Graduate Macro Theory II: The Basic New Keynesian Model

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1 Introduction

This set of notes lays and out and analyzes the canonical New Keynesian (NK) model. The NK model takes a real business cycle model as its backbone and adds to that sticky prices, a form of nominal rigidity that allows purely nominal shocks to have real effects, and which alters the response of the economy to real shocks in a way that gives rise to a non-trivial role for active stabilization policy.

To get price stickiness in the model, we have to have firms as price-setters, which means we need to move away from the perfectly competitive benchmark. To do so we introduce monopolistic competition. We split production into two sectors, where the final goods sector is perfectly competitive and aggregates intermediates into a final good for consumption. This generates a downward-sloping demand for intermediate goods. There are a continuum of intermediate goods producers who can set their own prices, but take all other prices as given. All the action in the model is at the level of the intermediate producers. We assume that they are not freely able to adjust their prices each period. In particular, the Calvo (1983) assumption posits that each period firms face a fixed probability of being allowed to change their price. This seems a little ridiculous in terms of its realism, but this assumption facilitates aggregation, and this is why it is so popular. With any price rigidity, any firm's price becomes a state variable. With a continuum of intermediate goods firms, we'd have a continuum of state variables. The Calvo (1983) assumption allows us to aggregate out this heterogeneity. Even though it seems somewhat bizarre on its surface, it has some normative implications that seem pretty reasonable (in particular, price stability ends up being an important policy goal).

The basic New Keynesian model that I'll lay out below (and which is laid out in Woodford (2003) and Gali (2007) textbook treatments) has no investment or capital. This simplifies the analysis quite a bit and permits us to get better intuition. It is not a completely innocuous omission, and we'll later look at how the inclusion of capital in the model affects things.

2 Households

There is a representative household that consumes, supplies labor, accumulates bonds, owns, and accumulates money. Its problem is:

$$\max_{C_t, N_t, B_{t+1}, M_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \theta \frac{N_t^{1+\chi}}{1+\chi} + \psi \ln\left(\frac{M_t}{P_t}\right)\right)$$

Here I have gone ahead and assumed that utility from real balances is logarithmic. As long as real balances are additively separable from consumption and labor, money in the utility function doesn't do much interesting here.[1] The nominal flow budget constraint is:

$$P_t C_t + B_{t+1} + M_t - M_{t-1} \leq W_t N_t + P_t D_t - P_t T_t + (1+i_{t-1}) B_t$$

Here money is the numeraire, and P_t is the price of goods in terms of money. B_t is the stock of nominal bonds a household enters the period with, and they pay out (known as of $t-1$) nominal interest rate i_{t-1}. The household also enters the period with a stock of money, M_{t-1}. Note that I'm not being super consistent with timing notation here: M_{t-1} and B_t are both known at $t-1$. The reason I write it this way is because the aggregate supply of money in period t, M_t, is not going to be predetermined but rather set by a central bank. W_t is the nominal wage (denominated in units of money, not goods). D_t denotes (nominal) profits remitted by firms, and T_t is a lump sum tax paid to a government (the government will have the role of setting the money supply and remitting any seignorage revenue back to the household lump sum).

A Lagrangian for the household is:

$$\mathbb{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \theta \frac{N_t^{1+\chi}}{1+\chi} + \psi \ln\left(\frac{M_t}{P_t}\right) + \lambda_t \left(W_t N_t + P_t D_t - P_t T_t + (1+i_{t-1}) B_t - P_t C_t - B_{t+1} - M_t + M_{t-1}\right)\right]$$

The FOC are:

$$\frac{\partial \mathbb{L}}{\partial C_t} = 0 \Leftrightarrow C_t^{-\sigma} = \lambda_t P_t$$

$$\frac{\partial \mathbb{L}}{\partial N_t} = 0 \Leftrightarrow \theta N_t^{\chi} = \lambda_t W_t$$

$$\frac{\partial \mathbb{L}}{\partial B_{t+1}} = 0 \Leftrightarrow \lambda_t = \beta \mathbb{E}_t \lambda_{t+1} (1+i_t)$$

$$\frac{\partial \mathbb{L}}{\partial M_t} = 0 \Leftrightarrow \theta \frac{1}{M_t} = \lambda_t - \beta \mathbb{E}_t \lambda_{t+1}$$

We can eliminate the multiplier and re-write these conditions as:

[1] If we were to assume that central bank policy focuses on an interest rate rather than a monetary aggregate, as we will do below, then we could ignore money altogether so long as utility from money is separable. This is sometimes referred to as a "cashless" economy.

$$\theta N_t^{\chi} = C_t^{-\sigma} w_t \tag{1}$$

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (1+i_t) \frac{P_t}{P_{t+1}} \right] \tag{2}$$

$$\psi \left(\frac{M_t}{P_t} \right)^{-1} = \frac{i_t}{1+i_t} C_t^{-\sigma} \tag{3}$$

(1) is a standard labor supply condition, where I have defined $w_t \equiv W_t / P_t$ as the real wage. (2) is the standard Euler equation for bonds, with $\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} = \Lambda_{t,t+1}$ the stochastic discount factor. (3) implicitly defines a demand function for real balances that is increasing in consumption and decreasing in the nominal interest rate.

3 Production

For the production side of things we split into two. There is a representative competitive final goods firm that aggregates intermediate inputs according to a CES technology. To the extent to which the intermediates are imperfect substitutes in the CES aggregator, this generates a downward-sloping demand for each intermediate, which gives these intermediate producers pricing power. There are a continuum (large number) of intermediates, so these producers behave as monopolistically competitive (they treat all prices but their own as given). These firms produce output using labor and are subject to an aggregate productivity shock. They are not freely able to adjust prices each period, in a way that we will discuss in more depth below.

3.1 Final Good Producer

The final output good is a CES aggregate of a continuum of intermediates:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

Here $\epsilon > 1$. This parameter measures how substitutable different varieties are. The profit maximization problem of the final good firm is:

$$\max_{Y_t(j)} \quad P_t \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_t(j) Y_t(j) dj$$

The FOC for a typical variety of intermediate j is:

$$P_t \frac{\epsilon}{\epsilon-1} \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}-1} \frac{\epsilon-1}{\epsilon} Y_t(j)^{\frac{\epsilon-1}{\epsilon}-1} = P_t(j)$$

This can be written:

$$\left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{1}{\epsilon-1}} Y_t(j)^{-\frac{1}{\epsilon}} = \frac{P_t(j)}{P_t}$$

Or:

$$\left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{-\frac{\epsilon}{\epsilon-1}} Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}$$

Making note of the definition of the aggregate final good, we have::

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t \tag{5}$$

(5) says that the demand for each intermediate is (i) proportional to total output, Y_t, and, (ii), decreasing in the relative price of the intermediate. ϵ has the interpretation of the price elasticity of demand.

We can derive an aggregate price index by defining nominal output as the sum of prices times quantities:

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$$

Plugging in the demand for each variety, we have:

$$P_t Y_t = \int_0^1 P_t(j)^{1-\epsilon} P_t^{\epsilon} Y_t dj$$

Pulling out of the integral things which don't depend on j:

$$P_t Y_t = P_t^{\epsilon} Y_t \int_0^1 P_t(j)^{1-\epsilon} dj$$

Simplifying, we get an expression for the aggregate price level:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{6}$$

3.2 Aside on the (Constant) Elasticity of Substitution

The elasticity of substitution for the production function is defined as the percentage change in relative demand for two intermediates (call them k and j) with respect to their relative prices:

$$\frac{\partial \left(Y_t(k)/Y_t(j)\right)}{\partial \left(P_t(j)/P_t(k)\right)} \times \frac{P_t(j)/P_t(k)}{Y_t(k)/Y_t(j)}$$

Take careful note of the indices here – this is measuring how the relative demand for intermediate k changes when the relative price of good j goes up.

From the demand function for each intermediate, (5), we have:

$$\frac{Y_t(k)}{Y_t(j)} = \left(\frac{P_t(k)}{P_t(j)}\right)^{-\epsilon} = \left(\frac{P_t(j)}{P_t(k)}\right)^{\epsilon}$$

The derivative part of the elasticity is then:

$$\frac{\partial\left(Y_t(k)/Y_t(j)\right)}{\partial\left(P_t(j)/P_t(k)\right)} = \epsilon\left(\frac{P_t(j)}{P_t(k)}\right)^{\epsilon-1}$$

This just says (naturally) that the relative demand for intermediate k will rise ($\epsilon > 0$) when the relative price of intermediate j increases.

The ratio of relative prices to relative demand is:

$$\frac{P_t(j)/P_t(k)}{Y_t(k)/Y_t(j)} = \frac{P_t(j)}{P_t(k)}\left(\frac{P_t(j)}{P_t(k)}\right)^{-\epsilon} = \left(\frac{P_t(j)}{P_t(k)}\right)^{1-\epsilon}$$

Therefore, the elasticity of substitution is:

$$\frac{\partial\left(Y_t(k)/Y_t(i)\right)}{\partial\left(P_t(k)/P_t(i)\right)} \times \frac{P_t(k)/P_t(j)}{Y_t(k)/Y_t(j)} = \epsilon\left(\frac{P_t(j)}{P_t(k)}\right)^{\epsilon-1}\left(\frac{P_t(j)}{P_t(k)}\right)^{1-\epsilon} = \epsilon$$

The elasticity of substitution is therefore a constant at ϵ. As $\epsilon \to \infty$, intermediates are perfect substitutes – the CES aggregator is just linear in varieties then.

3.3 Aside: More on CES Functions

The CES production function shown above looks kind of gnarly. There's an integral and complicated exponents.

It's useful to take a step back and note that we've actually been repeatedly using a special case of a CES function the entire course. That is the Cobb-Douglas production function. In particular, suppose that output is given by:

$$Y_t = \left(\alpha K_t^{\nu} + (1-\alpha)N_t^{\nu}\right)^{\frac{1}{\nu}}$$

This is a special case (two inputs, not an infinite number of inputs) with share parameters (α and $1-\alpha$) of what we did above (relabeling $\nu = \frac{\epsilon-1}{\epsilon}$. The function has constant returns to scale – if you scale both capital and labor by a factor ϑ, you scale output by the same factor.

