary log AR(1) processes:

$$\ln \epsilon_{w,t} = (1-\rho_w) \ln \epsilon_w + \rho_w \ln \epsilon_{w,t-1} + s_w e_{w,t}, \;\; \epsilon_w > 1 \tag{21}$$

$$\ln \epsilon_{p,t} = (1-\rho_p) \ln \epsilon_p + \rho_p \ln \epsilon_{p,t-1} + s_p e_{p,t}, \;\; \epsilon_p > 1 \tag{22}$$

The two kinds of preference shocks follow AR(1) processes in the log, with the mean of ν_t normalized to 1:

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + s_\nu e_{\nu,t} \tag{23}$$

$$\ln \psi_t = (1-\rho_\psi) \ln \psi + \rho_\psi \ln \psi_{t-1} + s_\psi e_{\psi,t} \tag{24}$$

There exists a balanced growth path in which most variables grow at rate $(1+g_a)^{\frac{1}{1-\alpha}}(1+g_z)^{\frac{\alpha}{1-\alpha}}$; exceptions include the capital stock, which grows faster due to growth in Z, and labor hours, N_t, which are stationary given assumptions on preferences. Most variables can be detrended such that $\widetilde{x}_t = \frac{X_t}{\Upsilon_t}$ is stationary, where $\Upsilon_t = (A_t^\tau)^{\frac{1}{1-\alpha}} (Z_t^\tau)^{\frac{\alpha}{1-\alpha}}$. An equilibrium in the detrended variables consists of a set of prices and allocations such that all household and firm optimality conditions

[12] Alternatively, one could define the gap in terms of the flexible price level of output. In steady state the efficient and flexible price levels of output differ only by a constant.

hold and all markets clear, given initial values of endogenous states and values of the exogenous shocks.

Given household heterogeneity in labor supply, there is no straightforward way to measure welfare. Following Erceg, Henderson, and Levin (2000), I assume that the welfare function of a central bank is equal to the sum of welfare across households. As shown in the Appendix, this expression can be written recursively without reference to household subscripts:

$$\mathbb{W}_t = \nu_t \ln(C_t - bC_{t-1}) - \psi_t v_t^w \frac{N_{d,t}^{1+\eta}}{1+\eta} + \beta E_t \mathbb{W}_{t+1} \tag{25}$$

v_t^w is a measure of wage dispersion, bound from below by unity. In a way analogous to how price dispersion drives a wedge between aggregate factor supply and aggregate output, wage dispersion drives a wedge between total labor supply and the quantity of labor used in production, and therefore is welfare-reducing. Provided $\beta\Upsilon < 1$, welfare is stationary in spite of the non-stationarity in consumption.

4 Quantitative Analysis in the Medium Scale Model

This section conducts quantitative analysis of optimal policy rule parameters in the medium scale model. The parameter values are conventional from the literature, loosely based on the structural estimations in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), and are shown in Table 2

The discount factor and Frisch labor supply elasticity are the same as in the model without capital, 0.99 and 1, respectively. The habit formation parameter is set to $b = 0.7$. Capital's share in production, α, is 1/3. The mean value of the scaling parameter on the disutility of labor, ψ, is set so that steady state labor hours are about 0.35. The investment adjustment cost parameter is $\kappa = 3$, capital depreciates at quarterly rate $\delta = 0.025$, and the parameter governing the convexity of the utilization adjustment cost function is set low at $\gamma = 0.05$. ϵ_p and ϵ_w are both set to 10, implying steady state price and wage markups of about 10 percent. The parameters governing wage and price rigidity, θ_p and θ_w, are both set to 0.65, implying average durations of price and wage contracts of 3 quarters. The steady state growth rates of neutral productivity and the marginal efficiency of investment are set to 0.002 and 0.004, respectively. This implies that output (per-capita, since there is no population growth) grows at about 2 percent per year at an annualized rate. The autoregressive parameters of the shock processes are all set to 0.95. The shock standard deviations are as shown. The parameterizations of the price- and wage-markup processes imply standard deviations of these markups of 2 percent. As a benchmark, I assume zero trend inflation. It would be equivalent to assume positive trend inflation with prices and wages fully indexed to trend inflation.[13] I also assume as a benchmark that there is no indexation to lagged inflation,

[13] As discussed in Coibion and Gorodnichenko (2011), and confirmed below, a positive level of trend inflation has a significant effect on the set of policy rule parameters giving rise to a determinate rational expectations equilibrium. Since the focus of this paper is not on determinacy per se, abstracting from trend inflation as a first pass seems to make the most sense.

which has been emphasized by some (e.g. Christiano, Eichenbaum, and Evans, 2005) as a means by which to generate nominal inertia following monetary shocks. Although they have the same effects on the long run properties of the model, trend indexation and indexation to lagged inflation have different short run effects.

The right column of the table gives some measures of goodness of fit. The standard deviation of the growth rate of output is 1.1 percent, similar to observed GDP volatility in the data. Investment and consumption growth are strongly procyclical, and the nominal interest rate and inflation are close to acyclical. Marginal efficiency of investment shocks account for about 40 percent of the variance of output growth. This is similar to the empirical findings in Justiniano, Primiceri, and Tambalotti (2010). Neutral productivity shocks account for around 15 percent of output fluctuations, while preference shocks to consumption account for around 20 percent. The labor supply preference shock generates 7 percent of output growth volatility, the price-markup shock about 15 percent, and the wage-markup shock about 1 percent.

4.1 Optimal Policy

I conduct similar analysis on the policy rule parameters as in Section 3. In particular, I numerically search for values of $\phi_\pi \in (1.01, 3)$, $\rho_i \in (0, 1)$, $\phi_x \in (0, 3)$, and $\phi_{\Delta_y} \in (0, 3)$ to maximize expected welfare in the sticky price/wage economy. I then present the welfare loss as the fraction of consumption with which households would need to be compensated in order to have the same expected welfare as in a hypothetical economy in which prices and wages are flexible and price- and wage-markups do not fluctuate.[14]

The first panel of Table 5 shows optimal policy rule parameters for the baseline specification of the model. The optimized policy rule features no interest smoothing, a large response to inflation, no response to the output gap, and a strong positive response to output growth. The optimal response to growth is significantly larger than in the simpler model without capital, with $\phi_{\Delta_y} = 0.65$ as opposed to 0.11. The optimized welfare loss is 1.7 percent of steady state consumption, which is also substantially higher than in the model without capital. The much larger welfare loss comes mostly from the combination of wage stickiness and trend growth, not endogenous capital accumulation.

Subsequent rows in the "Baseline" panel show restricted optimal policy rules, with the parameter restrictions described in the second column. Requiring only a modest response to the output gap, with $\phi_x = 0.25$, results in a welfare loss amounting to more than an additional full percent of steady state consumption. The optimal rule compensates with an even stronger response to growth and substantial interest smoothing. Constraining the rule to feature interest smoothing has only a small effect on welfare; the optimized rule continues to have a strong response to inflation and an even stronger response to output growth. The latter result is different than in the model without capital, where the desired response to output growth varied inversely with the amount of interest smoothing. Constraining there to be a much smaller response to inflation results in a significant

[14] This is an unconditional welfare metric. In the robustness subsection I also consider a conditional welfare metric which conditions on the same starting point in the state space. The construction of either welfare metric is described in more detail in the Appendix.

welfare loss, though the loss is not as large as when requiring a positive response to the output gap. Finally, requiring no response to output growth results in a rather small welfare loss, similarly to the model without capital.

