Problem Set 1

Graduate Macro II, Spring 2024
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due on Canvas by 5:00 pm on January 26.

1. Suppose we have a two-period endowment economy. There is no uncertainty. Time begins in t and ends after $t+1$. The stream of output is exogenously given, Y_t and Y_{t+1}. The representative household has standard preferences, and solve the following problem:

$$\max_{C_t, B_t, C_{t+1}} \quad \ln C_t + \beta \ln C_{t+1}$$

s.t.

$$C_t + B_t \leq Y_t - T_t$$
$$C_{t+1} \leq Y_{t+1} - T_{t+1} + (1+r_t)B_t$$

T_t and T_{t+1} are (lump-sum) taxes that the household must pay to a government. The government consumes an exogenous amount of output each period, G_t and G_{t+1}, financing this expenditure by running a balanced budget each period, $T_t = G_t$ and $T_{t+1} = G_{t+1}$.

(a) Derive the consumption Euler equation for the representative household.

(b) Using your Euler equations plus the budget constraints (holding with equality), derive a consumption function: i.e. an expression for C_t as a function of things the household takes as given.

(c) The market-clearing condition is that $B_t = 0$. Use this, in conjunction with the government's balanced budget condition, to derive the aggregate resource constraints for both t and $t+1$.

(d) Suppose that there is a one-period increase in G_t by a small amount. What is the "consumption multiplier" (i.e. dC_t/dG_t)? How does the equilibrium real interest rate react to an increase in G_t (i.e. does it go up, or does it go down)?

(e) Suppose, instead, that there is a two-period increase in government spending by a small amount (i.e. both G_t and G_{t+1} go up by the same amount. What is the "consumption multiplier" (i.e. dC_t/dG_t)? How does the equilibrium real interest rate react to a to two-period increase in G_t and G_{t+1} (i.e. does it go up, or does it go down)?

(f) Describe the intuition for why your answers on (d) and (e) do (or do not) differ.

2. Consider a deterministic endowment economy with two types of agents. Time lasts for two periods, t and $t+1$. There is no uncertainty. Type A agents are "rich" – they have an endowment stream of $Y_t(A) = Y_{t+1}(A) = 3/2$. Type B agents are "poor" – they have an endowment stream of $Y_t(B) = Y_{t+1}(B) = 1/2$. Assume that there is a unit mass of total

households in the economy, with type A being exactly 1/2 of that mass and type B also being 1/2 that mass. Either type of household, $i = A, B$, has preferences and faces budget constraints:

$$\mathbb{U}(i) = \ln C_t(i) + \beta \ln C_{t+1}(i)$$
$$C_t(i) + B_t(i) \leq Y_t(i)$$
$$C_{t+1}(i) \leq Y_{t+1}(i) + (1 + r_t)B_t(i)$$

The aggregate resource constraint is that $B_t(A) = -B_t(B)$.

(a) Given the endowment streams above, derive exact equilibrium expressions for $C_t(A)$, $B_t(A)$, $C_t(B)$, $B_t(B)$, and $1 + r_t$.

(b) Given this equilibrium allocation, provide an analytic expression for lifetime utility of each type of agent, $\mathbb{U}(A)$ and $\mathbb{U}(B)$.

(c) Show that the equilibrium real interest rate, $1 + r_t$, in the two-agent economy described above is identical to the equilibrium real interest rate in a representative agent version of the economy in which the endowment stream is $Y_t = Y_{t+1} = 1$.

(d) Suppose that there exists a government that can tax and/or transfer resources across agents in both periods. The budget constraints in both periods for either agent become:

$$C_t(i) + B_t(i) \leq Y_t(i) - T_t(i)$$
$$C_{t+1}(i) \leq Y_{t+1}(i) + (1 + r_t)B_t(i) - T_{t+1}(i)$$

$T_t(A)$ and $T_t(B)$ are taxes (positive) or transfers (negative values). The government must balance its budget each period, so $T_t(A) + T_t(B) = 0$ and $T_{t+1}(A) + T_{t+1}(B) = 0$. Argue that the equilibrium real interest rate, $1 + r_t$, is the same for any possible combination of taxes/transfers.

(e) Solve for the values of $T_t(A)$, $T_t(B)$, $T_{t+1}(A)$, and $T_{t+1}(B)$ that would maximize a utilitarian social welfare function, $\mathbb{U} = \mathbb{U}(A) + \mathbb{U}(B)$.

3. Consider a representative agent endowment economy that lasts for two periods. The future endowment is uncertain. There are two states of nature, 1 and 2. $Y_{t+1}(1) = 5/4$ with probability 1/2 and $Y_{t+1}(2) = 3/4$ with probability 1/2. The problem facing the representative household is:

$$\max_{C_t, B_t, C_{t+1}(1), C_{t+1}(2)} \quad \mathbb{E}_t \mathbb{U} = u(C_t) + \beta \left[\frac{1}{2} u(C_{t+1}(1)) + \frac{1}{2} u(C_{t+1}(2)) \right]$$

s.t.

$$C_t + B_t \leq Y_t$$
$$C_{t+1}(1) \leq Y_{t+1}(1) + (1 + r_t)B_t$$
$$C_{t+1}(2) \leq Y_{t+1}(2) + (1 + r_t)B_t$$

The market-clearing condition is $B_t = 0$.

(a) Derive the Euler equation characterizing an optimal consumption plan.

(b) Suppose that the utility function is quadratic, taking the following form:

$$u(C_t) = C_t - \frac{1}{4}C_t^2$$

With this utility function, solve for the equilibrium real interest rate, $1 + r_t$, and the expected value of lifetime utility, $\mathbb{E}_t\mathbb{U}$. When solving for the expected value of lifetime utility, to make your life easy, you may assume $\beta = 0.95$ (both here and in subsequent parts where it asks for lifetime utility).

(c) Suppose that $Y_t = 1$. Suppose that there is a mean-preserving spread on the future endowment, so that $Y_{t+1}(1) = 4/3$ and $Y_{t+1}(2) = 2/3$. Solve for the equilibrium real interest rate, $1 + r_r$, and the expected value of lifetime utility, $\mathbb{E}_t\mathbb{U}$.

(d) Instead, suppose that the utility function is the natural log:

$$u(C_t) = \ln C_t$$

Re-do (b) and (c) with this utility function, and comment on how your answers do (or do not) qualitatively differ.

4. Suppose that we have an endowment economy that lasts forever. There exists a representative household who can spend its endowment on non-durable consumption, C_t, or it can accumulate durable goods, D_t. The household receives a utility flow from its stock of durables. Durable goods depreciate at rate $0 < \delta < 1$. The household can also save via a one-period bond, B_t. The household's problem is:

$$\max_{C_t, C_t^D, B_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \theta \ln D_t\right]$$

s.t.

$$C_t + C_t^D + B_t \leq Y_t + (1 + r_{t-1})B_{t-1}$$
$$D_{t+1} = C_t^D + (1 - \delta)D_t$$

The second constraint is the law of motion for durables. Consumption on new durables does not impact the available stock of durables until the next period.

(a) Derive the first order conditions for the household problem.

(b) Re-write the first order conditions in such a way that you can interpret them both as asset pricing conditions (for bonds and durable goods). Verbally walk through what the re-written FOC say.

(c) The market-clearing condition is that $B_t = 0$. Assume that the non-stochastic steady state value of the endowment is Y. Derive an expression for θ that would be consistent with durable consumption being 20 percent of the endowment in steady state.