Suppose that $\nu = 1$. This corresponds to a case of perfect substitutes – the production function is just linear in capital and labor. The Cobb-Douglas case occurs when $\nu = 0$ (which would correspond to $\epsilon = 1$ as written above). To see why $\nu = 0$ corresponds to Cobb-Douglas, take logs:

$$\ln Y_t = \frac{1}{\nu}\ln\left(\alpha K_t^{\nu} + (1-\alpha)N_t^{\nu}\right)$$

I took logs because it allows me to invoke L'Hopital's rule. When $\nu = 0$, we get $\ln 1/0 = \frac{0}{0}$. To make L'Hopital's rule easier to use, rewrite the term in parentheses using exponents and logs:

$$\ln Y_t = \frac{1}{\nu} \ln \left(\alpha \exp(\nu \ln K_t) + (1-\alpha) \exp(\nu \ln N_t) \right)$$

L'Hopital's rule says (when both numerator and denominator go to 0):

$$\lim_{\nu \to 0} \frac{f(\nu)}{g(\nu)} = \frac{f'(\nu)}{g'(\nu)}$$

$g(\nu) = \nu$, so $g(0) = 0$ and $g'(\nu) = 1$ regardless of what value ν takes on. For the numerator, we have:

$$f(\nu) = \ln \left(\alpha \exp(\nu \ln K_t) + (1-\alpha) \exp(\nu \ln N_t) \right)$$

And:

$$f'(\nu) = \frac{1}{\alpha \exp(\nu \ln K_t) + (1-\alpha) \exp(\nu \ln N_t)} \left[\alpha \ln K_t \exp(\nu \ln K_t) + (1-\alpha) \ln N_t \exp(\nu \ln N_t) \right]$$

When $\nu = 0$, we get:

$$f(0) = \ln 1 = 0$$

Hence, we can use L'Hopital's rule. Evaluating the derivative at $\nu = 0$, we have:

$$f'(0) = \frac{1}{1} \left[\alpha \ln K_t + (1-\alpha) \ln N_t \right]$$

This is because $\exp(0) = 1$. We therefore have:

$$\ln Y_t = \alpha \ln K_t + (1-\alpha) \ln N_t$$

Or, taking the exponential function to undo the logs:

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

So, the Cobb-Douglas production function is a special case of a CES production function. In this case, the elasticity of substitution (defined above) is one – a one percent increase in the relative price of two factors results in a one percent increase in the relative use of those factors.

In the linear case, when the elasticity of substitution goes to ∞, goods are perfect substitutes – a one percent increase in the relative price of two factors results in an infinite shift in the relative use of those factors. In contrast, when the elasticity of substitution is less than one, we say that two factors are complements. In this case, a one percent increase in the relative price of two factors results in a less than one percent increase in the relative use of the cheaper factor. In the limiting case in which the elasticity of substitution goes to 0, we have a Leontief function in which goods are perfect complements.

The reason we assume $\epsilon > 1$ (but less than ∞) above is that in monopolistic competition goods are substitutes, but imperfectly so.

Note that sometimes CES functions are referred to as Armington aggregators. They are very common in international macro.

3.4 Aside: Putting the CES Aggregate in Preferences

As written, I have done this with a two-sector production model: the final good that is sold to the household, and intermediate goods. We could alternatively put the CES aggregator used by the final good in preferences. In particular, the household problem would be:

$$\max_{C_t, N_t, B_{t+1}, M_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \theta \frac{N_t^{1+\chi}}{1+\chi} + \psi \ln \left(\frac{M_t}{P_t} \right) \right)$$

s.t.

$$P_t C_t + B_{t+1} + M_t - M_{t-1} \leq W_t N_t + P_t D_t - P_t T_t + (1 + i_{t-1}) B_t$$

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$P_t C_t = \int_0^1 P_t(j) C_t(j) dj$$

This just says that the household gets utility from a composite consumption basket, that is a CES aggregate of the individual consumption varieties. $\epsilon > 1$ still has the interpretation as the elasticity of substitution. The household's FOC for C_t is:

$$C_t^{-\sigma} = \lambda_t P_t$$

If we think about the choice of an individual variety, we would have the FOC:

$$\frac{\partial \mathbb{L}}{\partial C_t(j)} = 0 \Leftrightarrow \frac{\epsilon}{\epsilon - 1} \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{(1-\sigma)\epsilon}{\epsilon-1} - 1} \frac{\epsilon - 1}{\epsilon} C_t(j)^{\frac{\epsilon-1}{\epsilon} - 1} = \lambda_t P_t(j)$$

Simplifying a bit, and substituting out $\lambda_t = C_t^{-\sigma} P_t^{-1}$, we have:

$$\left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{1-\sigma\epsilon}{\epsilon-1}} C_t(j)^{-\frac{1}{\epsilon}} = C_t^{-\sigma} \frac{P_t(j)}{P_t}$$

Which can be written:

$$\left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon(\sigma\epsilon-1)}{\epsilon-1}} C_t(j) = C_t^{\sigma\epsilon} \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon}$$

But note that:

$$C_t^{\sigma\epsilon} = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\sigma\epsilon^2}{\epsilon-1}}$$

So, we have:

$$\left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon(\sigma\epsilon-1)}{\epsilon-1}-\frac{\sigma\epsilon^2}{\epsilon-1}} C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}$$

Which is:

$$\left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}}\right)^{-\frac{\epsilon}{\epsilon-1}} C_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}$$

But the first part of the left hand side is just C_t^{-1}!. So we have:

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t \tag{7}$$

This is the same interpretation as the demand function for each intermediate from the final good producers, just written in terms of consumption instead of output.

3.5 Intermediate Producers

There are a continuum of intermediate goods producers, $j \in [0,1]$. A typical intermediate producer produces output according to a constant returns to scale technology in labor, with a common productivity shock, A_t:

$$Y_t(j) = A_t N_t(j) \tag{8}$$

Nominal profit for an intermediate producer is:

$$P_t D_t(j) = P_t(j) Y_t(j) - W_t N_t(j)$$

We can sub out for $N_t(j)$ using the production function:

$$P_t D_t(j) = P_t(j) Y_t(j) - \frac{W_t}{A_t} Y_t(j)$$

W_t/A_t is the ratio of the nominal wage (w_t is the real wage) to the marginal product of labor, which is just A_t. This is just (nominal) marginal cost – to produce an extra unit of output, a firm needs $1/A_t$ extra units of labor, which costs W_t. w_t/A_t is real marginal cost.

Plug in the demand function for each intermediate:

$$P_t D_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t - \frac{W_t}{A_t} \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

Which can be written:

$$P_t D_t(j) = P_t(j)^{1-\epsilon} P_t^{\epsilon} Y_t - \frac{W_t}{A_t} P_t(j)^{-\epsilon} P_t^{\epsilon} Y_t$$

We can also write this in real terms by dividing through by P_t:

$$D_t(j) = P_t(j)^{1-\epsilon} P_t^{\epsilon-1} Y_t - \frac{w_t}{A_t} P_t(j)^{-\epsilon} P_t^{\epsilon} Y_t$$

3.5.1 No Price Rigidity

If there were nothing more to the problem, we could solve for the $P_t(j)$ that would maximize profit. Take the derivative of real dividends with respect to $P_t(j)$:

$$\frac{\partial D_t(j)}{\partial P_t(j)} = (1-\epsilon) P_t(j)^{-\epsilon} P_t^{1-\epsilon} Y_t + \epsilon \frac{w_t}{A_t} P_t(j)^{-\epsilon-1} P_t^{\epsilon} Y_t$$

Setting this equal to zero and simplifying a bit:

$$(\epsilon - 1) P_t(j) = \epsilon \frac{w_t}{A_t} P_t$$

Or, since W_t/A_t is nominal marginal cost, MC_t:

$$P_t^{\#} = \frac{\epsilon}{\epsilon - 1} MC_t$$

$P_t^{\#}$ is the optimal price, *which does not vary across* j – it is the same for all intermediates. The optimal price is just a *markup* over marginal cost (this follows because all firms face the same wage and have the same productivity). Since we are assuming that $\epsilon > 1$, the term $\epsilon/(\epsilon - 1) > 1$. This is the desired *markup* of price over marginal cost.

Since all firms would choose the same price, $P_t^* = P_t$ (the aggregate price index). We could then write this as:

$$w_t = \frac{\epsilon - 1}{\epsilon} A_t \tag{9}$$

(9) is a labor demand condition in the aggregate. In an efficient allocation (i.e. what a planner would choose), we would have $w_t = A_t$ (i.e firms would hire labor up until the point at which the real wage equals the marginal product of labor, which is just A_t with a linear production technology). The term $\frac{\epsilon-1}{\epsilon}$, which is the inverse markup, is less than one. This says that the equilibrium allocation is going to be *distorted* relative to an efficient allocation. This distortion arises because of the monopoly power of intermediate firms, where the strength of their monopoly power depends on the size of ϵ. The more substitutable intermediates are (i.e. the bigger is ϵ), the smaller will be the distortion.

3.5.2 Staggered Price Setting

Let us instead assume that prices are "sticky." In particular, each period there is a fixed probability of $1-\phi$ that a firm can adjust its price. This means that the probability a firm will be stuck with a price one period is ϕ, for two periods is ϕ^2, and so on. Consider the pricing problem of a firm given the opportunity to adjust its price in a given period. Since there is a chance that the firm will get stuck with its price for multiple periods, the pricing problem becomes dynamic.

Because the pricing problem is dynamic, we need to discount future profit flows. Intermediate firms will use the stochastic discount factor of the household to do this – $\Lambda_{t,t+j} = \frac{\beta^j \lambda_{t+j}}{\lambda_t}$. In addition, when setting a price today, the firm will take into account the probability that a price chosen today will be in effect in the future – this will be given by ϕ^j.

The dynamic problem of an updating firm can therefore be written:

$$\max_{P_t(j)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} \left(\frac{P_t(j)}{P_{t+s}} \left(\frac{P_t(j)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} - \frac{w_{t+s}}{A_{t+s}} \left(\frac{P_t(j)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right)$$

Multiplying out, and for simplicity defining $mc_t = w_t/A_t$, we have:

$$\max_{P_t(j)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} \left(P_t(j)^{1-\epsilon} P_{t+s}^{\epsilon-1} Y_{t+s} - mc_{t+s} P_t(j)^{-\epsilon} P_{t+s}^{\epsilon} Y_{t+s} \right)$$

The first order condition can be written:

$$(1-\epsilon) P_t(j)^{-\epsilon} \mathbb{E}_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} P_{t+s}^{\epsilon-1} Y_{t+s} + \epsilon P_t(j)^{-\epsilon-1} \mathbb{E}_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} mc_{t+s} P_{t+s}^{\epsilon} Y_{t+s} = 0$$

Simplifying:

$$P_t(j) = \frac{\epsilon}{\epsilon-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} mc_{t+s} P_{t+s}^{\epsilon} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t,t+s} P_{t+s}^{\epsilon-1} Y_{t+s}}$$

First, note that since nothing on the right hand side depends on j, all updating firms will update to the same reset price, call it $P_t^{\#}$. We can write the expression more compactly as:

$$P_t^{\#} = \frac{\epsilon}{\epsilon-1} \frac{X_{1,t}}{X_{2,t}} \tag{10}$$

Here:

$$X_{1,t} = mc_t P_t^{\epsilon} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{1,t+1} \tag{11}$$

$$X_{2,t} = P_t^{\epsilon-1} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{2,t+1} \tag{12}$$

If $\phi = 0$, then the right hand side would reduce to $mc_t P_t = MC_t$. In this case, the optimal price would be a fixed markup, $\frac{\epsilon}{\epsilon - 1}$, over nominal marginal cost.

3.6 Alternative Setup: Retailers and Wholesalers

Above, we assumed that production firms were potentially heterogeneous. There is an alternative setup this is sometimes easier to use.