The second panel of Table 5 shows optimized policy rule coefficients conditional on each of the six shocks in the model one at a time. In all cases it is optimal to have a strong response to inflation, though the desired coefficient on inflation conditional on wage markup shocks is less than its constrained maximum value of 3. Large positive responses to output growth are desirable conditional on neutral productivity, marginal efficiency of investment, and price-markup shocks. It is welfare-reducing to react to growth conditional on preference or wage-markup shocks. The desired response to the gap is zero conditional on neutral productivity, marginal efficiency of investment, intertemporal preference, and price-markup shocks. It is welfare-improving to have a small positive response to the gap conditional on intratemporal preference and wage-markup shocks ($\phi_x = 0.07$ and 0.08, respectively).

Table 6 considers optimal policy parameters under several different assumptions about trends and the role of markup shocks. The first panel of Table 6 sets trend growth in both neutral productivity and the marginal efficiency of investment to zero. The optimized rule features a strong response to inflation, a near-zero response to growth, and a very small, but nevertheless positive, response to the output gap. Requiring a larger response to the gap results in a non-trivial welfare loss, but it is much milder than when trend growth is present. Requiring more interest smoothing, or a smaller response to inflation, results in welfare losses and the optimized rule compensates with a larger coefficient on output growth. Even though it is optimal to not respond to growth in the unrestricted version of the rule, requiring a strong positive response has very little effect on welfare. Overall, the welfare losses without trend growth are significantly smaller than with trend growth. This obtains because of an interaction between trend growth and wage rigidity, as documented in Amano, Moran, Murchison, and Rennison (2009). When wages are rigidity and trend growth is positive, wage dispersion becomes first order, which makes inflation variability very costly. Below I consider the effects of indexing wages to trend productivity growth as a robustness exercise.

In the simpler model without capital, the presence of price-markup shocks is critical in making responding to the output gap non-desirable. The overall importance of price- and wage-markup shocks is a matter of some heated debate (e.g. Justiniano, Primiceri, and Tambalotti, 2013). In the second panel of Table 6, I compute optimal policy rule parameters when price and wage markups are held fixed at their steady state levels – i.e., there are no markup shocks. The optimized policy rules, and welfare losses, are quite similar to the baseline specification. The optimized response to output growth is a little lower than in the baseline, and the welfare cost from requiring a moderate response to the gap is smaller. Nevertheless, even without markup shocks, responding to the output gap is welfare-reducing, which is different than in the model without trend growth.

The next panel of the Table, labeled "No trend growth or markup shocks," sets trend growth to zero and fixes markups at their steady state values. This results in a very different optimal policy rule relative to the baseline. In particular, the optimized policy rule features a small response to inflation ($\phi_\pi = 1.01$), and a very large response to the output gap, with $\phi_x = 3$. The optimal response to growth is close to zero, there is a small amount of interest smoothing, and the welfare loss evaluated at the optimal policy parameters is very small. Requiring a smaller response to the output gap results in a significant welfare loss. These conclusions follow from results dating back to Erceg, Henderson, and Levin (2000), who found that inflation targeting does poorly, and gap targeting well, in a simple model where price and wage rigidity are the only frictions relative to a neoclassical model. Their model does not feature trend growth or markup shocks.

The next several panels of Table 6 assume that there is positive trend inflation of 2 percent at an annualized frequency. Overall, positive trend inflation results in large welfare losses, as in Amano, Moran, Murchison, and Rennison (2009). The optimal policy rule features a large response to inflation, substantial interest smoothing, no response to the output gap, and a large positive response to output growth. The optimal coefficient on growth is substantially larger than when trend inflation is zero. Positive trend inflation effectively makes current price-setting more forward-looking, which makes the anchoring of inflation expectations more valuable. Requiring even a small positive response to the output gap when trend inflation is positive results in a very large welfare loss relative to the optimal rule. As shown by Coibion and Gorodnichenko (2011), responding strongly to the output gap with positive trend inflation can be de-stabilizing in the sense of reducing the likelihood of satisfying the modified Taylor principle. My exercises confirm this finding, where values of ϕ_x much in excess of 0.1 lead to indeterminacy. Unlike the case of zero trend inflation, interest smoothing is welfare-improving, and requiring less than the optimal amount of smoothing results in a mild welfare loss. Forcing no response to growth also results in a welfare loss relative to the optimal rule, though it is mild. A smaller response to inflation again leads to a significant increase in the welfare loss.

The next panel of Table 6 continues to assume that there is positive trend inflation, but sets trend growth to zero. The optimized rule features no smoothing, a strong response to inflation, and a moderate response to output growth ($\phi_{\Delta_y} = 0.38$). The response to growth is significantly larger than when trend growth and trend inflation are both zero, but is not as high as the case when trend growth is positive. The optimized welfare loss is significantly smaller than when trend growth is positive, but nevertheless quite a bit bigger than when there is no trend growth and no trend inflation. Requiring a positive response to the output gap is again welfare-reducing, but not nearly as much as when both trend inflation and growth are positive. The second to last panel of the table considers the case when trend inflation and growth are both positive, but there are no markup shocks. The optimized policy rule coefficients are fairly similar to earlier exercises – it is optimal to have substantial interest smoothing, a large response to inflation, no response to the gap, and a large response to growth, though the desired response to growth is slightly smaller than when there are markup shocks. The final panel assumes no trend growth or markup shocks, but with positive trend inflation. The optimized policy rule features no smoothing, no response to the gap, a large response to inflation, and a moderate response to output growth. Restricting other parameters leads to similar patterns as in previous exercises.

One can surmise from Tables 5 and 6 that trend growth, markup shocks, and trend inflation are key features in determining the desirability of responding to the output gap and output growth in the policy rule. The role of markup shocks is similar to the baseline model without capital, but the effects of trend growth and inflation are new. The only situation in which it is not welfare-reducing to respond vigorously to the output gap is when all three of these features are absent. If any one, or any combination of the three, is present, then reacting vigorously to the gap can lead to significant welfare losses. In contrast, each of these three features increase the welfare-desirability of responding to output growth. Price markup shocks magnify the inflation-gap tradeoff that is already present when both wages and prices are sticky, and make responding to growth desirable because it better anchors expected inflation, improving available inflation-gap tradeoffs. Trend growth and trend inflation make price-setting more forward-looking – either of these effectively increase the weight attached on future marginal costs when updating firms set prices, which makes pricing more forward-looking and increases the gains to be had from better anchoring inflation by responding to growth.

4.2 Robustness

This subsection considers a number of robustness exercises. These exercises are mainly focused on the key mechanisms which affect the desirability of responding to growth or the gap – markup shocks, trend growth, and trend inflation – though I also discuss overall parameter sensitivity as well as the measure of welfare.

Table 7 shows optimized policy rule parameters under different real and nominal indexation schemes. In the basic model, the reason that the welfare cost of price rigidity is so high is because of an interaction between trend productivity growth and wage dispersion. When trend growth is positive and wage contracts are staggered, wage dispersion becomes first order even without trend inflation. This has the effect of making inflation very costly. In the first panel of the table I consider the effects of fully indexing nominal wages to real productivity growth. When this is the only difference relative to the baseline analysis, the optimized welfare loss is substantially lower (0.4 percent of consumption versus 1.7 percent). The optimized rule is nevertheless fairly similar to the baseline analysis – it is desirable to have a strong response to inflation and a moderate response to output growth, though the optimal response to growth is smaller than when wages are not indexed to productivity growth (0.15 versus 0.65). Different from the baseline, it is optimal to have a positive coefficient on the output gap, though it is very small. Note that assuming wages are indexed to real growth is not identical to assuming that there is no trend growth. In addition to its effect on wage dispersion, trend growth effectively makes households more patient, which makes behavior more forward-looking. This means that responding to output growth has benefits over and above the interaction with wage dispersion highlighted earlier.

