Graduate Macro Theory II: Fiscal Policy in the RBC Model

Eric Sims
University of Notre Dame

Spring 2024

1 Introduction

This set of notes studies fiscal policy in the RBC model. Fiscal policy refers to government spending and finance. Government spending is a component of aggregate expenditure; we could model it as potentially solving some public goods provision problem or as being productive (by productive I mean that government spending helps firms produce output one way or another). In these notes, we will do neither and assume that government spending is pure consumption for the government. We allow for the possibility that the representative household gets utility from government spending, which is a reduced-form way to model the "usefulness" of government spending (which is to provide public goods that would be under-provided by private markets).

In our setup the government will choose spending exogenously; we could also model a Ramsey-type problem where spending is chosen to maximize social welfare. The government has a flow budget constraint that must hold and finances itself with some mixture of debt and taxes; in the long run, debt cannot explode, so debt today necessitates raising tax revenue at some point in the future. We consider both "lump-sum" taxes (the amount of tax an agent pays is independent of any choices it makes), as well as "distortionary" taxes that are tax rates on income or spending. We call these "distortionary" because they distorte inter- or intra-temporal first order conditions that would emerge in the solution to a planner's problem when the planner has access to lump sum taxes.

2 Lump-Sum Finance

We begin by assuming that all government revenue is raised through lump-sum taxes. While unrealistic, this is a good starting point. Among other things, it means that the competitive equilibrium can be supported as the solution to a social planner's problem where the amount of spending is exogenously given. We will analyze the effects of changes in government spending and talk about an important concept called Ricardian Equivalence, which says that, if taxes are lump sum, the mix between debt and tax finance is irrelevant.

Below I set up the problems of the different agents and then discuss equilibrium. I assume that the household owns the capital stock and leases it to firms.

2.1 Household

There is a representative household that solves a standard problem, owning capital and leasing it to firms. Its problem is:

$$\max_{C_t, N_t, K_{t+1}, B_{t+1}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} + h(G_t) \right)$$

s.t.

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} - B_t \leq w_t N_t + R_t^k K_t + \Pi_t - T_t + r_{t-1} B_t$$

The household can consume and supplies labor; it can save through capital or bonds. It receives profit from firms, Π_t, and potentially pays lum-sum taxes to the government, T_t. It takes both of these as given. We call T_t a "lump-sum" tax because the amount the household pays is independent of anything the household does – it pays T_t whether it works a lot or a little, etc. In the setup of the problem I suppose that the household may get some utility from government spending, G_t, through $h(G_t)$. The household takes G_t as given. As long as the utility from government spending is additively separable in the utility function, it will not affect household choices or the equilibrium dynamics. I put it there simply as a reduced-form way to justify why the government might choose to do any government spending in the first place.

The first order conditions for the household problem are standard:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \left(R_{t+1}^k + (1-\delta) \right) \right] \tag{1}$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{2}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} w_t \tag{3}$$

The above are the FOC for capital, bonds, and labor, and are entirely standard.

2.2 Firm

There is a representation firm that solves the standard static firm problem:

$$\max_{N_t, K_t} \quad \Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t^k K_t$$

The FOC are to equate factor prices with marginal products, again entirely standard:

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{4}$$

$$R_t^k = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{5}$$

2.3 Government

The government chooses spending, G_t, exogenously. It finances spending with the aforementioned lump-sum taxes, T_t, and by issuing new debt, D_{t+1}. The government inherits an existing stock of debt, D_t, from history. We will impose in the equilibrium that any debt issued by the government is held by the household. We do not model the government's problem, and as we will see below the exact mix between taxes and debt is both indeterminate and irrelevant for the equilibrium dynamics. All that we require is that the government's debt not explode.

The government's flow budget constraint is:

$$G_t + r_{t-1} D_t \leq T_t + D_{t+1} - D_t \tag{6}$$

Government spending plus interest payments on existing debt (the left-hand side) cannot exceed tax revenue plus new debt issuance. So if G_t increases in a period, there are two ways to pay for it – increase current taxes, T_t, or issue more debt, $D_{t+1} - D_t$.

2.4 Equilibrium

A competitive equilibrium is a set of prices (r_t, R_t^k, w_t) and allocations $(C_t, K_{t+1}, N_t, B_{t+1}, D_{t+1})$ such that (i) household and firm optimality conditions all hold, (ii) the firm hires all the labor and capital supplied by the household, (iii) the household and firm budget constraints hold with equality, and (iv) household bond-holdings equal government debt issuance in all periods (i.e. $B_{t+1} = D_{t+1}$, and we require that $B_t = D_t$ initially), given values and stochastic processes of G_t and A_t, as well as initial values of government debt and household bond-holdings, which must be equal (e.g. $B_t = D_t$).

The government budget constraint binding with equality means that:

$$T_t = G_t + (1 + r_{t-1}) D_t - D_{t+1}$$

Plug this, along with the definition of profit, into the household budget constraint at equality:

$$C_t + K_{t+1} - (1-\delta) K_t = A_t K_t^{\alpha} N_t^{1-\alpha} - G_t - (1 + r_{t-1}) D_t + D_{t+1} + (1 + r_{t-1}) B_t - B_{t+1}$$

Using the fact that $D_t = B_t$ and $D_{t+1} = B_{t+1}$, we have:

$$C_t + K_{t+1} - (1-\delta) K_t + G_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{7}$$

Defining $I_t = K_{t+1} - (1-\delta) K_t$ and $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$, we can summarize the equilibrium as:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(R_{t+1}^k + (1-\delta) \right) \right] \tag{8}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{9}$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{10}$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{11}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} w_t \tag{12}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{13}$$

$$Y_t = C_t + I_t + G_t \tag{14}$$

$$K_{t+1} = I_t + (1-\delta) K_t \tag{15}$$

I close the model with exogenous stochastic processes for government spending and productivity:

$$\ln G_t = (1-\rho_G) \ln(\omega Y) + \rho_G \ln G_{t-1} + s_G \varepsilon_{g,t} \tag{16}$$

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{a,t} \tag{17}$$

Where $\varepsilon_{a,t}$ and $\varepsilon_{g,t}$ are drawn from standard normal distributions. In writing these exogenous processes, I am assuming that steady-state government spending is a fraction, ω, of steady-state output, Y. I am also assuming that the non-stochastic steady state value of productivity is $A = 1$. I didn't write them down, but I am implicitly assuming non-explosion (i.e. the household doesn't over- or under-save, and the government's debt cannot explode). More on this in a minute.