In this setup, the final output good is still a CES aggregate, but it is an aggregate of retail output. Retailers face the same downward-sloping demand curve, but don't hire labor. Instead, they purchase wholesale output, $Y_{W,t}(j)$ at price $P_{W,t}$. There is a representative wholesale firm, indexed by W, that produces wholesale output according to:

$$Y_{W,t} = A_t N_t$$

The wholesaler just hires all the labor supplied by the household. It is competitive and takes its price, $P_{W,t}$, as given. It nominal profit is:

$$P_t D_{W,t} = P_{W,t} A_t N_t - W_t N_t$$

The first order condition is:

$$P_{W,t} = \frac{W_t}{A_t}$$

You will note: the right hand side is the same as nominal marginal cost above. We can define $p_{W,t} = P_{W,t}/P_t$ as the real price of wholesale output, and then have:

$$p_{W,t} = \frac{w_t}{A_t}$$

Where $w_t = W_t/P_t$ is the real wage. This is real marginal cost.

There are a continuum of retailers indexed by $j \in [0, 1]$. Their production function is simple: they purchase some wholesale output, $Y_{W,t}(j)$, and repackage this into retail output, $Y_t(j) = Y_{W,t}(j)$. Given that $Y_t(j) = Y_{W,t}(j)$, their nominal flow profit is:

$$P_t D_t(j) = P_t(j) Y_t(j) - P_{W,t} Y_t(j)$$

Plugging in the demand function for retail output that comes from the CES aggregator, this is the same problem we encountered before given what $P_{W,t}$ is equal to. We get exactly the same solution as above. This retailer-wholesaler distinction can be useful if there are features of production that might not aggregate well (which is not an issue with a linear production function).

4 Policy, Equilibrium, and Aggregation

Now we need to close the model. First, we need to say something about money supply. Suppose that the nominal money supply is set by the central bank and follows an AR(1) process in the growth rate, where $g_t^M = \ln M_t - \ln M_{t-1}$:

$$g_t^M = (1-\rho_M)g^M + \rho_M g_{t-1}^M + s_M \varepsilon_{M,t} \tag{13}$$

g^M is the steady state growth rate, $\varepsilon_{M,t}$ is drawn from a standard normal distribution, and s_M is the standard deviation of the shock.

The government's nominal budget constraint is:

$$i_{t-1}B_t^G \leq P_t T_t + M_t - M_{t-1} + B_{t+1}^G - B_t^G \tag{14}$$

I am assuming that the government does no spending; this is easy to add. It will turn out to be irrelevant whether the government issues any debt or not. But the government earns revenue (in nominal terms) from "printing" money.

Bond market clearing requires that $B_t = B_t^G$ in all periods. Combining (14) with the household's budget constraint at equality, we get:

$$P_t C_t = W_t N_t + P_t D_t$$

There are two sources of dividends: dividends from ownership in the final good firm and dividends from ownership in the intermediate goods firms. The final good firm nominal dividend is simple:

$$P_t D_t^F = P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj$$

Define the dividends from ownership in the intermediate firms as:

$$P_t D_t^I = \int_0^1 \left[P_t(j)Y_t(j) - W_t N_t(j)\right] dj = \int_0^1 P_t(j)Y_t(j)dj - W_t \int_0^1 N_t(j)dj$$

Market clearing for labor requires that $\int_0^1 N_t(j)dj = N_t$. We can therefore write this as:

$$P_t D_t^I = \int_0^1 P_t(j)Y_t(j)dj - W_t N_t$$

Total dividends received by the household are the sum of dividends from the two types of firms:

$$P_t D_t = P_t D_t^F + P_t D_t^I = P_t Y_t - \int_0^1 P_t(j)Y_t(j)dj + \int_0^1 P_t(j)Y_t(j)dj - W_t N_t = P_t Y_t - W_t N_t$$

Plugging this into the household's budget constraint, after making use of the market-clearing condition for bonds and imposing the government's budget constraint, gives the aggregate resource constraint:

$$Y_t = C_t \tag{15}$$

To get an aggregate production function, note that individual intermediate production functions are:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

The left hand side is:

$$A_t N_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

Now integrate both sides:

$$\int_0^1 A_t N_t(j) dj = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t dj$$

Which is:

$$A_t \int_0^1 N_t(j) dj = Y_t \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj$$

Using the market clearing condition for labor, this can be written:

$$A_t N_t = Y_t v_t^P \tag{16}$$

Where:

$$v_t^P = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj \tag{17}$$

v_t^P is a measure of price dispersion – if there were no price-setting friction, firms would all charge the same price, and this would be one. If prices are different, this expression will be bound from below by unity.

An equilibrium in this economy is a set of prices (w_t, i_t, P_t, $P_t^{\#}$, and Π_t) and allocations (C_t, N_t, Y_t) such that the households and firms behave optimally and markets clear. We need to close the model with a description of the exogenous productivity variable, which we assume follows a stationary AR(1):

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t} \tag{18}$$

Our full set of conditions can be written:

$$1 = \mathbb{E}_t \Lambda_{t,t+1}(1+i_t)\Pi_{t+1}^{-1} \tag{19}$$

$$\theta N_t^{\chi} = w_t C_t^{-\sigma} \tag{20}$$

$$\frac{M_t}{P_t} = \psi C_t^{\sigma}\left(\frac{1+i_t}{i_t}\right) \tag{21}$$

$$\Lambda_{t-1,t} = \beta\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma} \tag{22}$$

$$P_t^{1-\epsilon} = \left(\int_0^1 P_t(j)^{1-\epsilon}dj\right) \tag{23}$$

$$P_t^{\#} = \frac{\epsilon}{\epsilon-1}\frac{X_{1,t}}{X_{2,t}} \tag{24}$$

$$X_{1,t} = mc_t P_t^{\epsilon} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{1,t+1} \tag{25}$$

$$X_{2,t} = P_t^{\epsilon-1} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{2,t+1} \tag{26}$$

$$mc_t = \frac{w_t}{A_t} \tag{27}$$

$$Y_t = C_t \tag{28}$$

$$A_t N_t = Y_t v_t^P \tag{29}$$

$$v_t^P = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj \tag{30}$$

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t} \tag{31}$$

$$g_t^M = (1-\rho_M)g^M + \rho_M g_{t-1}^M + s_M \varepsilon_{M,t} \tag{32}$$

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{33}$$

$$g_t^M = \ln M_t - \ln M_{t-1} \tag{34}$$

As written, this is 16 equations and 16 variables (C_t, N_t, Y_t, $\Lambda_{t-1,t}$, i_t, Π_t, P_t, M_t, $P_t^{\#}$, $X_{1,t}$, $X_{2,t}$, mc_t, w_t, A_t, v_t^P, g_t^M). I could reduce this down if I wanted to.

There are a couple of issues with how I wrote this model. First, I have the price level showing up. To the extent to which the price level is growing over time, I want to write the model without reference to that, and so need to re-write things in terms of the inflation rate. Second, I still have j subscripts showing up in two places – first, in the definition of the aggregate price level, and second, in the price dispersion term. I need to get rid of the j subscripts if possible. The Calvo assumption allows us to do that.

Start with the definition of the price index. Note that exactly $1-\phi$ of firms will choose the same price, $P_t^{\#}$. Since these firms are random, we can order them however we want along the unit interval. We can therefore break up the integral as:

$$P_t^{1-\epsilon} = \int_0^{1-\phi} \left(P_t^{\#}\right)^{1-\epsilon} dj + \int_{1-\phi}^1 P_t(j)^{1-\epsilon} dj$$

Which becomes:

$$P_t^{1-\epsilon} = (1-\phi) \left(P_t^{\#}\right)^{1-\epsilon} + \int_{1-\phi}^1 P_t(j)^{1-\epsilon} dj$$

This follows because the reset price doesn't depend on j. For the firms who cannot change their price, their prices will just be whatever was charged in the previous period. So:

$$P_t^{1-\epsilon} = (1-\phi) \left(P_t^{\#}\right)^{1-\epsilon} + \int_{1-\phi}^1 P_{t-1}(j)^{1-\epsilon} dj$$

Because price adjusters and non-adjusters are randomly chosen, taking the sum of a subset of old prices is just proportional to summing over all old prices. Put another way:

$$\int_{1-\phi}^1 P_{t-1}(j)^{1-\epsilon} dj = \phi \int_0^1 P_{t-1}(j)^{1-\epsilon} dj$$

But the right hand side is just $P_{t-1}^{1-\epsilon}$!. Hence, we have:

$$P_t^{1-\epsilon} = (1-\phi) \left(P_t^{\#}\right)^{1-\epsilon} + \phi P_{t-1}^{1-\epsilon}$$

In effect, the current price level (raised to a power) is a convex combination of the reset price (common across all firms) and the old price level. The Calvo assumption means we don't need to keep track of individual firm prices, just the aggregate price level. To write this in terms of inflation rates, divide both sides by $P_t^{1-\epsilon}$. We get:

$$1 = (1-\phi) \left(\frac{P_t^{\#}}{P_t}\right)^{1-\epsilon} + \phi \left(\frac{P_{t-1}}{P_t}\right)^{1-\epsilon}$$

Define $p_t^{\#} = P_t^{\#}/P_t$ as the optimal relative reset price. The last term is just the inverse of gross price inflation. So we have:

$$1 = (1-\phi) \left(p_t^{\#}\right)^{1-\epsilon} + \phi \Pi_t^{\epsilon-1} \tag{35}$$

(35) describes the evolution of aggregate inflation dynamics, rather than the price level, and does so in terms of stationary variables. Now go to the price dispersion term. Using similar logic, we can break up the integral:

$$v_t^P = \int_0^{1-\phi} \left(\frac{P_t^{\#}}{P_t}\right)^{-\epsilon} dj + \int_{1-\phi}^1 \left(\frac{P_{t-1}(j)}{P_t}\right)^{-\epsilon} dj$$

Here, I have used the fact that non-updating firms charge their previous period's price. We can multiply and divide the right-hand term by $P_{t-1}^{-\epsilon}$ to get (since this doesn't vary with j, it can go inside or outside the integral):

$$v_t^P = (1-\phi)\left(p_t^{\#}\right)^{-\epsilon} + \int_{1-\phi}^{1}\left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon}\left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} dj$$

But this can be written:

$$v_t^P = (1-\phi)\left(p_t^{\#}\right)^{-\epsilon} + \Pi_t^{\epsilon}\int_{1-\phi}^{1}\left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon} dj$$

But now use the same trick as before. The integral is:

$$\int_{1-\phi}^{1}\left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon} dj = \phi\int_{0}^{1}\left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon} dj = \phi v_{t-1}^P$$

So we can write the price dispersion term just in terms of its own lag:

$$v_t^P = (1-\phi)\left(p_t^{\#}\right)^{-\epsilon} + \phi\Pi_t^{\epsilon}v_{t-1}^P \tag{36}$$

Now, we need to play with the price-setting terms. Define $\widehat{X}_{1,t} = X_{1,t}/P_t^{\epsilon}$ and $\widehat{X}_{2,t} = X_{2,t}/P_t^{\epsilon-1}$. Playing around:

$$\frac{X_{1,t}}{P_t^{\epsilon}} = mc_tY_t + \phi\mathbb{E}_t\Lambda_{t,t+1}\frac{X_{1,t+1}}{P_t^{\epsilon}} \tag{37}$$