The next row of Table 7 combines wage indexation to real growth with no markup shocks. The resulting optimal policy rule is similar to the rule with no trend growth and no markup shocks – it is optimal to respond strongly to the gap, not at all to growth, and little to inflation. The next two rows combine wage indexation to real growth with positive trend inflation (with and without markup shocks, respectively). In both cases it is optimal to respond strongly to growth and inflation, and not at all to the gap. The optimized responses to growth are nevertheless smaller than when trend inflation is positive and wages are not indexed to real growth.

The next panel of Table 7 considers nominal wage and price indexation. In particular, I set $\zeta_p = \zeta_w = 1$, so that prices and wages are fully indexed to lagged inflation.[15] This assumption leads to dramatic changes in the nature of the optimal policy rule. In particular, it becomes optimal to respond strongly to both the output gap and inflation, with no response to output growth. If markup shocks are absent, then the the desired response to inflation is also low. The third panel of the table combines wage indexation to real growth with full wage and price indexation to lagged inflation. If markup shocks are present, it is optimal to respond strongly to both inflation and the gap. If markup shocks are absent, then it is best to respond strongly only to the gap, with a small coefficient on inflation. In both cases it is best to not respond to output growth.

The last two panels of Table 7 make clear that the amount of nominal indexation to lagged inflation is important in determining the desirability of responding to the gap or growth. When there is complete indexation, it is best to respond strongly to the gap and not at all to growth. Perhaps curiously, the relationship between indexation and the desirability of gap or growth targeting is non-linear in the indexation parameters. For intermediate values of indexation, such as $\zeta_p = \zeta_w = 0.5$,[16] the optimal response to output growth is actually larger than with no indexation at all, and it is best to not react to the gap. Empirically, though some indexation to lagged inflation may play an important role in accounting for inertial, hump-shaped inflation dynamics (e.g. Christiano, Eichenbaum, and Evans, 2005), most structural estimates of these indexation parameters suggest far less than full indexation to lagged inflation. For example, Smets and Wouters (2007) estimate the indexation parameters to be 0.6 for wages and 0.2 for prices. Justiniano, Primiceri, and Tambalotti (2012) find these parameters to be even lower, around 0.1 for each. At either of these intermediate levels of nominal indexation, the desired responses to output growth are large.

As in the simpler model without capital, I also investigate the role of variations in the form of the policy rule. As written, the policy rule, (7), is one of partial adjustment, and ρ_i is restricted to be less than 1. With this specification, higher values ofρ_i make interest rates more persistent, but also reduce the short run responses of rates to deviations in the target variables, so that the "long run" responses of the interest rate to movements in the target variables are solely governed by the coefficients on the targets. Table 8 shows optimized policy rules when I drop the partial adjustment assumption, so that the policy rule can be written $i_t = \rho_i i_t + \phi_\pi(\pi_t - \pi^*) + \phi_x(x_t - x^*) + \phi_{\Delta_y}(\Delta y_t - \ln g_y)$. The value of ρ_i is only restricted to be non-negative; in principle, I allow it to be one or greater. In the baseline case, the optimized policy rule features a strong response to inflation, no

[15] Note that this form of indexation is quite different than indexation to trend inflation. Either indexation scheme will render trend inflation largely irrelevant (at least for the first order dynamics, which is why I do not present results for different levels of trend inflation), but indexation to trend inflation would not have as dramatic effects as indexation to lagged inflation does. This is because full indexation to lagged inflation almost completely eliminates price dispersion outside of the steady state altogether, where indexation to trend inflation only eliminates price dispersion in the steady state.

[16] With these indexation values and the other parameters at their benchmark values, the welfare-optimizing policy parameters are $\phi_\pi = 3.00$, $\phi_x = 0.00$, and $\phi_{\Delta_y} = 1.77$, with the minimized loss function equal to 1.62 percent of steady state consumption, lower than the 1.7 percent in the baseline analysis with no indexation.

response to the gap, a strong response to output growth, and an intermediate amount of smoothing, though significantly less than one. The optimized rule achieves a better welfare outcome than the partial adjustment specification, with a welfare loss of 1.55 percent of consumption as opposed to 1.7 percent, and the desired coefficient on output growth is larger. The better welfare outcome obtains because now the "long run" response of interest rates to target variables is increasing in ρ_i.

The remaining rows of Table 8 consider parameter restrictions on the non-partial adjustment rule specification. As earlier, requiring a positive response to the output gap results in a substantial welfare loss, and a larger desired response to output growth. Interestingly, restricting the coefficient on inflation to be significantly smaller than its optimal value results in a small welfare loss, differently than in the partial adjustment specification. Requiring no response to output growth leads to a larger welfare loss relative to the optimized rule than in the partial adjustment specification. Unlike the model without capital, "difference" and "super-inertial" policy rules are not welfare optimal in the medium scale model. Requiring that $\rho_i \geq 1$, the optimized coefficient on ρ_i is exactly 1, and achieves a slightly larger welfare loss than when $\rho_i < 1$. It is again optimal to respond to output growth and not to respond to the output gap. Restricting there to be no response to output growth results in an even larger welfare loss than when $\rho_i < 1$ (loss of about 0.3 percent of consumption, as opposed to 0.08 percent in the non-super-inertial specification).

Next, I qualitatively discuss how the optimized parameter values change with different values of other parameters in the model. Except in extreme cases already discussed, it is optimal to have a large response to inflation and no reaction to the gap. The optimal response to output growth is decreasing in ε_p and increasing in ε_w, which control steady state price- and wage-markups, respectively. The coefficient on growth is also increasing in ξ_p and ξ_w, which govern the amount of nominal rigidity in the economy. The amount of wage rigidity has much larger effects on welfare than does price rigidity. The desired response to growth is increasing in β. The intuition for this is that a higher discount factor (lower discount rate) makes forward-looking behavior more important. The desired response coefficient on growth is increasing in η, which is equal to the inverse Frisch labor supply elasticity. Finally, the optimal coefficient on output growth is decreasing in b and κ. Both of these parameters (habit formation and investment adjustment costs, respectively) induce real inertia. Real inertia (in the form of hump-shaped real responses) reduces the "anchoring" gains from responding to growth – the more output growth is positively autocorrelated, the less future policy will differ from current policy by reacting positively to growth, and hence the smaller will be the "anchoring" gains from reacting to growth.

Finally, I consider the nature of the welfare measure by which a central bank's policy rule is evaluated. In my baseline analysis, I assume that the central bank seeks to minimize the expected welfare loss from price rigidity. This is an unconditional metric. One could also consider a conditional metric in which the central bank seeks to minimize welfare losses starting from a particular point in the state space, such as the non-stochastic steady state. In a higher order approximation, unconditional and conditional welfare will not be the same because stochastic means of state variable will differ from their deterministic steady state values. Optimal policy rule parameters which obtain from minimizing conditional welfare are nevertheless very similar to those based on the unconditional measure of welfare. The conditional welfare losses (evaluated at the non-stochastic steady state) are a little smaller than the unconditional ones – for example, in the baseline analysis, the unconditional welfare loss is 1.70 percent of consumption, while the conditional loss is 1.56. This results from the fact that the unconditional metric ignores some transition costs that the unconditional metric implicitly ignores.