The above, (8)-(17) is 10 equations with 10 variables: $\{C_t, N_t, Y_t, I_t, K_t, G_t, A_t, R_t^k, w_t, r_t\}$. Note that D_t,B_t, and T_t *do not appear*. We do not need to know the values of government debt and taxes, as long as we know spending, G_t, in order to solve the model. This is the basic gist of an important concept called *Ricardian Equivalence*.

3 Ricardian Equivalence

The basic idea of Ricardian Equivalence is this: the method of government budget finance (the mix between taxes and debt) is *irrelevant* conditional on a time path of government spending, G_t. This will hold under conditions that are specified here: there exists an infinitely-lived representative agent and all taxes are lump sum.

We see the implications of Ricardian Equivalence in the above representation of equilibrium. Looking at (8)-(17), the *only* thing that is different relative to a standard RBC model is the presence of government spending, which shows up additively in the aggregate resource constraint and which follows an exogenous stochastic process. Debt and taxes are indeterminate – we don't need to know them to solve the model, and different dynamic combinations of them would be consistent with the same equilibrium outcome.

To see all this more concretely, we can add an equation to (8)-(17): the government's flow budget constraint:

$$G_t + r_{t-1} D_t = T_t + D_{t+1} - D_t \tag{18}$$

The model, as written, would be indeterminate: we would have 11 equations but 12 variables (we'd be adding D_t and T_t). We could, for example, suppose that the government issues no debt, so $D_t = 0$ at all times. This would require that $T_t = G_t$ (i.e. the government balances its budget each period). But we could also have non-zero debt and the outcome would be the same: D_t could be set according to some rule, and T_t would endogenous adjust to make (18) hold each period. Since D_t and T_t don't appear in (8)-(17), how exactly D_t and T_t are set doesn't not matter for the equilibrium allocations and prices. The one caveat is that we need to impose a no-Ponzi condition for the government – its debt can't explode.

What drives this result is that government bonds are not wealth to the household that holds them. It therefore does not matter how much debt the government issues or does not. What is driving this is that, conditional on a time path for government spending, a non-explosive path for government debt means that current debt must be paid for by future lump-sum taxes.

To see this a bit more clearly, let's make a couple of simplifying assumptions on both the household and government problems. First, let's assume that the real interest rate is constant, $r_{t+j} = r$ for any j (this is, of course, not consistent with general equilibrium, but it makes the exposition to follow a bit cleaner and is not necessary). Second, let's just assume that household income is given by Y_t and there is no investment so we don't have to keep writing factor prices. From the household's perspective, this is like an endowment economy model. Further, let us assume that the initial values of household saving and government debt are both zero, $D_t = B_t = 0$. None of these assumptions are necessary; they just make things cleaner.

The household's flow budget constraint is:

$$C_t + B_{t+1} = Y_t - T_t$$

In period $t+1$, the budget constraint will be:

$$C_{t+1} + B_{t+2} = Y_{t+1} - T_{t+1} + (1+r)B_{t+1}$$

So:

$$B_{t+1} = \frac{1}{1+r} C_{t+1} - \frac{1}{1+r}\left[Y_{t+1} - T_{t+1}\right] + \frac{1}{1+r} B_{t+2}$$

But B_{t+2} will look similarly:

$$B_{t+2} = \frac{1}{1+r} C_{t+2} - \frac{1}{1+r}\left[Y_{t+2} - T_{t+2}\right] + \frac{1}{1+r} B_{t+3}$$

But then plugging this in for the B_{t+1} expression, we have:

$$B_{t+1} = \frac{1}{1+r}C_{t+1} + \left(\frac{1}{1+r}\right)^2 C_{t+2} - \frac{1}{1+r}(Y_{t+1} - T_{t+1}) - \left(\frac{1}{1+r}\right)^2 (Y_{t+2} - T_{t+2}) + \left(\frac{1}{1+r}\right)^2 B_{t+3}$$

If we keep doing this, we are going to end up with the following:

$$B_{t+1} = \sum_{j=1}^{\infty} \frac{C_{t+j}}{(1+r)^j} - \sum_{j=1}^{\infty} \frac{Y_{t+j} - T_{t+j}}{(1+r)^j} + \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1}$$

The transversality condition eliminates the last condition. So we are left with:

$$B_{t+1} = \sum_{j=1}^{\infty} \frac{C_{t+j}}{(1+r)^j} - \sum_{j=1}^{\infty} \frac{Y_{t+j} - T_{t+j}}{(1+r)^j}$$

Plugging this in to the period-t budget constraint, we get the intertemporal budget constraint for the household:

$$\sum_{j=0}^{\infty} \frac{C_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{Y_{t+j}}{(1+r)^j} - \sum_{j=0}^{\infty} \frac{T_{t+j}}{(1+r)^j} \tag{19}$$

In other words, the presented discounted value of consumption must equal the present discounted value of net income. Now go to the government's budget constraint assuming no initial debt:

$$G_t + D_{t+1} = T_{t+1}$$

In $t+1$, this will be:

$$G_{t+1} + D_{t+2} = T_{t+1} + (1+r)D_{t+1}$$

Re-arranging terms:

$$D_{t+1} = \frac{T_{t+1}}{1+r} - \frac{G_{t+1}}{1+r} + \frac{D_{t+2}}{1+r}$$

Solving forward one period, we would have:

$$D_{t+1} = \frac{T_{t+1}}{1+r} + \frac{T_{t+2}}{(1+r)^2} - \frac{G_{t+1}}{1+r} - \frac{G_{t+2}}{(1+r)^2} + \frac{D_{t+3}}{(1+r)^2}$$

If we keep going, we are left with:

$$D_{t+1} = \sum_{j=1}^{\infty} \frac{T_{t+j} - G_{t+j}}{(1+r)^j} + \lim_{T\to\infty} \frac{D_{t+1+T}}{(1+r)^T}$$