Multiply and divide the last term by P_{t+1}^{ϵ}:

$$\widehat{X}_{1,t} = mc_tY_t + \phi\mathbb{E}_t\Lambda_{t,t+1}\frac{X_{1,t+1}}{P_{t+1}^{\epsilon}}\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} \tag{38}$$

So:

$$\widehat{X}_{1,t} = mc_tY_t + \phi\mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{\epsilon}\widehat{X}_{1,t+1} \tag{39}$$

Do the same with $X_{2,t+1}$:

$$\frac{X_{2,t}}{P_t^{\epsilon-1}} = Y_t + \phi\mathbb{E}_t\Lambda_{t,t+1}\frac{X_{2,t+1}}{P_t^{\epsilon-1}}$$

So:

$$\widehat{X}_{2,t+1} = Y_t + \phi\mathbb{E}_t\Lambda_{t,t+1}\frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}}\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1}$$

And finally:

$$\widehat{X}_{2,t+1} = Y_t + \phi\mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{\epsilon-1}\widehat{X}_{2,t+1} \tag{40}$$

Now go to the reset price condition. Re-write it so it is in terms of $\widehat{X}_{1,t}$ and $\widehat{X}_{2,t}$:

$$P_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{X_{1,t}/P_t^{\epsilon}}{X_{2,t}/P_t^{\epsilon-1}} \frac{P_t^{\epsilon}}{P_t^{\epsilon-1}}$$

But this is:

$$P_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{\widehat{X}_{1,t}}{\widehat{X}_{2,t}} P_t$$

Which can be written in terms of $p_t^{\#}$:

$$p_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{\widehat{X}_{1,t}}{\widehat{X}_{2,t}} \tag{41}$$

Now, let's re-write things in terms of real balances, $m_t = M_t/P_t$. For the money demand specification, this is straightforward:

$$m_t = \psi C_t^{\sigma} \left(\frac{1 + i_t}{i_t} \right) \tag{42}$$

For the definition of nominal money growth, to write this in terms of real balance growth, add and subtract logs of the price level as necessary:

$$g_t^M = (\ln M_t - \ln P_t) + \ln P_t - (\ln M_{t-1} - \ln P_{t-1}) - \ln P_{t-1}$$

Which is then:

$$g_t^M = \ln m_t - \ln m_{t-1} + \ln \Pi_t \tag{43}$$

Okay, now I'm done. The equilibrium is now characterized by:

$$1 = \mathbb{E}_t \Lambda_{t,t+1} (1 + i_t) \Pi_{t+1}^{-1} \tag{44}$$

$$\theta N_t^{\chi} = w_t C_t^{-\sigma} \tag{45}$$

$$m_t = \psi C_t^{\sigma} \left(\frac{1 + i_t}{i_t} \right) \tag{46}$$

$$\Lambda_{t-1,t} = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \tag{47}$$

$$1 = (1 - \phi) \left(p_t^{\#} \right)^{1-\epsilon} + \phi \Pi_t^{\epsilon - 1} \tag{48}$$

$$p_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{\widehat{X}_{1,t}}{\widehat{X}_{2,t}} \tag{49}$$

$$\widehat{X}_{1,t} = mc_t Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} \widehat{X}_{1,t+1} \tag{50}$$

$$\widehat{X}_{2,t} = Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} \widehat{X}_{2,t+1} \tag{51}$$

$$mc_t = \frac{w_t}{A_t} \tag{52}$$

$$Y_t = C_t \tag{53}$$

$$A_t N_t = Y_t v_t^P \tag{54}$$

$$v_t^P = (1-\phi)\left(p_t^{\#}\right)^{-\epsilon} + \phi \Pi_t^{\epsilon} v_{t-1}^P \tag{55}$$

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t} \tag{56}$$

$$g_t^M = (1-\rho_M) g^M + \rho_M g_{t-1}^M + s_M \varepsilon_{M,t} \tag{57}$$

$$g_t^M = \ln m_t - \ln m_{t-1} + \ln \Pi_t \tag{58}$$

This is the same system of equations, but it is now 15 variables and 15 equations (I have dropped P_t and re-written things in terms of m_t instead of M_t). Importantly, nothing here inherits any nominal trend, and there are no j subscripts – this is just aggregate variables!

5 The Steady State

In the non-stochastic steady state, $A = 1$, real variables are constant, and g^M is a constant parameter (the average growth rate of the nominal money supply). To solve for the steady state, note that this requires:

$$\ln \Pi = g^M$$

Hence:

$$\Pi = \exp(g^M)$$

If $g^M = 0$, for example, then steady state gross inflation will be 1 (so net inflation will be zero). The steady state stochastic discount factor is:

$$\Lambda = \beta$$

Therefore, the steady state nominal interest rate is:

$$1 + i = \Pi/\beta$$

Given Π, we then have steady state reset price inflation as:

$$1 - \phi \Pi^{\epsilon-1} = (1-\phi)\left(p^{\#}\right)^{1-\epsilon}$$

So:

$$p^{\#} = \left(\frac{1-\phi\Pi^{\epsilon-1}}{1-\phi}\right)^{\frac{1}{1-\epsilon}}$$

Next, in steady state, note that:

$$\widehat{X}_1 = \frac{mcY}{1-\phi\beta\Pi^{\epsilon}}$$

$$\widehat{X}_2 = \frac{Y}{1-\phi\beta\Pi^{\epsilon-1}}$$

Hence:

$$\frac{\widehat{X}_1}{\widehat{X}_2} = mc\frac{1-\phi\beta\Pi^{\epsilon-1}}{1-\phi\beta\Pi^{\epsilon}}$$

Therefore, we can solve for mc (which is in turn equal to the real wage):

$$mc = \frac{\epsilon-1}{\epsilon}p^{\#}\frac{1-\phi\beta\Pi^{\epsilon}}{1-\phi\beta\Pi^{\epsilon-1}}$$

Now we can solve for steady state price dispersion:

$$v^P = \frac{(1-\phi)\left(p^{\#}\right)^{-\epsilon}}{1-\phi\Pi^{\epsilon}}$$

Now we can solve for steady state N from the first order condition for labor supply. Since $C = Y = N/v^P$, we have:

$$\theta N^{\chi} = mc\left(\frac{N}{v^P}\right)^{-\sigma}$$

So:

$$N^{\sigma+\chi} = (v^p)^{\sigma} mc$$

So:

$$N = \left(\frac{(v^p)^{\sigma} mc}{\theta}\right)^{\frac{1}{\sigma+\chi}}$$

Then:

$$Y = Nv^P = C$$

Once we have Y and C, we can solve for steady state m from the money demand relationship:

$$m = \psi C^{\sigma}\left(\frac{1+i}{i}\right)$$

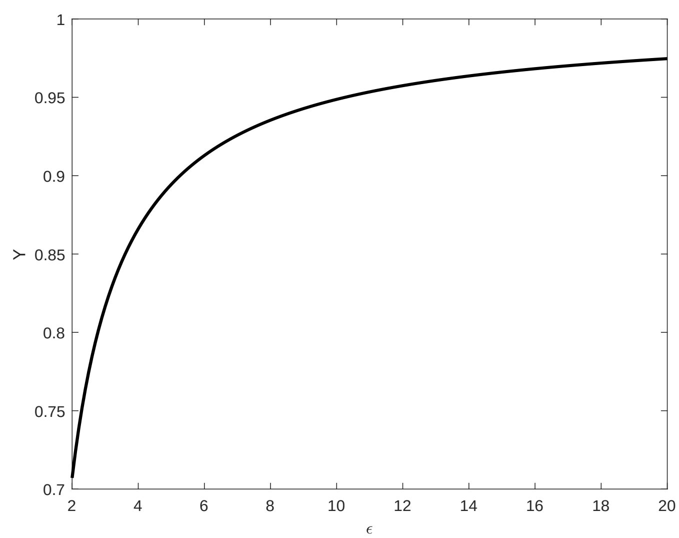
5.1 Long-Run Distortions

There are two distortionary features of this model in the long run: monopolistic competition and inflation. To focus on the monopolistic competition distortion, it is easiest to think about a world in which $g^M = 0$ (since $\Pi = 1$, so there is no net inflation in the long run). It is easy to see that $mc = w = \frac{\epsilon-1}{\epsilon} < 1$. The steady state wage without monopolistic competition would be $w = mc = 1$ (which coincides with $\epsilon \to \infty$). Monopolistic competition will have the effect of giving lower hours and output in the long run.

The second distortion is inflation. When $g^M = 0$ so that $\Pi = 1$, $v^P = 1$. But for any $g^M \neq 0$, $v^P > 1$. This is like having lower productivity – for a given N, you get less Y. This arises because of staggered price-setting. If trend inflation is non-zero ($\Pi \neq 1$), then relative prices of intermediates will not all be the same. But this means that some output is "lost" when aggregating up in the CES aggregator.

To get a sense of these factors, I solve for the steady state using some different values of these parameters. First, I fix $\beta = 0.99$, $\theta = 1$, $\chi = 1$, $\phi = 3/4$, and $\psi = 1$. Then, fixing $g^M = 0$ (so no trend inflation), I solve for steady state output as a function of ϵ, for values of ϵ ranging from 2 to 20. This is shown below.

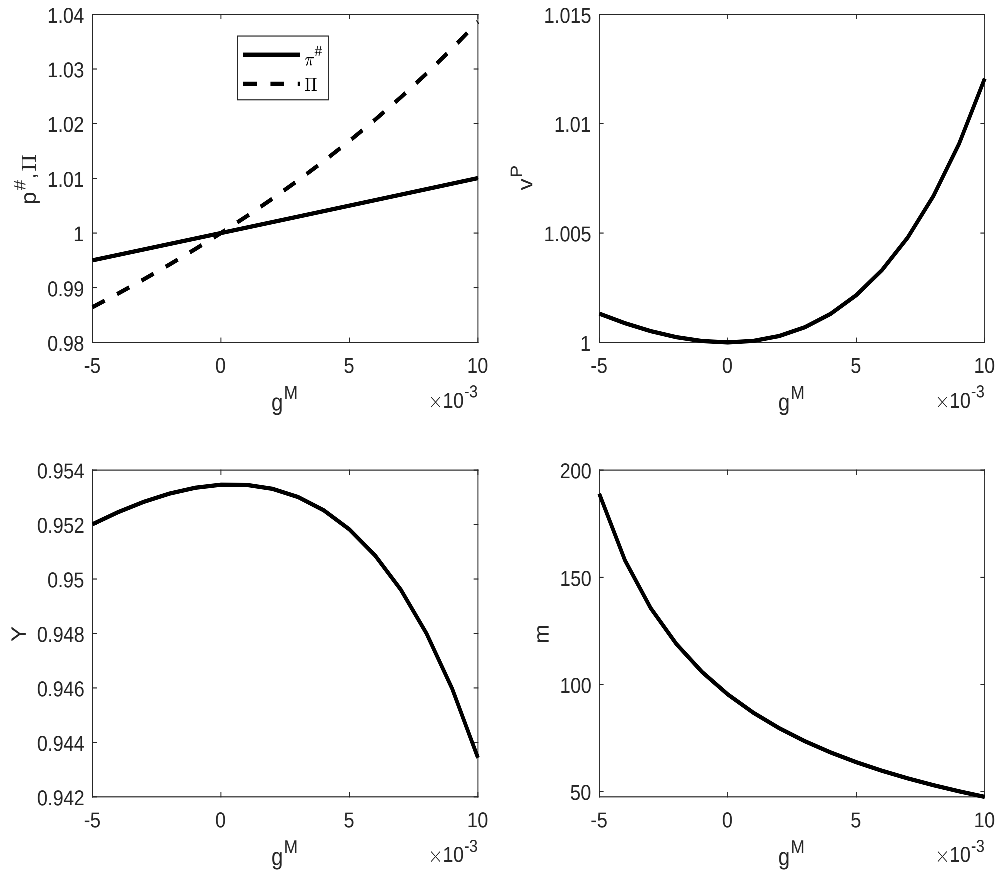
Figure 1: Steady-State Y and ϵ



We can see that output is everywhere increasing in ϵ. The bigger ϵ is, the weaker is monopoly power, and the less distorted is the economy.