5 Conclusion

Simple interest rate rules have become a ubiquitous feature of models of the business cycle with monetary non-neutralities. Though there is substantial agreement that such rules should react strongly to inflation, there appears to be no practical consensus on what measure of economic activity, if any, ought to appear in these rules.

This paper has sought to provide an answer to this unresolved question. Two principle conclusions emerge. First, if "inefficient" shocks to the price and/or wage Phillips curves are important, if trend inflation is positive, and/or if trend productivity growth is positive, then responding to the output gap can lead to substantial welfare losses. Second, moving interest rates in reaction to output growth often has beneficial welfare effects, with the welfare benefits to growth targeting largest under exactly the same circumstances in which gap targeting is most costly. Several authors have previously argued in favor of targeting growth over the gap on the grounds that the gap is difficult to observe in real time (e.g. McCallum, 2001; Orphanides, 2002; and Orphanides and Williams, 2006). The results in this paper suggest that it is likely better to respond to output growth than the gap, even if the latter is observed perfectly. My results about the desirability of gap versus growth targeting in interest rate rules dovetails nicely with a separate strand of the literature, which argues that growth targeting is likely better than focusing on the gap from the perspective of equilibrium determinacy.

The results of this paper are not merely of interest for academic discussions of what kind of rule to write down in a DSGE model. Rather, they are of critical importance for understanding desirable policy responses in the current zero lower bound environment. As shown in Figure 1, while the implied target interest rate under the gap version of the rule is negative, the current desired interest rate under a growth rate policy rule is actually positive. Since the results of this paper suggest that the growth rate rule performs better from a welfare perspective in normal times, it may be that the current Federal Reserve policy of keeping the Federal Funds rate at or near zero may actually be too stimulative.

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Table 1: Benchmark Parameter Values and Model Fit

Basic NK Model

Parameter	Fit
$\sigma = 1$	std($\Delta \ln y_t$)= 0.012
$\eta = 1$	$\text{corr}(\Delta \ln y_t, \pi_t) = -0.045$
$\theta_p = 0.75$	$\text{corr}(\Delta \ln y_t, i_t) = -0.0167$
$\epsilon_p = 10$	$\text{var}(\Delta \ln y_t \mid A_t) = 26.8$
$\rho_a = \rho_\nu = \rho_{up} = 0.95$	$\text{var}(\Delta \ln y_t \mid \nu_t) = 57.1$
$s_a = 0.0075$	$\text{var}(\Delta \ln y_t \mid u_t^p) = 16.1$
$s_{up} = 0.001$	
$s_\nu = 0.015$	
$\beta = 0.99$	

Notes: This table shows the benchmark values of the parameters in the basic NK model without capital. In the right column are some measures of model fit. The entries labeled "var($\Delta \ln y_t, \mid X$)" show the unconditional contribution of each exogenous shock to the variance of output growth. These measures of goodness of fit are generated under a policy rule in which $\rho_i = 0.7$, $\phi_\pi = 1.5$, and $\phi_x = \phi_{\Delta_y} = 0$.

Table 2: Benchmark Parameter Values and Model Fit

Medium Scale Model

Parameter		Fit
$\eta = 1$	$\alpha = 1/3$	std($\Delta \ln y_t$)= 0.011
$b = 0.7$	$\beta = 0.99$	$\text{corr}(\Delta \ln y_t, \Delta \ln c_t) = 0.598$
$\psi = 6.5$	$g_a = 0.002$	$\text{corr}(\Delta \ln y_t, \Delta \ln I_t) = 0.7055$
$\kappa = 3$	$g_z = 0.004$	$\text{corr}(\Delta \ln y_t, \pi_t) = 0.045$
$\delta = 0.025$	$\pi^* = 0.00$	$\text{corr}(\Delta \ln y_t, i_t) =$ -0.029
$\gamma_1 = 0.05$	$s_a = 0.005$	$\text{var}(\Delta \ln y_t \mid A_t) = 15.6$
$\epsilon_p = \epsilon_w = 10$	$s_z = 0.02$	$\text{var}(\Delta \ln y_t \mid Z_t) = 41.1$
$\theta_p = \theta_w = 0.65$	$s_\nu = 0.03$	$\text{var}(\Delta \ln y_t \mid \nu_t) = 21.2$
$\zeta_p = \zeta_w = 0.00$	$s_\psi = 0.02$	$\text{var}(\Delta \ln y_t \mid \psi_t) = 7.3$
$\rho_a = \rho_z = \rho_\nu = \rho_\psi = \rho_{up} = \rho_{uw} = 0.95$	$s_{up} = s_{uw} = 0.05$	$\text{var}(\Delta \ln y_t \mid u_t^p) = 14.1$
		$\text{var}(\Delta \ln y_t \mid u_t^w) = 0.5$

Notes: This table shows the benchmark values of the parameters in the medium scale model. In the right column are some measures of model fit. The entries labeled "var($\Delta \ln y_t \mid X$)" show the unconditional contribution of each exogenous shock to the variance of output growth. These measures of goodness of fit are generated under a policy rule in which $\rho_i = 0.7$, $\phi_\pi = 1.5$, and $\phi_x = \phi_{\Delta_y} = 0$.

Table 3: Optimal Policy Rule Parameters

Basic NK Model

Specification	L	ρ_i	ϕ_π	ϕ_x	ϕ_{Δ_y}
Baseline	0.039	0.576	3.000	0.000	0.110
$\phi_x = 0.50$	0.147	0.796	3.000	n/a	0.106
$\rho_i = 0.00$	0.039	n/a	3.000	0.000	0.141
$\phi_\pi = 1.50$	0.080	0.812	n/a	0.000	0.010
$\phi_{\Delta_y} = 0.00$	0.039	0.632	3.000	0.000	n/a
Shock-Specific					
Productivity	0.001	0.000	3.000	3.000	0.534
Preference	0.001	0.894	3.000	3.000	0.000
Price Markup	0.038	0.000	3.000	0.000	0.567

Note: The upper panel of the table shows optimized parameter values (and optimized values welfare losses) for the basic NK model without capital. Parameter restrictions are indicated in the left column as well as by "n/a" in appropriate columns. The lower panel shows optimized parameter values conditional on each of the three shocks one at a time.

Table 4: Optimal Policy Rule Parameters

Basic NK Model, Robustness

	L	ρ_i	ϕ_π	ϕ_x	ϕ_{Δ_y}
Rule specification					
Super-inertial	0.034	1.069	2.241	0.015	0.047
Difference	0.034	n/a	2.285	0.011	0.051
Backward-looking	0.038	0.476	3.000	0.000	0.087
Forward-looking	0.040	0.286	3.000	0.000	0.138
Arbitrary welfare weight					
$\lambda = 0.05$	n/a	n/a	3.000	0.000	0.239
$\lambda = 0.15$	n/a	n/a	3.000	0.061	0.394
$\lambda = 0.25$	n/a	n/a	3.000	0.129	0.474
$\lambda = 0.50$	n/a	n/a	3.000	0.299	0.578
$\lambda = 1.00$	n/a	n/a	3.000	0.641	0.665

Note: The upper panel of this table shows optimized parameter values (and optimized values welfare losses) for the basic NK model without capital under different versions of the policy rule, described in the left column. The second panel of the table shows optimized policy rule parameters using arbitrary values of the welfare weight on the variance of the output gap, λ. For these exercises the coefficient on the lagged interest rate is restricted to be $\rho_i = 0$, indicated by "n/a" in the appropriate column.