This says that the government's new debt issued today, in period t (which is D_{t+1}), equals the present discounted value of primary surpluses (primary surpluses are defined as tax revenue minus spending, not counting any interest payments). Plugging this into the period t government budget constraint to eliminate debt, we have:

$$\sum_{j=0}^{\infty} \frac{G_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{T_{t+j}}{(1+r)^j} \tag{20}$$

In other words, the present discounted value of government spending must equal the present discounted value of taxes. This is the government's intertemporal budget constraint. We can combined (20) with (19):

$$\sum_{j=0}^{\infty} \frac{C_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{Y_{t+j}}{(1+r)^j} - \sum_{j=0}^{\infty} \frac{G_{t+j}}{(1+r)^j} \tag{21}$$

In other words, (21) means that the household will behave as though the government's budget is balanced – what is relevant for the household is the time path of government spending, not the mix between taxes and debt. This follows because (i) taxes are lump sum (i.e. additive), (ii) the household lives forever, and (iii) there are no frictions to the household borrowing or saving.

Here we have the gist of Ricardian Equivalence. All that matters for the equilibrium of the model is the time path of government spending. Taxes and debt are irrelevant.

Intuitively, what is driving this result is the following. If the government issues a lot of debt in the present, taking spending as given, it must pay for this with future lump-sum taxes. These future lump-sum taxes must be equal (in present value) to the tax revenue it would have to raise in the present to avoid issuing any debt. In a present value sense, it does not matter whether tax revenue is raised today or in the future.

4 Equilibrium Effects of Government Spending Shocks

To solve for equilibrium dynamics, we need to first solve for the non-stochastic steady state of the model, noting that $A = 1$ and $\frac{G}{Y} = \omega$, as well as noting that we don't need to worry about T or D. I denote steady state values with the absence of a time subscript. From the Euler equation for capital, we have:

$$R^k = \frac{1}{\beta} - (1-\delta) \tag{22}$$

Combining this with the first order condition for capital demand allows us to solve for the steady state capital-labor ratio:

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta)} \right)^{\frac{1}{1-\alpha}} \tag{23}$$

Note that this is the same expression for the capital-labor ratio that we would get in a model without government spending. We then know that the wage satisfies:

$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha} \tag{24}$$

From the production function, we then have:

$$\frac{Y}{N} = \left(\frac{K}{N}\right)^{\alpha}$$

From the accumulation equation, we know that $I = \delta K$. We can then solve for the steady state consumption-labor ratio from the resource constraint as:

$$\frac{C}{N} = (1-\omega)\left(\frac{K}{N}\right)^{\alpha} - \delta\frac{K}{N}$$

Now, this expression is impacted by the presence of government spending (where I am assuming $G = \omega Y = \omega\left(K/N\right)^{\alpha}$). More government spending (i.e. higher ω), means a lower consumption-labor ratio.

Armed with this expression for the consumption-labor ratio, go to the FOC for labor. Multiply both sides by N, we can write:

$$\theta N^{1+\chi} = \frac{N}{C}(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

Solving:

$$N = \left(\frac{1}{\theta}\frac{(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}}{(1-\omega)\left(\frac{K}{N}\right)^{\alpha} - \delta\frac{K}{N}}\right)^{\frac{1}{1+\chi}} \tag{25}$$

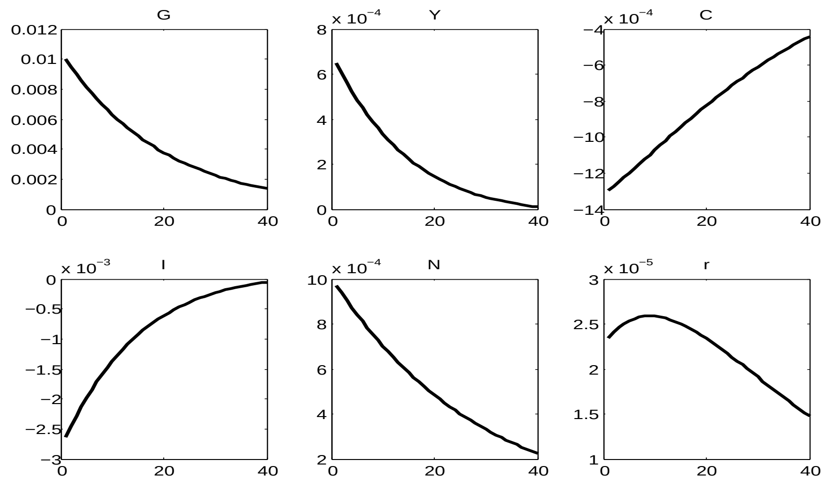
Since $\frac{K}{N}$ is independent of ω, we see here that N is increasing in ω – more government spending in steady state, more N. Since $\frac{K}{N}$ is unaffected by ω but N is increasing in ω, we see that K must be increasing in ω. Since $I = \delta K$, this means that steady state investment is higher the bigger is the government spending share. This is perhaps somewhat non-intuitive, since we often think of government spending "crowding out" private investment, but we can see this is not the case here. The intuition for what is going on is that higher ω makes people feel poorer, so they work more. More N raises the marginal product of capital, which makes it optimal to accumulate more capital, which necessitates more investment.

Since N is higher and $\frac{K}{N}$ is unaffected, we see that Y will be higher when ω is higher. Since $\frac{K}{N}$ is unaffected and N is higher, from the FOC for labor supply we see that C must be lower if ω is bigger.

4.1 Quantitative Simulations

I solve the model via log-linearizing about the non-stochastic steady state. I use the following parameter values: $\alpha = 1/3$, $\beta = 0.99$, $\chi = 1$, $\delta = 0.025$, $\theta = 4$, $\rho_A = 0.97$, $\rho_A = 0.95$, $\omega = 0.2$, and set the standard deviations of both the productivity and government spending shocks to 0.01, or 1 percent.