Next, to get a sense of the role of trend inflation, I fix $\epsilon = 11$, and consider values of g^M ranging from -0.005 up to 0.01. I plot steady state values of the optimal reset price, inflation, price dispersion, output, and real money balances as a function of trend growth in money.

Figure 2: Steady-State Inflation and Other Variables



Since steady state gross inflation is the exponential of steady state money growth, inflation plotted against money growth is not exactly a 45-degree line (it has some curvature), though for some levels of trend growth in money (which I am considering), net inflation would be approximately equal to money growth. When $g^M = 0$, we get $p^{\#} = \Pi = 1$. For negative values of money growth, reset price inflation lies below inflation; for positive values, it lies above. Here is what is going on. If trend inflation is positive, when given a chance to reset its price, a firm has to *overadjust* to factor in trend inflation during the period in which it will be (in expectation) stuck with the price it chooses today (and vice-versa on the downside). Essentially, firms *want* a constant markup of price over marginal cost of $\epsilon/(\epsilon - 1)$. If the aggregate price level is trending up due to inflation, they have to over-adjust to get that desired markup on average over the duration of the time when their price will be in effect.

The right plan plots price dispersion. This bottoms out when $\Pi = 1$ at $v^P = 1$. It lies everywhere else above unity. If there is trend inflation (or deflation), then prices are going to be disperse. This effectively acts like a negative productivity shifter in the production function, lowering output. Indeed, steady state output is essentially the mirror image of steady state price dispersion. Steady state output peaks at $g^M = 0$ so $\Pi = 1$ and $v^P = 1$.

The mechanisms above call for $\Pi = 1$ (i.e. no steady state inflation). Any steady state inflation ends up distorting relative prices of intermediate goods, resulting in lost output, which is bad. But, as we see in the lower right panel, there is still the Friedman rule logic at play – deflation results in higher steady state real money balances, which increases welfare. Any discussion of optimal trend inflation must balance the Friedman rule logic with money in the utility function (lower-right panel) against the bad effects of price dispersion (lower-left panel).

6 Impulse Responses

Next, I solve the model in Dynare and produce impulse responses. I need to parameterize the model. I pick $\beta = 0.99$, $\sigma = 3$, $\chi = 1$, and $\theta = 1$. I set $\phi = 0.75$ (as we will see below, this implies that the average duration between price changes is four quarters). I set $\epsilon = 11$. This implies that average markups of price over marginal cost are 10 percent. I set $\psi = 1$ (this parameter really has no influence on the model dynamics). I set $g^M = 0$, so there is zero steady state inflation ($\Pi = 1$, or $\pi = 0$). I set $\rho^M = 0.5$ and $\rho_A = 0.9$. I set the standard deviations of both the monetary and productivity shocks to one percent (0.01).

6.1 Aside: Calibrating the Calvo Parameter

How does one come up with a reasonable value for ϕ? This is a really important parameter in the model – the bigger it is (the stickier are prices), the bigger will be the effects of nominal shocks and the more distorted will be the response of variables to real shocks.

It turns out that there exists a close mapping between ϕ and the expected duration of a price change. Consider a firm that gets to update its price in a period. In expectation, how long will it be stuck with that price? The probability of getting to adjust its price one period from now is $1 - \phi$. The probability of adjusting in two periods is $\phi(1 - \phi)$: ϕ is the probability it doesn't adjust after one period, and $1 - \phi$ is the probability it can adjust in two periods. The probability of adjusting in three periods is $\phi^2(1 - \phi)$: ϕ^2 is the probability it gets to the third period with its initial price, and $1 - \phi$ is the probability it can adjust in that period. And so on. So the expected duration of a price chosen today is:

$$\text{Expected Duration} = (1 - \phi) \sum_{j=1}^{\infty} \phi^{j-1} j$$

The summation part on the inside can be written:

$$
\begin{aligned}
S &= 1 + 2\phi + 3\phi^2 + 4\phi^3 + \dots \\
S\phi &= \phi + 2\phi^2 + 3\phi^3 + \dots \\
(1-\phi)S &= 1 + \phi + \phi^2 + \phi^3 + \dots \\
(1-\phi)S &= \frac{1}{1-\phi} \\
S &= \frac{1}{(1-\phi)^2}
\end{aligned}
$$

The second to last line above uses the fact that $1 + \phi + \phi^2 + \dots = \frac{1}{1-\phi}$ as long as $\phi < 1$. Combining this with the $1 - \phi$ outside the summation term above, we have:

$$\text{Expected Duration} = \frac{1}{1-\phi}$$

Bils and Klenow (2004, *JPE*) analyze micro data on pricing and computing the average length of time between prices changes. Though there is substantial heterogeneity across types of goods (e.g. the price of newspapers rarely changes, while gasoline changes daily), for most goods, prices change on average once every six months, which would suggest that $\phi \approx 1/2$ at a quarterly frequency (average duration of two quarters). For these models to produce realistic responses to monetary policy shocks you need ϕ much higher (more like the 0.75 value I've been using). So an important area of research essentially involves ways to "flatten" the Phillips Curve without assuming counterfactually large levels of price rigidity. I will typically assume something like $\phi = 3/4$ or $\phi = 2/3$.

6.2 IRFs to Monetary and Productivity Shocks

Consider first impulse responses to a monetary shock. I'm plotting the log level of the money supply (the cumulated response of g_t^M. There is a permanent effect on the money supply that grows over time (given that I assumed $\rho_M > 0$). Output increases, inflation increases, the price level increases, real marginal cost (w_t/A_t) increases, and the nominal interest rate decreases (although not quite on impact, though it does subsequent to that). These look kind of "standard" from the perspective of most people's basic intuition. The increase in real marginal cost is because firms have to pay workers more to get them to work more, which is necessary for output to increase.

Effectively, price stickiness ($\phi > 0$) is limiting how much the price level can react to the increase in M_t in the short run. This means that m_t goes up in the short run (but ultimately the long-run increase in M_t is met by a proportional increase in P_t). It is easiest to think about the real effects of a nominal shock by focusing on the money demand relationship, and assuming that the nominal rate is approximately constant. Then, in log-linear terms, we would have:

$$\widetilde{m}_t = \sigma y_t$$

For ease of notation (see discussion below), I am just going to use lowercase variables to denote

Figure 3: IRFs to a Monetary Shock

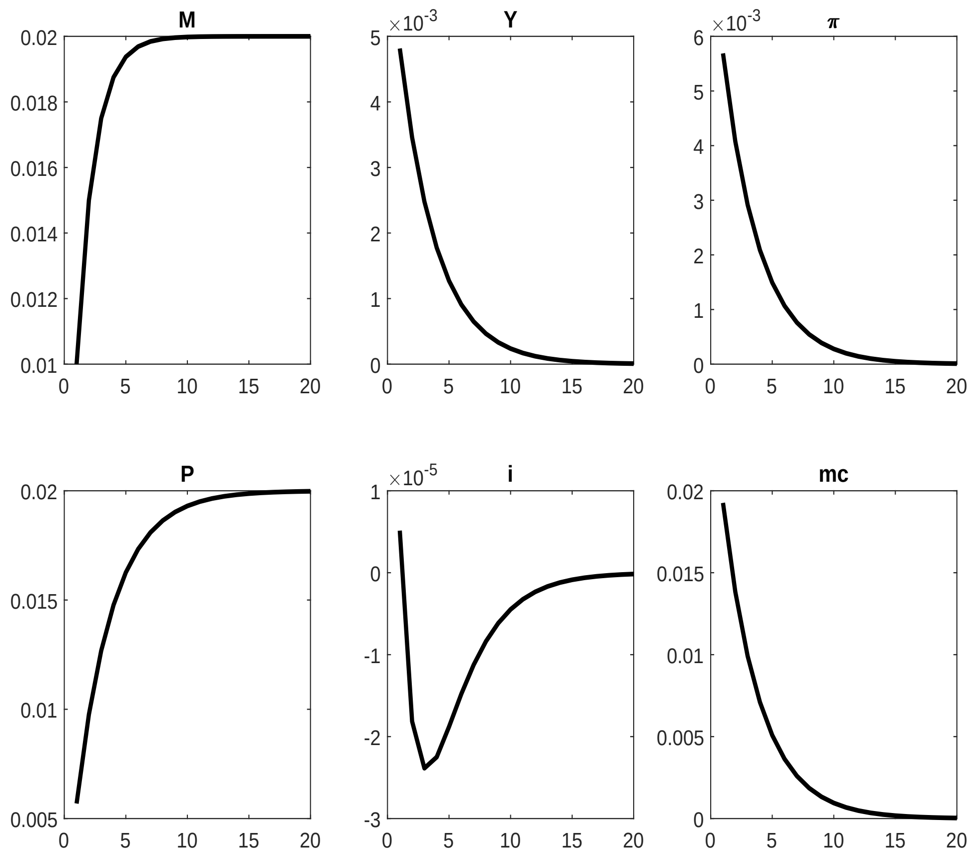
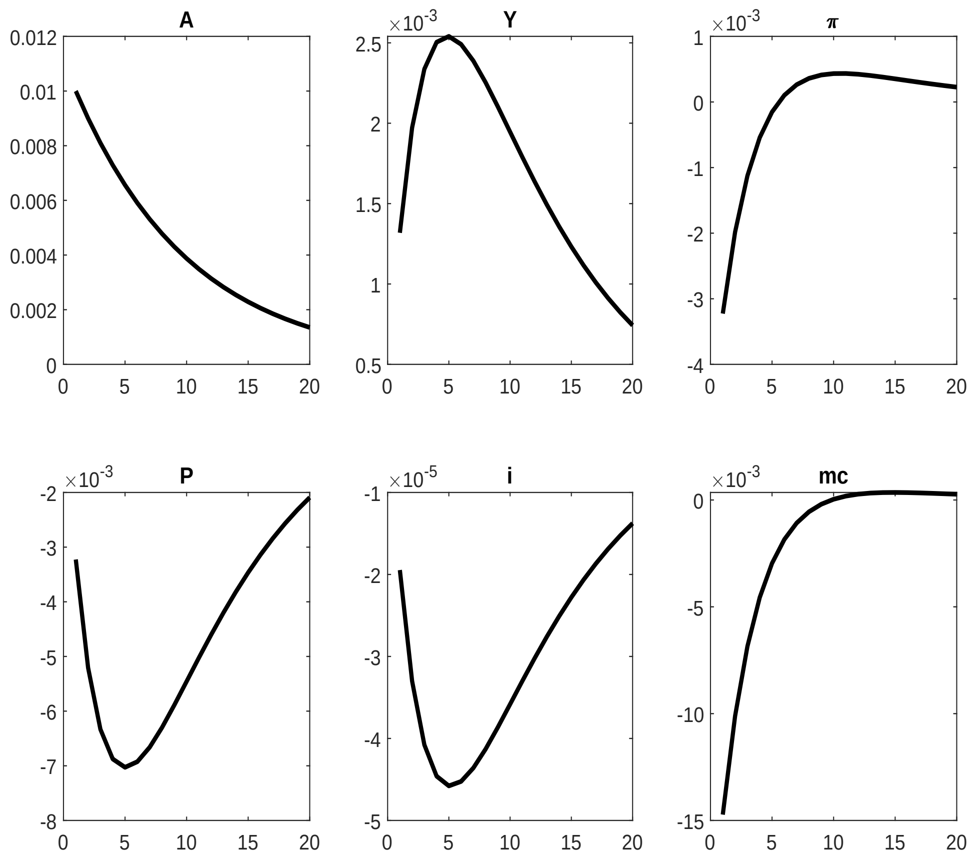