Table 5: Optimal Policy Rule Parameters

Medium Scale Model

	L	ρ_i	ϕ_π	ϕ_x	ϕ_{Δ_y}
Baseline	1.703	0.000	3.000	0.000	0.652
$\phi_x = 0.25$	2.843	0.990	3.000	n/a	3.000
$\rho_i = 0.75$	1.735	n/a	3.000	0.000	1.228
$\phi_\pi = 1.50$	2.345	0.989	n/a	0.011	0.678
$\phi_{\Delta_y} = 0.00$	1.734	0.607	3.000	0.059	n/a
Shock Specific					
Neutral Productivity	0.998	0.000	3.000	0.000	0.832
Investment	1.191	0.000	3.000	0.000	2.572
Intertemporal preference	0.995	0.000	3.000	0.000	0.000
Intratemporal preference	1.025	0.000	3.000	0.071	0.000
Price markup	1.062	0.000	3.000	0.000	2.571
Wage markup	1.056	0.000	2.311	0.083	0.000

Note: This first panel of this table shows optimized parameter values (and optimized values welfare losses) for the medium scale model. Parameter restrictions are also indicated in that column and by "n/a" in appropriate columns. The second panel, labeled "Shock specific," shows optimized parameter values (and optimized values welfare losses) for the medium scale model, conditional on one shock at a time. In particular, in each row, the standard deviations of the five non-listed shocks are set to zero.

Table 6: Optimal Policy Rule Parameters

Robustness Medium Scale Model

Specification	L	ρ_i	ϕ_π	ϕ_x	ϕ_{Δ_y}
No trend growth	0.391	0.109	3.000	0.037	0.001
$\phi_x = 0.25$	0.519	0.000	3.000	n/a	0.268
$\rho_i = 0.75$	0.400	n/a	3.000	0.085	0.465
$\phi_\pi = 1.50$	0.618	0.000	n/a	0.000	0.520
$\phi_{\Delta_y} = 0.50$	0.402	0.000	3.000	0.000	n/a
No markup shocks	1.469	0.000	3.000	0.000	0.407
$\phi_x = 0.25$	1.555	0.000	3.000	n/a	2.459
$\rho_i = 0.75$	1.485	n/a	3.000	0.000	0.954
$\phi_\pi = 1.50$	2.063	0.989	n/a	0.022	0.727
$\phi_{\Delta_y} = 0.00$	1.481	0.000	3.000	0.000	n/a
No trend growth or markup shocks	0.030	0.116	1.010	3.000	0.001
$\phi_x = 0.00$	0.235	0.103	3.000	n/a	0.000
$\rho_i = 0.75$	0.037	n/a	2.204	3.000	0.158
$\phi_\pi = 3.00$	0.039	0.000	n/a	3.000	0.000
$\phi_{\Delta_y} = 0.50$	0.036	0.187	1.526	3.000	n/a
$\pi^* = 2.00$	7.172	0.959	3.000	0.000	2.87
$\phi_x = 0.10$	11.478	0.999	3.000	n/a	3.000
$\rho_i = 0.75$	7.229	n/a	3.000	0.000	1.812
$\phi_\pi = 1.50$	8.089	0.999	n/a	0.000	0.86
$\phi_{\Delta_y} = 0.50$	7.293	0.00	3.000	0.000	n/a
$\pi^* = 2.00$, no trend growth	1.741	0.000	3.000	0.000	0.384
$\phi_x = 0.10$	1.921	0.975	3.000	n/a	2.313
$\rho_i = 0.75$	1.787	n/a	3.000	0.000	1.018
$\phi_\pi = 1.50$	2.513	0.988	n/a	0.000	0.967
$\phi_{\Delta_y} = 0.00$	1.751	0.00	3.000	0.000	n/a
$\pi^* = 2.00$, no markup shocks	6.322	0.961	3.000	0.000	2.761
$\phi_x = 0.10$	7.267	0.999	3.000	n/a	2.453
$\rho_i = 0.75$	6.394	n/a	3.000	0.000	1.719
$\phi_\pi = 1.50$	7.154	0.99	n/a	0.000	0.000
$\phi_{\Delta_y} = 0.25$	6.519	0.00	3.000	0.000	n/a
$\pi^* = 2.00$, no trend growth or markup shocks	1.507	0.000	3.000	0.000	0.216
$\phi_x = 0.10$	1.581	0.979	3.000	n/a	2.273
$\rho_i = 0.75$	1.529	n/a	3.000	0.000	0.741
$\phi_\pi = 1.50$	2.216	0.99	n/a	0.016	1.016
$\phi_{\Delta_y} = 0.25$	1.510	0.00	3.000	0.000	n/a

Note: This table shows optimized parameter values (and optimized welfare losses) for different versions of the medium scale model. Versions of the model are indicated in the left column. Parameter restrictions are also indicated in that column and by "n/a" in appropriate columns.

Table 7: Optimal Policy Rule Parameters

Medium Scale Model, Real and/or Nominal Indexation

Indexation	Specification	L	ρ_i	ϕ_π	ϕ_x	ϕ_{Δ_y}
Wages to productivity:	Baseline	0.412	0.029	3.000	0.044	0.148
	No markup shocks	0.033	0.157	1.010	3.000	0.001
	$\pi^* = 2.00$	1.795	0.000	3.000	0.000	0.665
	$\pi^* = 2.00$, no markup shocks	1.559	0.00	3.000	0.000	0.439
Prices/Wages to inflation:	Baseline	1.366	0.000	2.529	2.696	0.000
	No trend growth	0.254	0.000	2.826	2.014	0.000
	No markup shocks	1.015	0.000	1.01	3.000	0.000
	No trend growth or markup shocks	0.039	0.000	1.010	3.000	0.000
Prices/Wages to inflation &:	Baseline	0.264	0.000	3.000	2.524	0.000
Wages to productivity	No markup shocks	0.042	0.000	1.010	3.000	0.000

Note: This table shows optimal parameter values (and optimized welfare losses) for the medium scale model under different assumptions about price and wage indexation as described in the far left column.

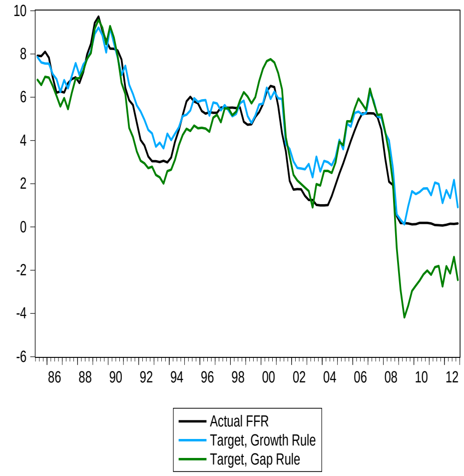
Table 8: Optimal Policy Rule Parameters

Medium Scale Model, Different Rule Specifications

	L	ρ_i	ϕ_π	ϕ_x	ϕ_{Δ_y}
No partial adjustment	1.550	0.629	2.873	0.000	0.949
$\phi_x = 0.25$	2.159	0.990	3.000	n/a	1.335
$\phi_\pi = 1.50$	1.555	0.869	n/a	0.000	0.541
$\phi_{\Delta_y} = 0$	1.637	0.305	3.000	0.000	n/a
Super-inertial	1.557	1.000	1.010	0.000	0.376
$\phi_x = 0.25$	1.914	1.500	3.000	n/a	1.254
$\phi_\pi = 3.00$	1.560	1.000	n/a	0.043	1.102
$\phi_{\Delta_y} = 0$	1.868	1.000	1.010	0.025	n/a

Note: This table is constructed similarly to Table 5, but drops the partial adjustment specification. The first panel restricts $\rho_i \in (0, 1)$, while the second panel, labeled "super-inertial,", restricts $\rho_i \geq 1$.