Below are the impulse responses to a shock to government spending:



Here we see that Y_t and N_t both go up, but I_t and C_t both fall; the real interest rate rises. The government spending multiplier is often defined as $\frac{dY_t}{dG_t}$. We can calculate this from the IRFs by dividing the impact response of output by the impact response of goverment spending, and then multiplying by the inverse of the steady state government spending to output ratio (multiplication by the inverse of this ratio transforms the impact responses in the IRFs, which are percentage deviations from steady state, into actual deviations from steady state). For this setup the multiplier comes out to be 0.3237 – i.e. a one unit increase in government spending raises output by about 0.33 units; output rises by less than G because both C and I fall.[1]

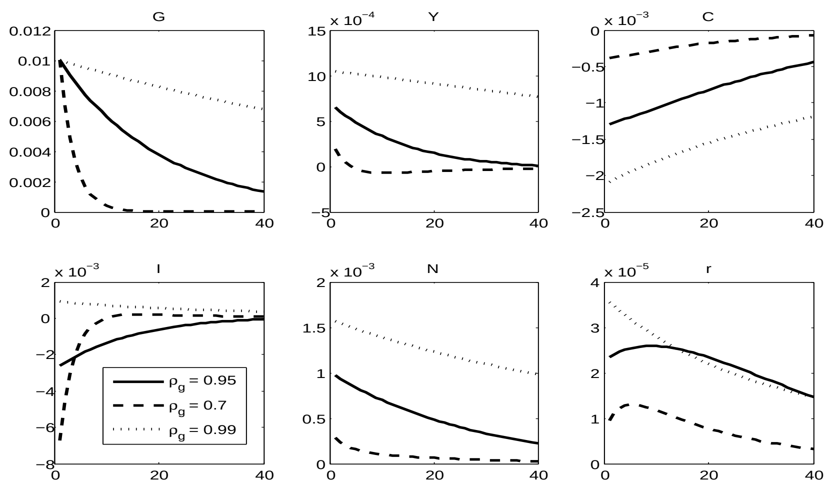
The fall in investment after the government spending shock is not consistent with the steady state analysis, which showed that in the long run investment must be higher if government spending is higher. It is useful to think about the mechanism by which government spending impacts output in this model. When G goes up, the household feels poorer (because it has to pay more in taxes, either now or in the future). This makes it want to consume less and work more. Working more raises output. So the mechanism through which government spending impacts output in this model is not by stimulating "demand," but rather through a wealth effect channel wherein people feel poorer and supply more labor.

Using our logic from phase diagrams, we know that if the change in government spending is very transitory, this is very little wealth effect, and consumption should not change by much. If consumption does not jump by much, then the labor supply curve does not shift by much, so there is little change in equilibrium hours, N_t. But if hours don't change by much, output doesn't change by much (i.e. the multiplier will be lower). From the aggregate resource constraint, if Y_t and C_t don't change by much, mechanically I_t must fall by approximately the increase in G_t. Effectively,

[1]Note you often see the multiplier defined as the sum the output responses divided by the sum of the government spending response (basically the ratio of two integrals under the IRFs), and other times as a present discounted value multiplier (a discounted sum of the of the response of output to the response of government spending).

for a very transitory change, a shock to government spending will just crowd out private investment one-for-one. As the shock to government spending gets more persistent, the wealth effect gets bigger – consumption would jump down by more in a phase diagram, which means labor supply would shift out by more, and hence Y_t would rise by more. But Y_t rising by more, and C_t falling by more in response to the same change in government spending, means that I_t will fall by less. If the change in G_t is permanent, we would expect I_t to actually rise – this is because we know that K must be higher in the new steady state, so investment will have to go up to get us to transition to that new higher steady state. We can infer, then, that there will be some persistence level, ρ_G, where investment will be "crowded in" and increase on impact.

Bottom line: we would expect the persistence of changes in government spending, governed by the magnitude of ρ_G, to have very important effects on the equilibrium response to change in government spending. In particular, the bigger ρ_G is, we'd expect (i) output to rise by more, (ii) consumption to fall by more, (iii) hours to rise by more, and (iv) investment to fall by less, and for some sufficiently high value of ρ_G to actually rise. Below are impulse responses to a change in government spending for three different values of ρ_G:



The responses are exactly in line with what we would expect – Y rises by more, N by more, and C falls by more the more persistent is the change in government spending. I falls by less the more persistent is the change in government spending, and as we see for $\rho_G = 0.99$ investment actually rises.

5 Distortionary Taxes

Instead of assuming that taxes are all lump sum, I now allow for distortionary tax rates on capital and labor income, τ_t^k and τ_t^n. I'll assume that there is a stochastic component to the tax rates, so that we can analyze the equilibrium effects of changes in tax rates. I'll consider two setups: one in which the government can still use lump sum taxes, and other in which there are no lump sum taxes. In the setup with lump sum taxes, the tax rates will show up in the equilibrium conditions, but debt will not again. When I prohibit the use of lump sum taxes, we get more interesting stuff.

5.1 Tax Shocks, Lump Sum Finance

We assume that the government now can finance its spending decisions via issuing debt, lump sum taxes, and distortionary tax rates on labor and capital income on households. The household's budget constraint is now:

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} - B_t \leq (1-\tau_t^n)w_t N_t + (1-\tau_t^k)R_t^k K_t + \Pi_t - T_t + r_{t-1}B_t$$

τ_t^n and τ_t^k are potentially time-varying tax rates on labor and capital income, respectively. The first order conditions for the household problem can now be written:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left((1-\tau_{t+1}^k)R_{t+1}^k + (1-\delta) \right) \right] \tag{26}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1+r_t) \right] \tag{27}$$

$$\theta N_t^{\chi} = \frac{1}{C_t}(1-\tau_t^n)w_t \tag{28}$$

We can see why these tax rates are considered "distortionary" – the labor income tax throws a wedge between the wage the firm pays and the wage the household receives, while the capital tax throws a wedge between the rental rate the firm pays for capital and the rental rate the household receives from leasing capital. The firm problem is unaffected, so the FOC are still to hire capital and labor up until the point at which factor prices equal the marginal product.

The government's budget constraint is now given by:

$$G_t + r_{t-1}D_t \leq \tau_t^k R_t^k K_t + \tau_t^n w_t N_t + T_t + D_{t+1} - D_t \tag{29}$$

Here, the government earns revenue from taxing both capital and labor income. It can still levy lump-sum taxes and issue debt.