Figure 4: IRFs to a Productivity Shock

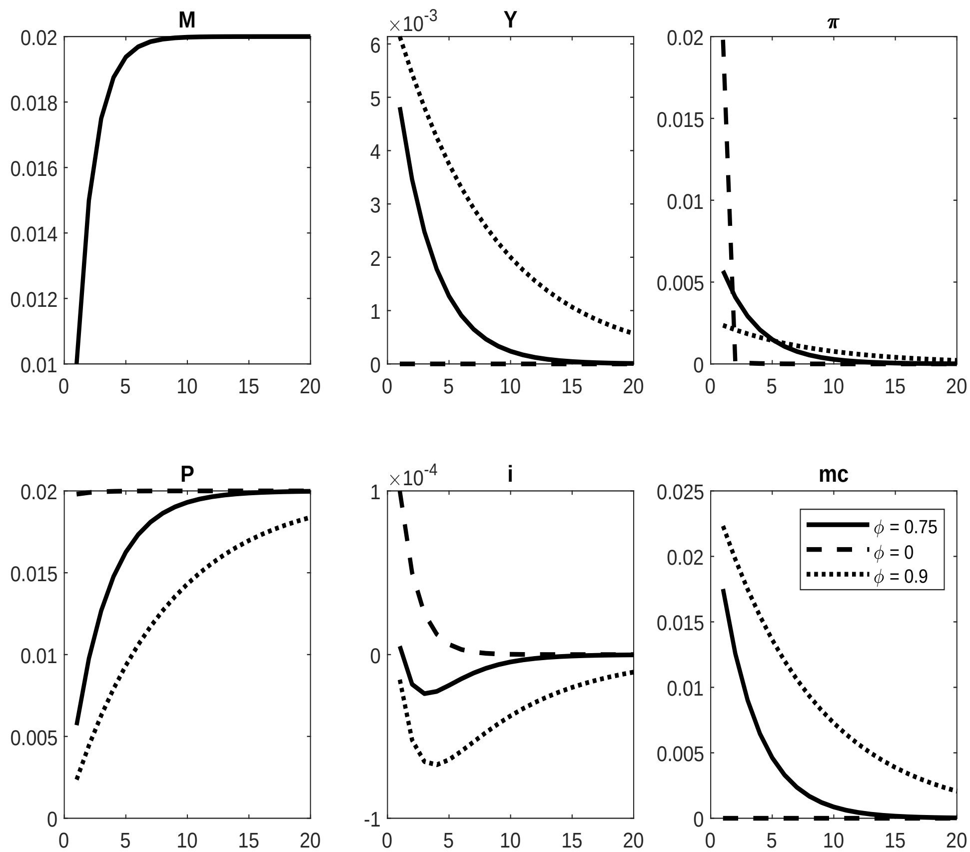


log deviations of variables that are already uppercase. This says that output and real balances must move one-to-one. This is effectively quantity theory logic – since $M_tV_t = P_tY_t$, we have $m_t = V_t^{-1}Y_t$. If velocity is constant, as assumed in quantity theory (and as would be the case with a constant nominal interest rate), then real balances and output move one-to-one. Price stickiness allows the increase in the nominal money supply to cause real balances to rise in the short run, which necessitates a temporary increase in output until price adjustment has taken place.

Next, consider a positive productivity shock. The impulse responses are shown below. Output increases, but by substantially less than the increase in A_t. Mechanically, this means that hours, N_t are going *down* (which is not shown). Inflation falls, as does the price level. For this parameterization, the nominal interest rate declines. Real marginal cost falls – this is saying that the real wage increases less than the increase in productivity.

In terms of intuition, it is easiest to think about what is going in with the quantity theory logic. When productivity goes up, output wants to increase. But it can only increase if m_t increases. Because prices are sticky, and there is no change in the money supply, the price level falls by "too

Figure 5: IRFs to a Monetary Shock, Different ϕ



little" relative to what it would do without price stickiness, which limits the increase in Y_t.

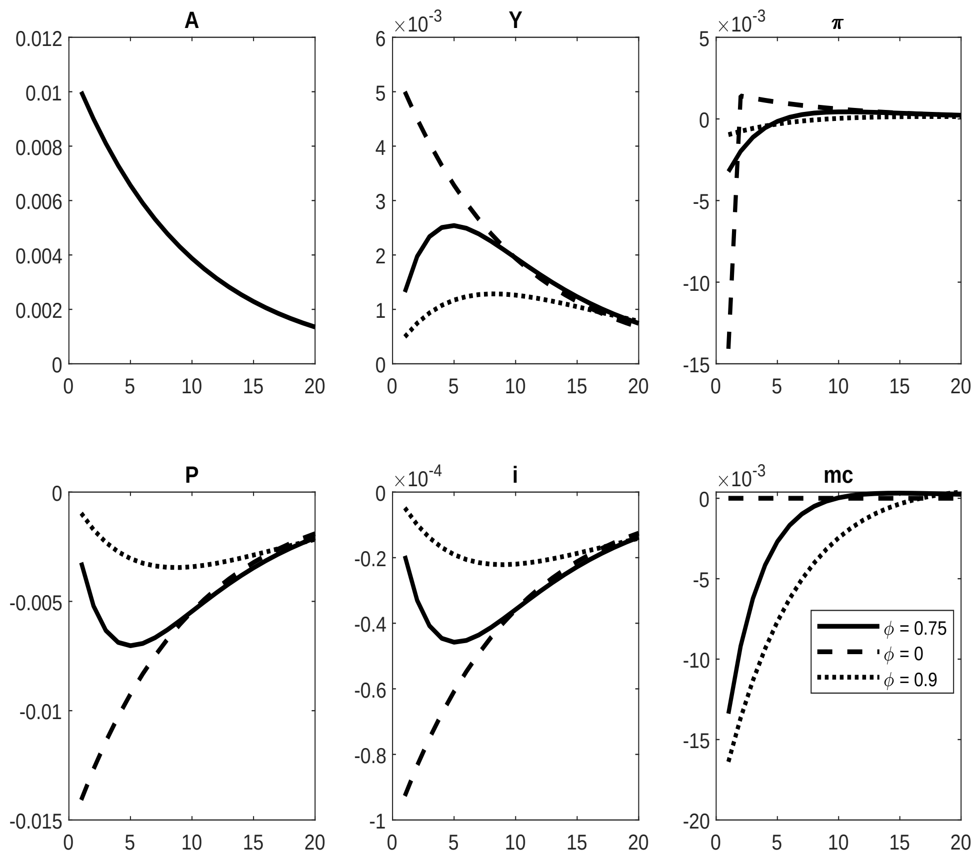
6.3 The Role of Price Stickiness

We can solve the model for different values of ϕ. Recall that $\phi \to 0$ means that prices are flexible, whereas $\phi \to 1$ means that prices are very sticky.

Below, I show IRFs to a monetary and productivity shock for three different values of ϕ: 0.75 (what I showed above), $\phi = 0$ (prices flexible), and $\phi = 0.9$ (prices more sticky).

If prices are flexible (dashed line), then the monetary shock has no effect on output or real marginal cost. The price level jumps up almost all the way to its new higher steady state value. This means that, on impact, real money balances decline (the price level jumps more than the nominal money supply initially). To accommodate real money balances declining, the nominal interest rate actually increases. Inflation jumps up a lot. There is no impact on output or real marginal cost (nor on the real interest rate or hours). In contrast, when prices are stickier (bigger ϕ), output reacts more, and the price level reacts less. Thus, naturally, the real effects of monetary

Figure 6: IRFs to a Productivity Shock, Different ϕ



expansion are bigger the stickier are prices.

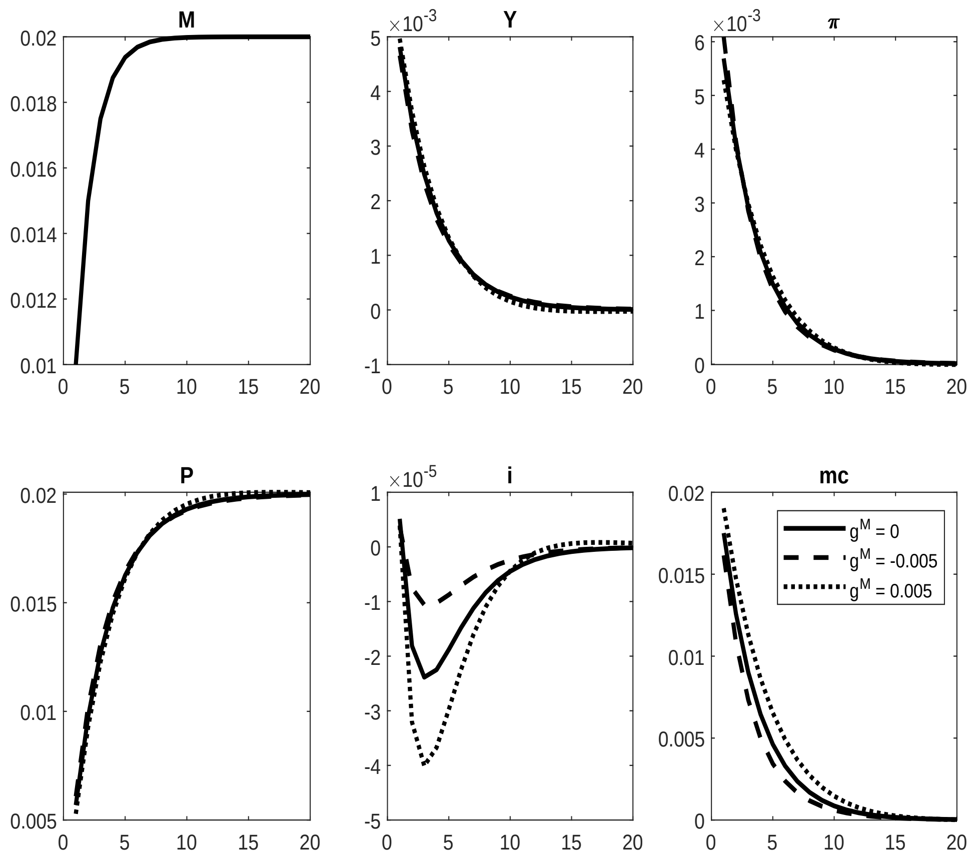
Next, consider a productivity shock.