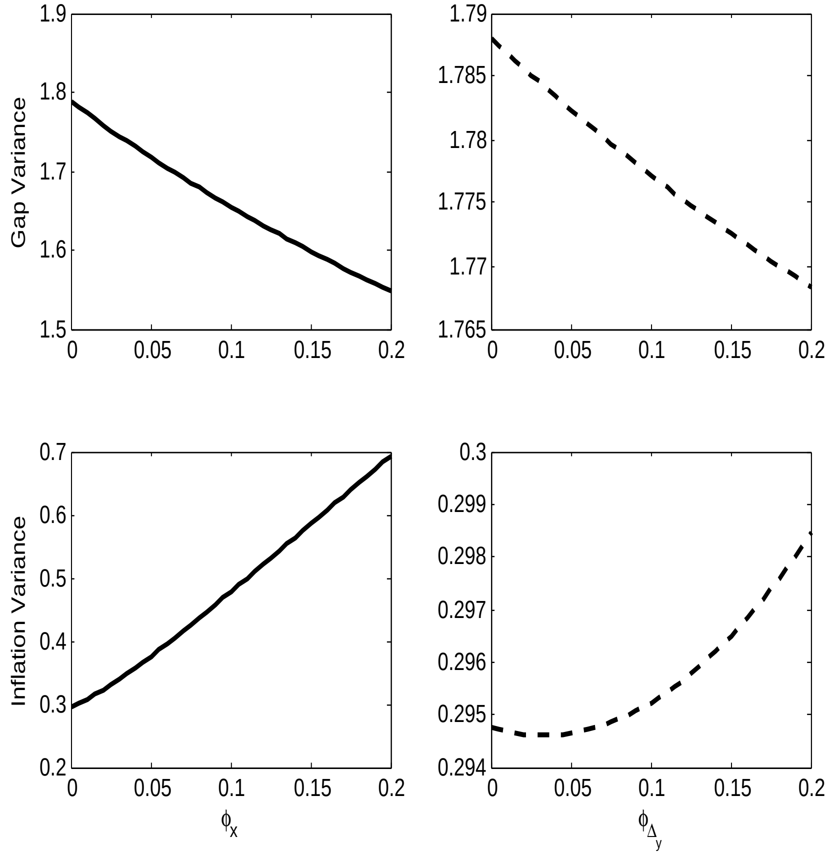
Figure 1: Actual and Target FFR Under Different Hypothetical Rules



Notes: The black line plots the actual effective Federal Funds rate, averaged to a quarterly frequency and expressed at an annualized rate. The green line plots the implied target funds rate under the rule $i_t = (1-\rho_i)\bar{i} + \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi(\pi_t - \bar{\pi}) + \phi_x(x_t - \bar{x})$. π_t is quarter-over-quarter inflation as defined by the GDP price deflator and x_t is the CBO measure of the output gap. I use parameter values $\rho_i = 0.7$, $\phi_\pi = 1.5$, and $\phi_x = 0.5$. Variables with a "bar" denote sample averages over the period 1985-2006. The blue line plots the implied target funds rate using a growth rule, where the $\phi_x(x_t - \bar{x})$ term is replaced by $\phi_{\Delta_y}(\Delta y_t - \Delta\bar{y}_t)$, and I use a value of $\phi_{\Delta_y} = 0.5$.

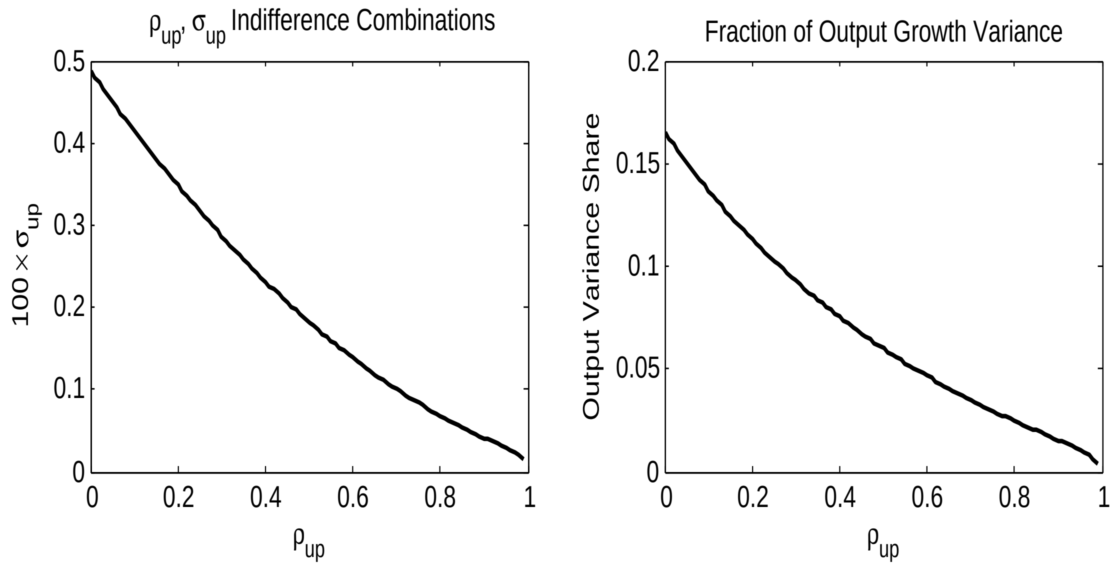
Figure 2: Inflation and Output Gap Variances

Basic NK Model



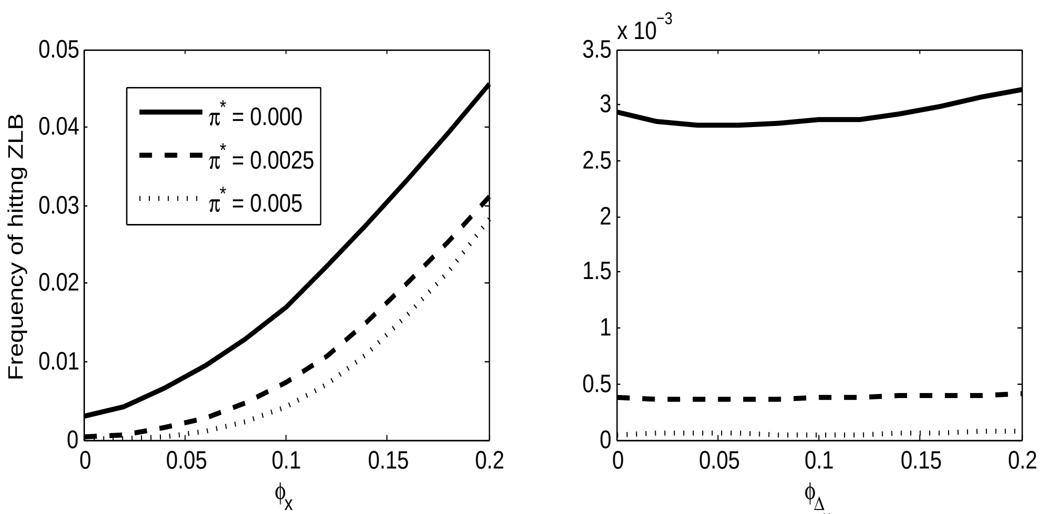
Note: This figure shows how the variance of the output gap (times 100, upper row) and the variance of inflation (times 100, lower row) change as ϕ_x (left column, solid line) and ϕ_{Δ_y} (right column, dashed line) vary relative to a benchmark policy rule with $\rho_i = 0.7$ and $\phi_\pi = 1.5$.