In equilibrium, we require that $D_t = B_t$ and $D_{t+1} = B_{t+1}$. Since $\Pi_t = Y_t - w_t N_t - R_t K_t$, the aggregate resource constraint still boils down to:

$$Y_t = C_t + I_t + G_t \tag{30}$$

The equilibrium conditions are then:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left((1-\tau_{t+1}^k)R_{t+1}^k + (1-\delta) \right) \right] \tag{31}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1+r_t) \right] \tag{32}$$

$$w_t = (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha} \tag{33}$$

$$R_t^k = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{34}$$

$$\theta N_t^{\chi} = \frac{1}{C_t}(1-\tau_t^n)w_t \tag{35}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{36}$$

$$Y_t = C_t + I_t + G_t \tag{37}$$

$$K_{t+1} = I_t + (1-\delta)K_t \tag{38}$$

These are the same as we had before, with the exception that the capital tax rate shows up in the Euler equation for capital and the labor tax rate shows up in the first order condition for labor supply. We assume that both G_t and A_t follow the same log AR(1) processes given above. I assume that the tax rates obey stationary AR(1) processes with shocks, with tax rates without a time subscript denoting exogenous steady state values:

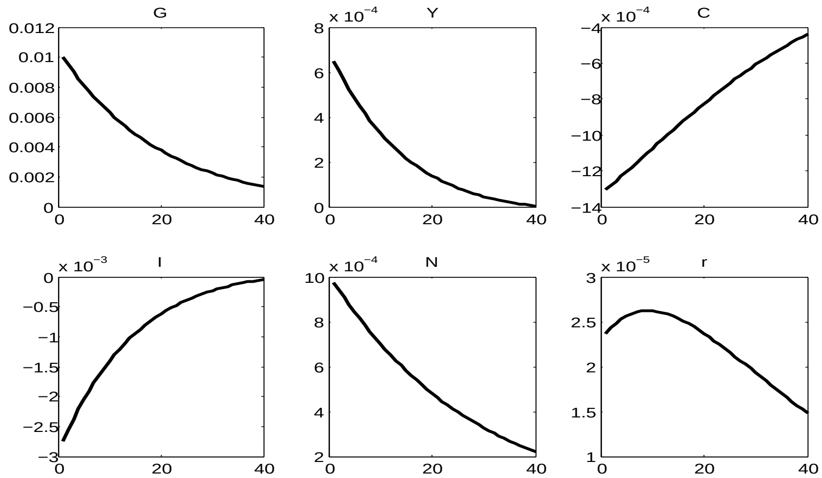
$$\tau_t^k = (1-\rho_k)\tau^k + \rho_k \tau_{t-1}^k + \epsilon_{k,t} \tag{39}$$

$$\tau_t^n = (1-\rho_n)\tau^n + \rho_n \tau_{t-1}^n + \epsilon_{n,t} \tag{40}$$

Even though we have added distortinary taxes, we again here have the result that D_t and T_t do not appear in the equilibrium conditions. It is therefore without loss of generality to assume that the government issues no debt and that T_t simply adjusts so as to make the government budget constraint hold every period. We could alternatively assume some exogenous process for lump sum taxes (with debt adjusting to make the government's flow budget constraint hold), or an exogenous process for debt (with lump-sum taxes adjusting to make the government's flow budget constraint hold). As long as government debt does not explode, the exact nature of these processes will not matter.

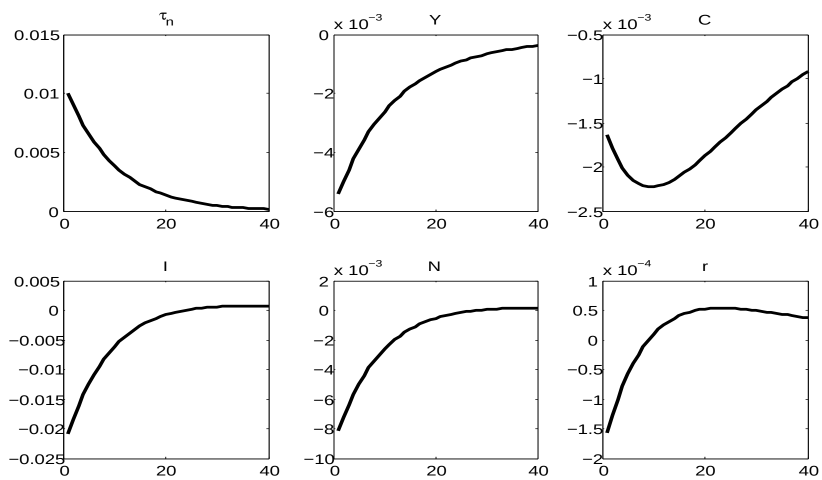
The model is nevertheless different in important ways with the presence of distortionary taxes. For example, since w_t, N_t, R_t, and K_t show up in the government's budget constraint, there is an endogenous reaction of government revenue to a change in government spending – it's not just all coming through an indeterminate mix between lump sum taxes and debt. This means that the impulse responses to a government spending shock will not necessarily be the same here as in the model with lump sum taxes only.

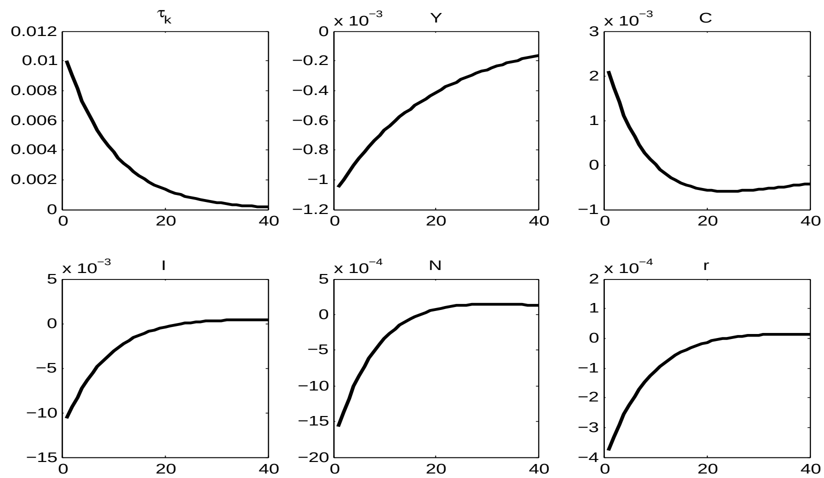
I compute impulse responses to shocks using the parameterization from the previous section (with $\rho_G = 0.95$), and assume that $\rho_n = \rho_k = 0.90$. I also assume that the standard deviations of the two tax shocks are 0.01. I assume that the steady state labor income tax is 0.2 and the steady state capital tax is 0.1. Below are impulse responses to a government spending shock.