6.4 The Relative Unimportance of Steady State Inflation

Above, I solved the model with $g^M = 0$, which means $\Pi = 1$ (gross inflation, which equivalently means π, net inflation, is zero in the steady state). Obviously, we live in a world where inflation is, on average, positive.

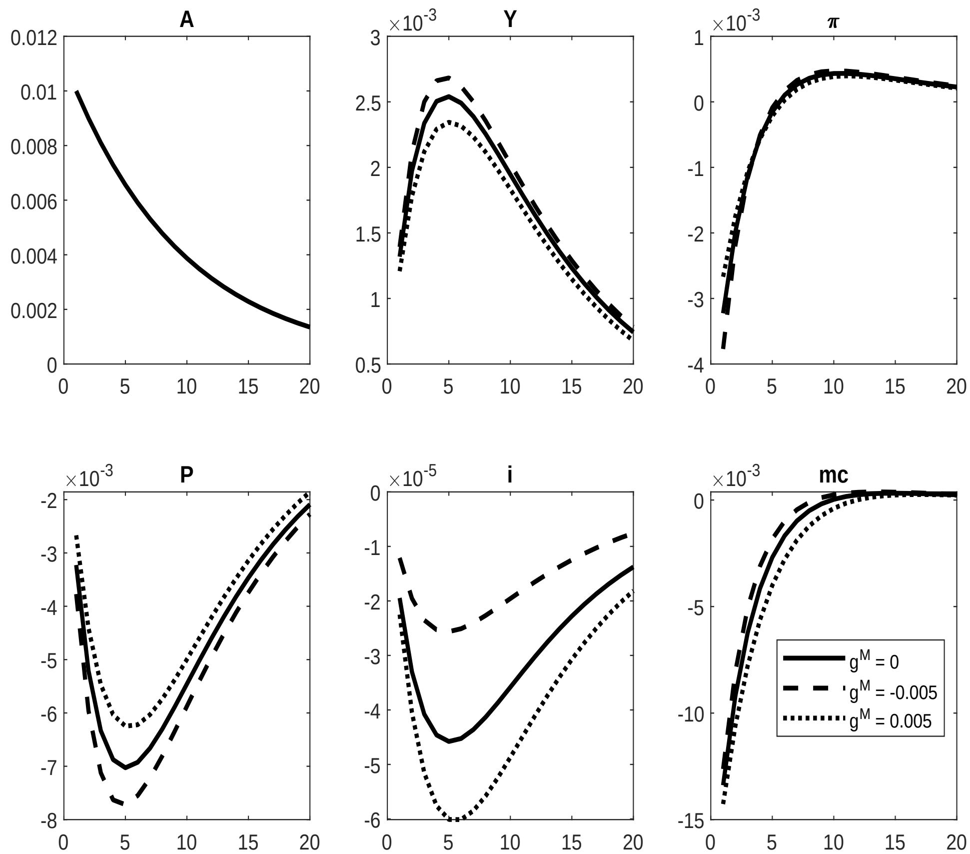
It turns out that for low values of g^M (equivalently, values of Π near one) the model dynamics are not much affected. And it is *much* easier to linearize the model about a zero inflation steady state (as we will see in a bit). Below, I show impulse responses to monetary and productivity shocks for different parameterizations of steady state inflation. I'm using a quarterly calibration. Inflation is typically quote at an annual frequency (so approximately four times the quarterly number). Hence, $g^M = -0.005$ corresponds to $\pi^A = -2$ percent and $g^M = 0.005$ corresponds to $\pi^A = 2$ percent, where the A superscript stands for annualized.

Figure 7: IRFs to a Monetary Shock, Different g^M



Qualitatively, the impulse responses for to both shocks are the same regardless of the level of trend inflation. There are some (small) quantitative differences, particularly in response to the productivity shock. But, ignoring trend inflation is typically not much of a big deal for understanding equilibrium dynamics, at least in a small-scale model. This does not mean that trend inflation is irrelevant, particularly for welfare – as noted above, higher trend inflation leads to price dispersion, which works like a reduction in aggregate productivity. It also does not mean that substantially higher levels of inflation (like 4 percent or more at an annualized frequency) will not have substantive impacts (indeed, if trend inflation is high or low enough, it will not be possible to solve the model).

Figure 8: IRFs to a Productivity Shock, Different g^M



6.5 A Taylor Rule Formulation

I have written the model with an exogenous rule for the money supply. This is not the way that the model is typically written down. Modern central banks pay little attention to the money supply, and instead focus on targeting short-term interest rates.

We can replace the money growth rule with an interest rate rule of the sort proposed by Taylor (1993). In particular:

$$i_t = i + \phi_\pi(\ln \Pi_t - \ln \Pi) + \phi_y(\ln Y_t - \ln Y) + u_t \tag{59}$$

$$u_t = \rho_u u_{t-1} + s_u \varepsilon_{u,t} \tag{60}$$

One can alternatively write this in levels; taking logs gives the expression above.

$$1 + i_t = (1 + i) \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp(u_t) \tag{61}$$

In this formulation, the (nominal) interest rate reacts to deviations of inflation from its long-run level and to output from its steady state. For a determinate equilibrium (more on this below), we need ϕ_π and ϕ_y to be sufficiently big (more on this later). I will always assume that $\phi_\pi > 1$ – this is typically referred to as the "Taylor principle." The condition needed for determinacy is actually slightly different than that, but if $\phi_y = 0$, then $\phi_\pi > 1$ is necessary for a unique equilibrium. i is the steady state interest rate (which is a function of the steady state real interest, $\frac{1}{\beta} - 1$, and the steady state inflation rate, Π, which we can treat as an exogenous policy parameter). u_t is an autocorrelated shock – given that there are no endogenous state variables in the model, for persistent effects of a monetary shock, we need an autocorrelated shock. Alternatively, we could get persistence by assuming some form of interest rate smoothing (more on this later).

To solve the model with a Taylor rule, we simply replace the money growth rule with the rule for i_t. This makes the nominal money supply an endogenous variable – it reacts to meet money demand given the interest rate target from the policy rule. Impulse responses to a monetary shock (u_t) and a productivity shock with a Taylor rule formulation are shown below. In generating these, I assume $\phi_\pi = 1.5$ and $\phi_y = 0.5$.

Figure 9: IRFs to a Monetary Shock, Taylor Rule

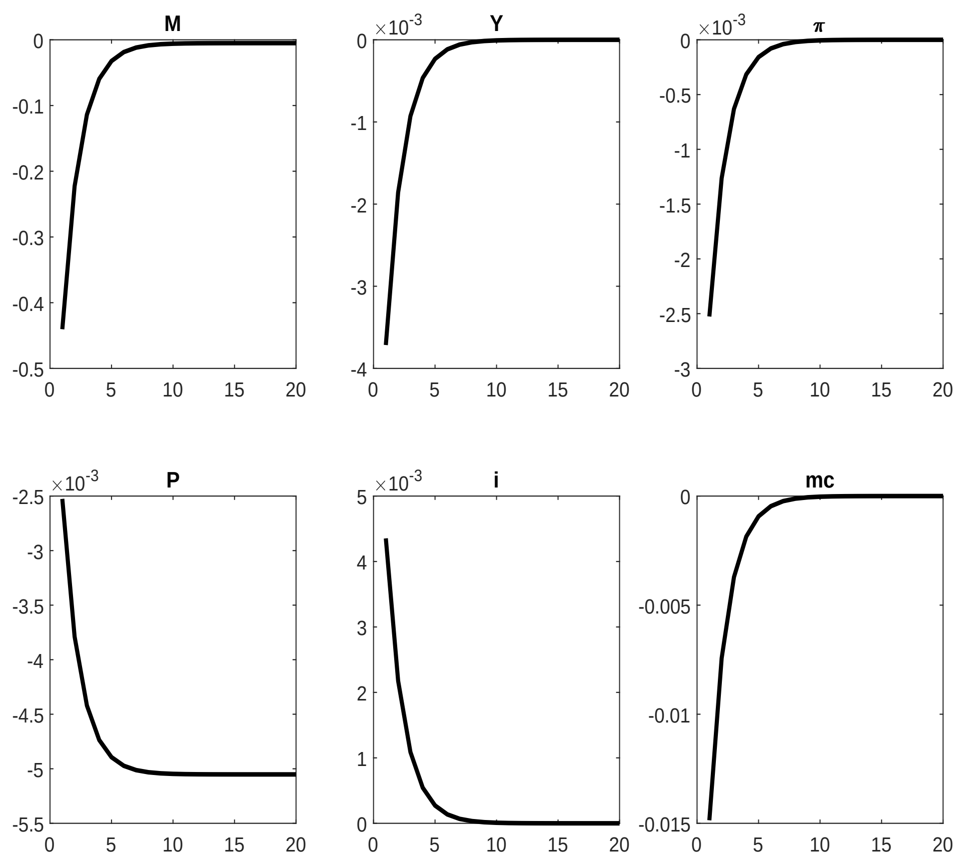
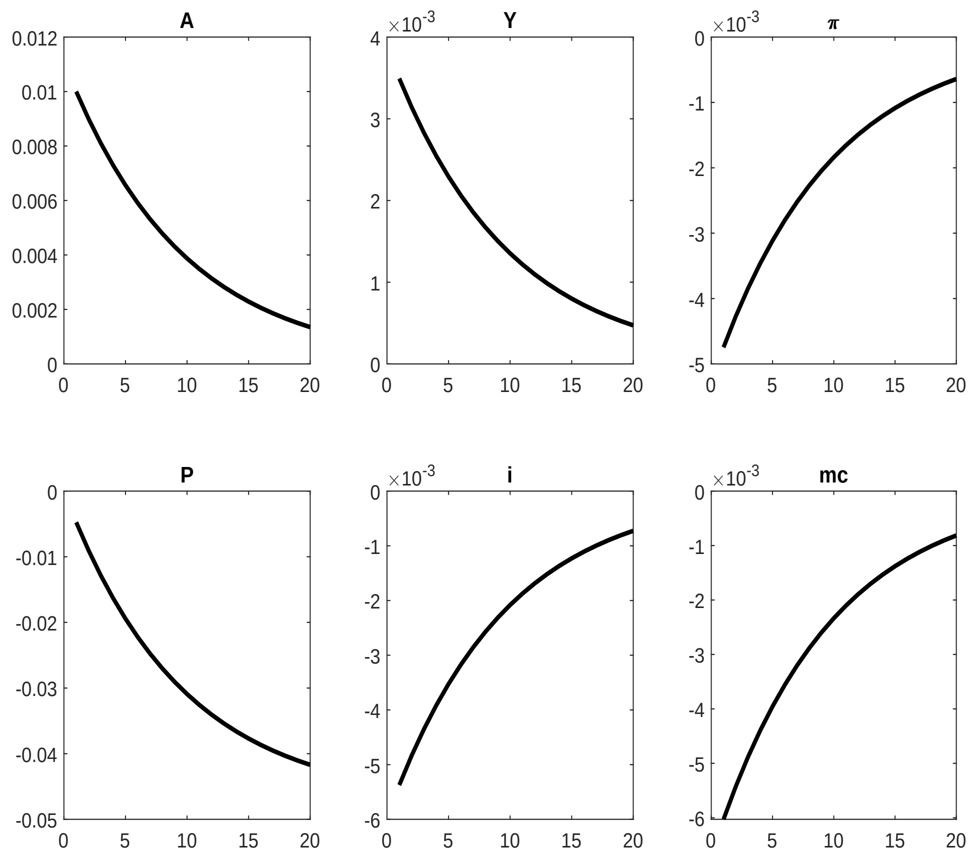