Figure 3: Price Markup Shocks and Responding to Gap
Basic NK Model



Note: The left panel plots the combinations of ρ_{up} and σ_{up} (the persistence and innovation standard deviation of the price markup shock), for which welfare is unaffected by a movement from $\phi_x = 0$ to $\phi_x = 0.05$, relative to a Taylor rule in which $\rho_i = 0.7$, $\phi_\pi = 1.5$, $\phi_x = 0$, and $\phi_{\Delta_y} = 0$. The plot can be thought of as an indifference map – for (ρ_{up}, s_{up}) pars below the curve, welfare is (locally) increasing in ϕ_x; above the curve, welfare is decreasing in ϕ_x. The right panel plots the fraction of the total variance of output growth (standard deviation of 0.012 at baseline parameterization) generated by price markup shocks at the (ρ_{up}, s_{up}) combinations from the left panel.

Figure 4: Incidence of Hitting the Zero Lower Bound
Basic NK Model



Note: This figure plots the frequency of hitting the zero lower bound as a function of the response coefficients on the output gap (left panel) and output growth (right panel) in the basic New Keynesian model without capital. The baseline monetary policy rule is taken to be $\rho_i = 0.7$, $\phi_\pi = 1.5$, and $\phi_x = \phi_{\Delta_y} = 0$. The solid lines show the incidence when trend inflation is zero, the dashed lines when trend inflation is 0.0025 (1 percent annualized), and the dotted line when trend inflation is 0.005 (2 percent annually). The frequencies are generated by simulating 200,000 periods at the baseline parameter values and calculating the frequency of time that the nominal interest rate is less than or equal to zero.

A Appendix

This appendix lists the full set of equilibrium conditions for the medium scale model:

$$\mu_t = \nu_t(C_t - \gamma C_{t-1})^{-1} - \beta\gamma E_t \nu_{t+1}(C_{t+1} - \gamma C_t)^{-1} \tag{A.1}$$

$$R_t = \frac{1}{Z_t}\left(\gamma_0 + \gamma_1(u_t - 1)\right) \tag{A.2}$$

$$1 = q_t Z_t \left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 - \kappa\left(\frac{I_t}{I_{t-1}} - g_I\right)\frac{I_t}{I_{t-1}}\right) + \beta E_t \frac{\mu_{t+1}}{\mu_t} q_{t+1} Z_{t+1} \kappa \left(\frac{I_{t+1}}{I_t} - g_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{A.3}$$

$$q_t = \beta E_t \frac{\mu_{t+1}}{\mu_t}\left(R_{t+1}u_{t+1} - \frac{1}{Z_{t+1}}\left(\gamma_0(u_{t+1} - 1) + \frac{\gamma_1}{2}(u_{t+1} - 1)^2\right) + (1-\delta)q_{t+1}\right) \tag{A.4}$$

$$\mu_t = \beta E_t \mu_{t+1}(1 + i_t)(1 + \pi_{t+1})^{-1} \tag{A.5}$$

$$W_t^{\#,1+\epsilon_{w,t}\eta} = \frac{\epsilon_{w,t}}{\epsilon_{w,t} - 1}\frac{f_{1,t}}{f_{2,t}} \tag{A.6}$$

$$f_{1,t} = \psi_t W_t^{\epsilon_{w,t}(1+\eta)} N_{d,t}^{1+\eta} + \theta_w \beta E_t (1 + \pi_t)^{-\epsilon_{w,t}\zeta_w(1+\eta)}(1 + \pi_{t+1})^{\epsilon_{w,t}(1+\eta)} f_{1,t+1} \tag{A.7}$$

$$f_{2,t} = \mu_t W_t^{\epsilon_{w,t}} N_{d,t} + \theta_w \beta E_t (1 + \pi_t)^{\zeta_w(1-\epsilon_{w,t})}(1 + \pi_{t+1})^{\epsilon_{w,t}-1} f_{2,t+1} \tag{A.8}$$

$$W_t = (1-\alpha) mc_t A_t \left(\frac{K_{s,t}}{N_{d,t}}\right)^{\alpha} \tag{A.9}$$

$$R_t = \alpha mc_t A_t \left(\frac{K_{s,t}}{N_{d,t}}\right)^{\alpha-1} \tag{A.10}$$

$$1 + \pi_t^{\#} = \frac{\epsilon_{p,t}}{\epsilon_{p,t} - 1}(1 + \pi_t)\frac{x_{1,t}}{x_{2,t}} \tag{A.11}$$

$$x_{1,t} = mc_t \mu_t Y_t + \theta_p \beta E_t (1 + \pi_t)^{-\zeta_p \epsilon_{p,t}}(1 + \pi_{t+1})^{\epsilon_{p,t}} x_{1,t+1} \tag{A.12}$$

$$x_{2,t} = \mu_t Y_t + \theta_p \beta E_t (1 + \pi_t)^{\zeta_p(1-\epsilon_{p,t})}(1 + \pi_{t+1})^{\epsilon_{p,t}-1} x_{2,t+1} \tag{A.13}$$

$$i_t = (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi(\pi_t - \pi^*) + \phi_x(\ln Y_t - \ln Y_t^e) + \phi_y(\ln Y_t - \ln Y_{t-1} - \ln g_y)\right) \tag{A.14}$$

$$1 + \pi_t = \left((1-\theta_p)(1 + \pi_t^{\#})^{1-\epsilon_{p,t}} + \theta_p(1 + \pi_{t-1})^{\zeta_p(1-\epsilon_{p,t})}\right)^{\frac{1}{1-\epsilon_{p,t}}} \tag{A.15}$$

$$W_t = \left((1-\theta_w) w_t^{\#,1-\epsilon_w} + \theta_w W_{t-1}^{1-\epsilon_{w,t}}(1 + \pi_t)^{\epsilon_{w,t}-1}(1 + \pi_{t-1})^{\zeta_w(1-\epsilon_{w,t})}\right)^{\frac{1}{1-\epsilon_{w,t}}} \tag{A.16}$$

$$v_t^p Y_t = A_t(u_t K_t)^{\alpha} N_{d,t}^{1-\alpha} - g_y F \tag{A.17}$$

$$Y_t = C_t + I_t + \left(\gamma_0(u_t - 1) + \frac{\gamma_1}{2}(u_t - 1)^2\right)\frac{K_t}{Z_t} \tag{A.18}$$

$$v_t^p = (1 + \pi_t)^{\epsilon_{p,t}}\left((1-\theta_p)(1 + \pi_t^{\#})^{-\epsilon_{p,t}} + \theta_p(1 + \pi_{t-1})^{-\epsilon_{p,t}\zeta_p} v_{t-1}^p\right) \tag{A.19}$$