Qualitatively, these look similar to what we had before. Y_t rises by roughly the same amount as earlier – the government spending multiplier now 0.3256, or just a little bit higher than what we had before. While not identical to the only lump-sum tax case, the responses to a government spending shock are nevertheless quite similar with distortionary tax rates.

Below are impulse responses to the two tax shocks:





We see that output, hours, consumption, and investment all decline when the tax rate on labor income goes up. People want to work less because they get to keep a smaller fraction of their labor; they want to consume less because the higher tax rate is like a negative shock to their wealth. Investment goes down because the marginal product of capital is lower with less employment. Note that the "Barro-King curse" does not hold with shocks to the labor tax rate. Changes in the labor income tax show up directly in the equilibrium condition for labor (like productivity), so it is possible that consumption and labor can move in the same direction in response to shocks to the labor tax rate.

The responses to the capital tax shock are a little bit different. When the capital tax rate goes up, output goes down, but consumption rises. The higher tax on capital income encourages people to save less (so consume more) via a substitution effect – they substitute away from future consumption (which is current investment) into current consumption. Hence investment goes down. Since there is a smaller incentive to accumulate capital, the household wants to consume more leisure as well – in a mechanical sense, consuming more pushes labor supply in, so N falls and therefore output falls too.

5.2 No Lump Sum Taxes

Now let's continue to assume that there are distortionary tax rates on labor and capital income, but let's not allow the government to use lump sum taxes. Without lump sum taxes, government debt is not going to drop out of the equilibrium conditions. It is going to matter.

The household and firm first order conditions are the same as before. The government's budget constraint without lump sum taxes is now:

$$G_t + r_{t-1} D_t \leq \tau_t^k R_t K_t + \tau_t^n w_t N_t + D_{t+1} - D_t \tag{41}$$

We will still require that $D_t = B_t$ and $D_{t+1} = B_{t+1}$ in equilibrium; hence the aggregate resource constraint will still be $Y_t = C_t + I_t + G_t$. But we will not be able to ignore the government's budget constraint in the equilibrium conditions. The reason we could do so earlier is that either T_t or D_{t+1} would adjust to make it hold; it didn't matter which, and the levels of T_t or D_t didn't affect anything else. This will no longer be the case. To see that, it is easiest to think about the government's budget constraint in the steady state.

Let's suppose that the government sets an exogenous long run (i.e. steady state) target for the debt-GDP ratio. This, combined with the long run level of government spending, will have bearing on the level of tax rates in the long run. Let's evaluate the government budget constraint at equality in the "long run" (e.g. at steady state, which I denote with the absence of a time subscript). Dividing both sides by Y, we have:

$$\frac{G}{Y} = \tau^k \frac{RK}{Y} + \tau^n \frac{wN}{Y} - r\frac{D}{Y}$$

Let's assume that $\frac{G}{Y} = \omega$ and that $\frac{D}{Y}$ is also exogenously given. We know that $r = \frac{1}{\beta} - 1$ from the household's Euler equation for bonds. Given assumptions on the production function plus the fact that factors are paid their marginal products, we know that $\frac{RK}{Y} = \alpha$ and $\frac{wN}{Y} = 1 - \alpha$. Using this we have:

$$\omega + \left(\frac{1}{\beta} - 1\right)\frac{D}{Y} = \alpha\tau^k + (1-\alpha)\tau^n \tag{42}$$

This must hold in the long run; if we make ω and $\frac{D}{Y}$ exogenous parameters, this means that we cannot freely choose the steady state values of both τ^k and τ^n. They must be set such that this constraint holds.

We can also see from this equation why a large debt-GDP ratio is costly. Since $\frac{1}{\beta} - 1$ is positive, a bigger debt-gdp ratio necessitates some combination of higher tax rates – either τ^k or τ^n have to be bigger, or both. These higher tax rates lower the steady state level of output and also lower welfare. Alternatively, if we wanted to fix tax rates, higher debt-gdp would have to be met by a lower share of government spending in output; this may also be undesirable if households get utility from government spending. The bottom line here is that the higher is the steady state debt-gdp ratio, you must have some combination of higher taxes or lower spending for the government's budget constraint to hold. This is also true with lump sum taxes, but if finance comes via lump sum taxes there are no distortions arising in the first order conditions for the household and firm.

We also need to worry about debt and its relation with taxes outside of the steady state. There will in general not exist an equilibrium with a non-explosive path of government debt if we simply assume that government spending and the two tax rates follow exogenous processes. To ensure stability of debt, we need to write down processes which have some degree of tax rates rising in response to debt being too high relative to steady state (or government spending falling in response to debt being higher than steady state). I'm going to assume that this comes in through the tax rates, with government spending following the same exogenous AR(1) process in the log as before. I write down processes for the tax rates as follows:

$$\tau_t^k = (1-\rho_k)\tau^k + \rho_k \tau_{t-1}^k + (1-\rho_k)\gamma_k \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{k,t} \tag{43}$$

$$\tau_t^n = (1-\rho_n)\tau^n + \rho_n \tau_{t-1}^n + (1-\rho_n)\gamma_n \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{n,t} \tag{44}$$

These processes are the same as we had before, except for the coefficients γ_k and γ_n multiplying the deviation of the debt-gdp ratio from steady state. We require that these coefficients be such that debt be non-explosive in equilibrium – this will require that one or both of these coefficients be sufficiently positive so that debt doesn't explode upward (but also not so positive that debt doesn't explode downward). I pre-multiply both these coefficients by one minus the respective AR parameter because this gives these coefficients a "long run" interpretation – if the debt-GDP ratio were to permanently rise by 0.01, then the tax rate would permanently rise by γ_k or γ_n times this rise.