$$v_t^w = (1-\theta_w)\left(\frac{W_t^{\#}}{W_t}\right)^{-\epsilon_{w,t}(1+\eta)} + \theta_w\left(\frac{W_{t-1}}{W_t}\right)^{-\epsilon_{w,t}(1+\eta)}\left(\frac{(1 + \pi_{t-1})^{\zeta_w}}{1 + \pi_t}\right)^{-\epsilon_{w,t}(1+\eta)} v_{t-1}^w \tag{A.20}$$

$$K_{t+1} = Z_t \left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t + (1-\delta)K_t \tag{A.21}$$

$$K_{s,t} = u_t K_t \tag{A.22}$$

$$\mathbb{W}_t = \nu_t \ln(C_t - bC_{t-1}) - \psi_t v_t^w \frac{N_{d,t}^{1+\eta}}{1+\eta} + \beta E_t \mathbb{W}_{t+1} \tag{A.23}$$

$$\ln \widetilde{A}_t = \rho_a \ln \widetilde{A}_{t-1} + s_a e_{a,t} \tag{A.24}$$

$$\ln \widetilde{Z}_t = \rho_z \ln \widetilde{Z}_{t-1} + s_z e_{z,t} \tag{A.25}$$

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + s_\nu e_{\nu,t} \tag{A.26}$$

$$\ln \psi_t = (1-\rho_\psi)\ln \psi + \rho_\psi \ln \psi_{t-1} + s_\psi e_{\psi,t} \tag{A.27}$$

$$\ln \epsilon_{w,t} = (1-\rho_w)\ln \epsilon_w + \rho_w \ln \epsilon_{w,t-1} + s_w e_{w,t}, \ \ \epsilon_w > 1 \tag{A.28}$$

$$\ln \epsilon_{p,t} = (1-\rho_p)\ln \epsilon_p + \rho_p \ln \epsilon_{p,t-1} + s_p e_{p,t}, \ \ \epsilon_p > 1 \tag{A.29}$$

Expressions (A.1)-(A.5) are household optimality conditions for non-labor choices. (A.6)-(A.8) are the optimality conditions related to wage-setting, expressed in recursive form. (A.9)-(A.10) are the optimality conditions for cost-minimization by firms. (A.11)-(A.13) are the first order conditions related to optimal price-setting, also expressed in recursive form. (A.14) is the policy rule. (A.15) describes the evolution of prices, while (A.16) shows the evolution of real wages. (A.17) is the aggregate production function, while (A.18) is the aggregate resource constraint. (A.19) shows the evolution of price dispersion and (A.20) the evolution of wage dispersion. The capital accumulation equation is given by (A.21). Expression (A.22) defines capital services as the product of utilization and physical capital. (A.23) is the recursive representation of aggregate welfare. (A.24)-(A.29) are laws of motion for exogenous states. μ_t is the marginal utility of consumption (i.e. the Lagrange multiplier on the budget constraint), and q_t is the ratio of the Lagrange multiplier on the capital accumulation equation to the multiplier on the budget constraint.

As noted in the text, most variables can be stationarized under the transformation $\widetilde{X}_t = \frac{X_t}{\Upsilon_t}$, where $\Upsilon_t = (A_t^\tau)^{\frac{1}{1-\alpha}} (Z_t^\tau)^{\frac{\alpha}{1-\alpha}}$. Exceptions include the capital stock, for which $\widetilde{K}_t = \frac{K_t}{\Upsilon_t Z_t^\tau}$, $\widetilde{R}_t = R_t Z_t$, $\widetilde{q}_t = q_t Z_t$, and $\widetilde{\mu}_t = \mu_t \Upsilon_t$ are stationary. Given assumptions on preferences, labor input is stationary along any balanced growth path. Provided $\beta g_\Upsilon < 1$, welfare will be stationary in the non-stationary variables.

The welfare of an individual household in the model can be written recursively (omitting explicit dependence on the state):

$$V_t(h) = \nu_t \ln\left(C_t(h) - bC_{t-1}(h)\right) - \psi_t \frac{N_t(h)^{1+\eta}}{1+\eta} + \beta E_t V_{t+1}(h)$$

Define the aggregate welfare function as the sum of household welfare:

$$\mathbb{W}_t = \int_0^1 V_t(h)dh \tag{A.30}$$

Using the fact that consumption will be equal across households, this can be written:

$$\mathbb{W}_t = \nu_t \ln\left(C_t - bC_{t-1}\right) - \frac{\psi_t}{1+\eta} \int_0^1 N_t(h)^{1+\eta} dh + \beta E_t \mathbb{W}_{t+1}$$

Using the expression for labor demand (10), this can be written without the integral as:

$$\mathbb{W}_t = \nu_t \ln(C_t - bC_{t-1}) - \psi_t v_t^w \frac{N_{d,t}^{1+\eta}}{1+\eta} + \beta E_t \mathbb{W}_{t+1} \tag{A.31}$$

In terms of detrended consumption, $\widetilde{C}_t = \frac{C_t}{\Upsilon_t}$, this can be written:

$$\mathbb{W}_t = \nu_t \ln(\widetilde{C}_t - bg_{\Upsilon}^{-1}\widetilde{C}_{t-1}) + \nu_t \ln \Upsilon_t - \psi_t v_t^w \frac{N_{d,t}^{1+\eta}}{1+\eta} + \beta E_t \mathbb{W}_{t+1} \tag{A.32}$$

Where:

$$\Upsilon_t = (A_t^{\tau})^{\frac{1}{1-\alpha}} (Z_t^{\tau})^{\frac{\alpha}{1-\alpha}}$$
$$g_{\Upsilon} = \frac{\Upsilon_t}{\Upsilon_{t-1}} = (1+g_a)^{\frac{1}{1-\alpha}} (1+g_z)^{\frac{\alpha}{1-\alpha}}$$

Picking policy rule parameters involves maximizing the expected value of (A.32), or, in the case of the conditional metric, maximizing it conditional on a particular realization of the state vector. To present consumption equivalent metrics, consider a hypothetical economy in which prices are flexible and markups are constant at their steady state value. Denote this economy with super-script f:

$$\mathbb{W}_t^f = \nu_t \ln(\widetilde{C}_t^f - bg_{\Upsilon}^{-1}\widetilde{C}_{t-1}^f) + \nu_t \ln \Upsilon_t - \psi_t v_t^w \frac{N_{d,t}^{f,1+\eta}}{1+\eta} + \beta E_t \mathbb{W}_{t+1}^f \tag{A.33}$$

Let ω_c denote the fraction of consumption a household would need each period in the sticky price economy to have the same welfare as in the hypothetical flexible price economy. One can show that it solves:

$$\frac{1}{1-\beta} \ln(1+\omega_c) + \mathbb{W}_t = \mathbb{W}_t^f$$

Hence:

$$\omega_c = \exp\Big((1-\beta)(\mathbb{W}_t^f - \mathbb{W}_t)\Big) \tag{A.34}$$

The unconditional metric, the baseline used in the text, evaluates the value functions on the right hand side at their unconditional means. A conditional metric evaluates each value function at a particular realization of the state. The numbers in the table are multiplied by 100 and multiplied by -1 (even though, since $\mathbb{W}_t^f \geq \mathbb{W}_t > 0$, ω_c is positive as defined) to interpret them as a loss.