The full set of equilibrium conditions, excluding the exogenous process for A_t and G_t, are:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left((1-\tau_{t+1}^k)R_{t+1}^k + (1-\delta)\right)\right] \tag{45}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}}(1+r_t)\right] \tag{46}$$

$$w_t = (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha} \tag{47}$$

$$R_t^k = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{48}$$

$$\theta N_t^{\chi} = \frac{1}{C_t}(1-\tau_t^n)w_t \tag{49}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{50}$$

$$Y_t = C_t + I_t + G_t \tag{51}$$

$$K_{t+1} = I_t + (1-\delta)K_t \tag{52}$$

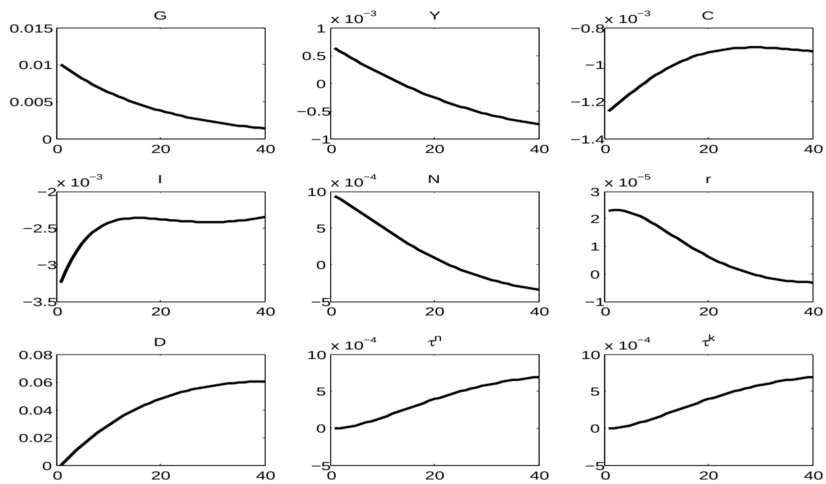
$$G_t + r_{t-1}D_t \leq \tau_t^k R_t^k K_t + \tau_t^n w_t N_t + D_{t+1} - D_t \tag{53}$$

$$\tau_t^k = (1-\rho_k)\tau^k + \rho_k \tau_{t-1}^k + (1-\rho_k)\gamma_k \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{k,t} \tag{54}$$

$$\tau_t^n = (1-\rho_n)\tau^n + \rho_n \tau_{t-1}^n + (1-\rho_n)\gamma_n \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right) + \epsilon_{n,t} \tag{55}$$

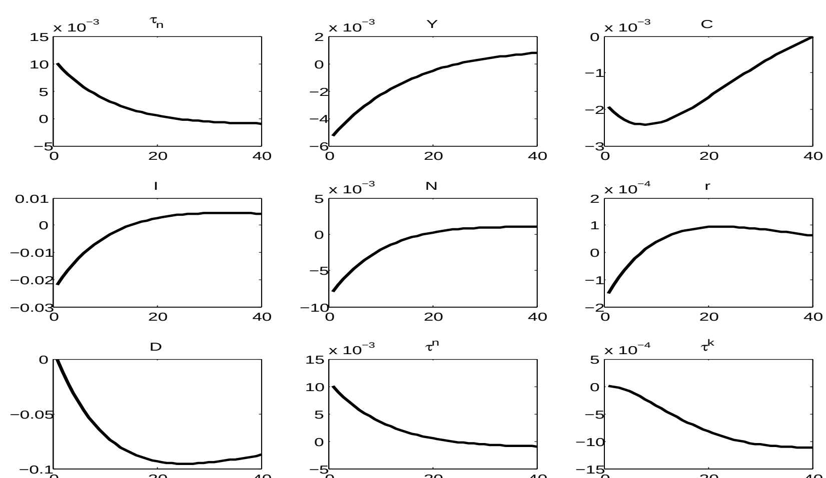
These are the same as we had before, except now I have the deviation of the debt-gdp ratio from steady state in the two tax processes. Given that I've added another endogenous variable, D_t, I have to include the government budget constraint as an equilibrium condition.

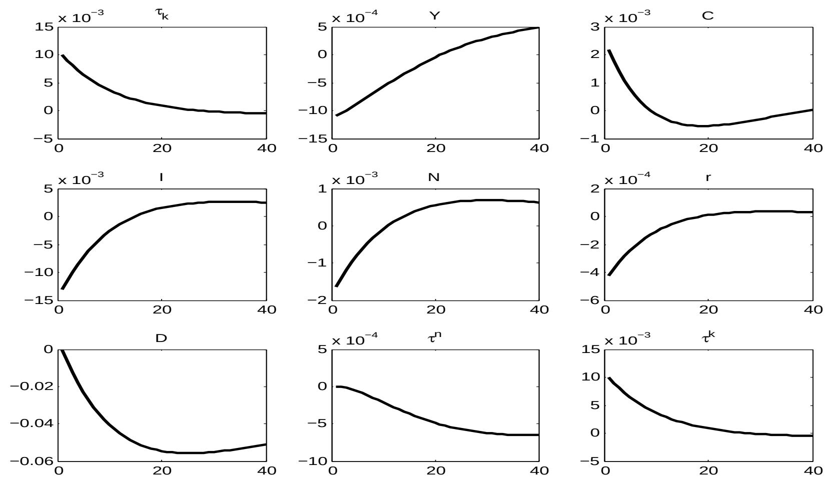
I use the same parameter values as before, this time setting $\gamma_k = \gamma_n = 0.025$. I assume that the steady state debt-gdp ratio is $\frac{D}{Y} = 0.5$. The impulse responses are shown below:



First, let's look at the impulse responses to a government spending shock. The immediate impact effects on endogenous variables are fairly similar to what we had before – here the output multiplier is 0.3131, so very similar to what we had earlier. But the responses out at longer forecast horizons are quite different. As we see in the lower left-hand corner, the increase in government spending causes debt to rise. After a few periods, this triggers increases in both capital and labor taxes, which exert contractionary effects on overall economic activity. After several periods, the increase in government spending has mostly faded away, but tax rates are high, so output is actually lower than where it started before the shock.

Next, consider the impulse responses to the tax shocks.





In both cases, the immediate impact effects are pretty similar to what we had before, but again the responses are different in important ways out at further forecast horizons. The increase in either tax rate causes debt to fall. Falling debt automatically works to lower tax rates out at some horizon, which is relatively stimulative, so we see output eventually higher than where it started in both cases after a sufficiently long forecast horizon.

There are a lot of different experiments you could run here – you can fiddle with the γ and ρ coefficients to generate different debt-dynamics, which can induce pretty interesting dynamics for the rest of the variables in the model.

6 Optimal Taxation in the Long Run

Rather than focusing on short run equilibrium dynamics assuming exogenous values of taxes and spending, let's briefly think about optimal taxes and government spending in the long run (i.e. the steady state).

Consider the model where we do not permit the government to use lump sum taxes (otherwise the problem of choosing optimal tax rates is pretty easy – it doesn't want to have those taxes). We will focus on the steady state. Suppose that steady state flow utility for the representative household is given by:

$$U = \ln C - \theta \frac{N^{1+\chi}}{1+\chi} + \xi \ln G$$

While it is not necessary to think about utility from government spending in terms of equilibrium dynamics (so long as utility from government spending is additively separable), for thinking about welfare it is important.

Taking the steady state government spending-output ratio, ω, and the steady state debt-gdp ratio, $\frac{D}{Y}$, as given, we can solve for the steady state values of C and N in terms of tax rates as:

$$\frac{K}{N} = \left(\frac{(1-\tau^k)\alpha}{\frac{1}{\beta} - (1-\delta)} \right)^{\frac{1}{1-\alpha}} \tag{56}$$

$$\frac{C}{N} = (1-\omega) \left(\frac{K}{N} \right)^{\alpha} - \delta \frac{K}{N} \tag{57}$$

$$N = \left(\frac{1}{\theta} \frac{(1-\tau^n)(1-\alpha) \left(\frac{K}{N}\right)^{\alpha}}{(1-\omega) \left(\frac{K}{N}\right)^{\alpha} - \delta \frac{K}{N}} \right)^{\frac{1}{1+\chi}} \tag{58}$$

$$Y = \left(\frac{K}{N} \right)^{\alpha} N \tag{59}$$

$$C = \frac{C}{N} N \tag{60}$$

$$G = \omega Y \tag{61}$$

In steady state, the tax rates must satisfy:

$$\omega + \left(\frac{1}{\beta} - 1 \right) \frac{D}{Y} = \alpha \tau^k + (1-\alpha)\tau^n \tag{62}$$

I'm going to do a couple of experiments. In particular, I first want to find the steady state tax rates on capital and labor, τ^k and τ^n, that maximize steady state utility, taking $\frac{D}{Y}$ and ω as given. In doing this, I want to impose a non-negativity constraint – the tax rates cannot be negative. In principle, I could do this analytically, but it's pretty laborious. So I do it numerically by searching over τ^k and τ^n to maximize U subject to all of the above conditions holding, as well as the restriction that the tax rates be non-negative. I assume that $\frac{D}{Y} = 0.5$ and $\omega = 0.2$, and use the other parameter values with which I've been working. I set $\xi = 0.3$.

I find that the optimal steady state tax rates are $\tau^n = 0.3076$ and $\tau^k = 0.0000$; this results in a steady state utility level of -1.0349 (as opposed to -1.0923 when I fixed $\tau^n = 0.20$ and find the steady state capital tax rate consistent with the government budget constraint holding). Note that the utility level being negative is fine, as utility is ordinal; as we would expect, steady state utility is higher when I optimally choose the tax rates.

I do a second experiment in which I simultaneously choose τ^n and τ^k, but also choose the steady state government spending-gdp ratio, ω, instead of fixing it at 0.2. Here I find that the optimal government spending share is $\omega = 0.1679$ (a little lower than I assumed before), and the optimal tax rates are $\tau^n = 0.2621$ and $\tau^k = 0.0000$. The steady state utility level associated with these values is -1.0291. This is naturally higher than what I get when I fixed ω.

What we see coming out here is a result in the literature dating back to Chamley (1985) and Judd (1986). This literature states that in the steady state of a neoclassical growth model capital should not be taxed. Different intuitions have been offered for this result. My own is simple Econ 101 type reasoning. The standard optimal taxation literature says that you should tax different things depending inversely on how elastically they are supplied/demanded. If a good is in either inelastic supply or demand, then the equilibrium quantity of that good does not depend on the price of the good. Taxes just drive a wedge between the "buy" and "sell" prices of a good. But if equilibrium quantity doesn't depend on price, then a positive tax doesn't distort quantities produced – it represents a transfer from buyers to sellers (or vice versa depending on whether demand or supply are inelastic), but it doesn't actually distort a productive margin. For something which is in either completely inelastic supply or demand, a distortionary tax is isomorphic to a lump sum tax. The more elastic the supply or demand of a good is, the bigger are the quantity distortions from taxes. You can see this by drawing "deadweight" triangles in simple supply and demand graphs.

The simple insight with respect to capital is this. In the long run (e.g. the steady state), capital is in perfectly elastic supply. The long run rental rate on capital satisfies $(1-\tau^k)R^k = \frac{1}{\beta} - (1-\delta)$. This steady state rental rate depends on how the household discounts the future and how fast capital depreciates; it does not depend on the level of the capital stock itself. This relationship is the long run capital supply curve – it is perfectly elastic. Capital demand is downward-sloping in K and equates the marginal product of K with the rental rate. With perfectly elastic supply of capital, the deadweight losses from capital taxation are large. For labor, neither demand nor supply of labor are perfectly elastic in the long run. This means, intuitively, that the welfare losses from labor taxation are smaller than for capital taxation, which is why we see that it is apparently optimal to have a relatively low tax on capital income in the long run.

The interesting side note to this discussion is that this idea of zero capital tax only applies to the long run. Whereas capital is in perfectly elastic supply in the long run, in the short run capital is perfectly inelastic – it is pre-determined, after all. This means that if a government wanted to raise money in an efficient way in a dynamic sense, it should levy a very large tax on capital at the beginning of time with a promise to eliminate the capital tax in the future. This is because capital is inelastic in the short run, so that a capital tax is functionally like a lump sum tax, whereas in the long run it is perfectly elastic and carries with it large distortions. Of course, there is time inconsistency issue at play here – it may be hard to commit to a high capital tax in the present and a low capital tax in the future.