Figure 10: IRFs to a Productivity Shock, Taylor Rule

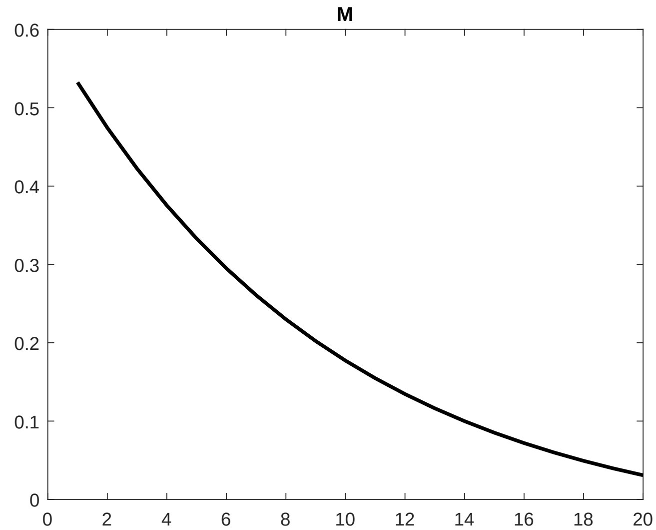


The responses to a monetary shock are qualitatively similar to what we get with a money growth rule, except that the nominal interest rate response is more conventional. Output, inflation, the price level, and the money supply all fall. Output and inflation eventually revert to where they started, and the price level ends up permanently lower. In response to the productivity shock, we also get a similar pattern – output rises, but undershoots the increase in productivity. Inflation and the price level fall. One difference relative to the money growth rule – the price level ends up permanently lower in response to the productivity shock (it was mean-reverting in the money growth specification).

As noted, a Taylor rule makes the money supply endogenous – it adjusts to meet money demand given the interest rate target. Below, I show the impulse response of the nominal money supply to a productivity shock with a Taylor rule. The money supply increases when there is a positive productivity shock. This means that the central bank is (partially) accommodating the productivity shock when it follows a Taylor rule. This is what allows output to respond more (relative to the money growth rule) in response to the productivity shock.

As long as utility from real money balances is additively separable, the evolution of the money

Figure 11: Endogenous Money Response to Productivity Shock, Taylor Rule



supply with a Taylor rule is actually irrelevant for the equilibrium dynamics of output, inflation, and the interest rate to any kind of shock. We could solve the model without reference to money demand altogether and get the same exact impulse responses we have above. For this reason, it is quite common to consider the economy to be "cashless" in New Keynesian models. In the background, there is a demand for money and an endogenous reaction of the money supply to meet that demand at the desired interest rate, but the exact specification of money demand is irrelevant for the equilibrium dynamics of other variables. So, many people ignore it altogether.

7 Log Linearization

The basic New Keynesian model is most often presented in log-linear form. I'm going to use the notation that $x_t = \ln X_t - \ln X = \frac{dX_t}{X}$ for variables that are already capitalized, and $\widetilde{x}$ notation for variables that are lowercase (with the exception that I'm doing to treat $i_t = di_t$ and $\pi_t d \ln \Pi_t$). This just eases things up a bit.

Take logs of the Euler equation on bonds. We get:

$$\ln 1 = \ln \Lambda_{t,t+1} + \ln(1 + i_t) - \ln \Pi_{t+1}$$

Note that we can write the stochastic discount factor in terms of consumption:

$$\ln 1 = \ln \beta - \sigma \ln C_{t+1} + \sigma \ln C_t + \ln(1 + i_t) - \ln \Pi_{t+1}$$

Totally differentiating:

$$0 = -\sigma \mathbb{E}_t c_{t+1} + \sigma c_t + i_t - \mathbb{E}_t \pi_{t+1}$$

Here, I am using the notation that $c_t = dC_t/C$, $i_t \approx \ln(1+i_t)$, and $\pi_t \approx \ln \Pi_t$. Imposing the resource constraint (i.e. $c_t = y_t$), we have:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \tag{62}$$

(62) is often referred to as the "IS" equation. This sounds a little odd, because there is no investment (nor saving) in the model, but it's the same idea of imposing asset market-clearing and taking the optimal condition (Euler equation) and imposing the aggregate resource constraint. Since $r_t = i_t - \mathbb{E}_t \pi_{t+1}$, this is showing a negative relationship between current output and the real interest rate, taking the expected future level of output as given.

Next, note that the labor supply condition is already log-linear:

$$\chi n_t = \widetilde{w}_t - \sigma c_t$$

Again, just to be clear $n_t = dN_t N$ and $c_t = C_t/C$, but $\widetilde{w}_t = dw_t/w$ since I'm already using a lowercase variable. But we can substitute out $c_t = y_t$ from the resource constraint, leaving:

$$\chi n_t = \widetilde{w}_t - \sigma y_t \tag{63}$$

Next, log-linearize the money demand relationship:

$$\ln m_t = \ln \psi + \sigma \ln C_t + \ln(1+i_t) - \ln i_t$$

Totally differentiating, we have:

$$\widetilde{m}_t = \sigma c_t + i_t - \frac{i_t}{i}$$

Again, keep the notation in mind: $i_t = di_t$. We can write this as:

$$\widetilde{m}_t = \sigma y_t + \frac{i-1}{i} i_t \tag{64}$$

Since $i < 1$, the coefficient on i_t is negative. This just says that money demand is increasing in output and decreasing in the nominal interest rate.

I'm going to proceed a bit out of order. The marginal cost condition and production function are already log-linear:

$$\widetilde{mc}_t = \widetilde{w}_t - a_t \tag{65}$$

$$a_t + n_t = y_t + \widetilde{v}_t^P \tag{66}$$

Again, take note of my notation: for example, $a_t = d \ln A_t = \frac{dA_t}{A}$. Given this, exogenous processes are straightforward:

$$a_t = \rho_A a_{t-1} + s_A \varepsilon_{A,t} \tag{67}$$

$$\widetilde{g}_t^M = \rho_M \widetilde{g}_{t-1}^M + s_M \varepsilon_{M,t} \tag{68}$$

$$\widetilde{g}_t^M = \widetilde{m}_t - \widetilde{m}_{t-1} + \pi_t \tag{69}$$

The nasty part involves linearizing the pricing conditions. To do this, we're going to assume we are linearizing about a zero inflation steady state: $\Pi = 1$ (so $g^M = 0$). For small steady state money growth rates, this isn't going to be a big deal (see above), and it makes the math *much easier*.

Start with the evolution of inflation condition. Take logs:

$$\ln 1 = \ln \left[(1-\phi) \left(p_t^{\#} \right)^{1-\epsilon} + \phi \Pi_t^{\epsilon - 1} \right]$$

Now, totally differentiate. This is pretty easy because, in steady state, the term in brackets is one. So we have:

$$0 = (1-\phi)(1-\epsilon) \left(p^{\#} \right)^{-\epsilon} dp_t^{\#} + (\epsilon - 1)\phi \Pi^{\epsilon-1} d\Pi_t$$

Or:

$$0 = (1-\phi)(1-\epsilon) \left(p^{\#} \right)^{1-\epsilon} \frac{dp_t^{\#}}{p^{\#}} + (\epsilon - 1)\phi \Pi^{\epsilon-1} d\Pi_t$$

Now, since $p^{\#} = 1$ if steady state inflation is zero, and because $\Pi = 1$ and $d\Pi_t = \pi_t$, we have:

$$0 = (1-\phi)(1-\epsilon)\widetilde{p}_t^{\#} + \phi(\epsilon - 1)\pi_t$$

Hence, we have:

$$\pi_t = \frac{1-\phi}{\phi} \widetilde{p}_t^{\#} \tag{70}$$

Next, go to the price dispersion term. Take logs:

$$\ln v_t^P = \ln \left[(1-\phi) \left(p_t^{\#} \right)^{-\epsilon} + \phi \Pi_t^{\epsilon} v_{t-1}^P \right]$$

With zero trend inflation, we know that $v_t^P = 1$ in steady state. This makes the total differentiation easier. We have:

$$\frac{dv_t^P}{v^P} = \frac{1}{v^P} \left[-\epsilon(1-\phi) \left(p^{\#} \right)^{-\epsilon-1} dp_t^{\#} + \epsilon \phi \Pi^{\epsilon-1} v^P d\Pi_t + \phi \Pi^{\epsilon} dv_{t-1}^P \right]$$

Now, again using the fact that $p^{\#} = \Pi = v^P = 1$, we can write this as:

$$\widetilde{v}_t^P = -\epsilon(1-\phi)\widetilde{p}_t^{\#} + \epsilon\phi\pi_t + \phi\widetilde{v}_{t-1}^P$$

Or:

$$\widetilde{v}_t^P = -\epsilon\left((1-\phi)\widetilde{p}_t^{\#} - \phi\pi_t\right) + \phi\widetilde{v}_{t-1}^P$$

But, using (70), the terms involving inflation and the relative reset price drop out, so we are left with:

$$\widetilde{v}_t^P = \phi\widetilde{v}_{t-1}^P \tag{71}$$

But since there is nothing to shock $\widetilde{v}_t^P$ out of steady state (the point of approximation), we have $\widetilde{v}_t^P = 0$ – it drops out of the linearization (including in the production function). This would not be true if trend inflation were non-zero, but around a zero inflation steady state, we can ignore price dispersion.

Now we need to log-linearize the reset price expression. Since this is multiplicative, it is already log-linear:

$$\widetilde{p}_t^{\#} = \widehat{x}_{1,t} - \widehat{x}_{2,t}$$

Where $\widehat{x}_{1,t} = d\widehat{X}_{1,t}/\widehat{X}_1$. Now we need to linearize those two things. We have:

$$\ln\widehat{X}_{1,t} = \ln\left[mc_tY_t + \phi\mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{\epsilon}\widehat{X}_{1,t+1}\right]$$

Totally differentiate:

$$\frac{d\widehat{X}_{1,t}}{\widehat{X}_t} = \frac{1}{\widehat{X}_{1,t}}\left[Y dmc_t + mcdY_t + \phi\Lambda\Pi^{\epsilon}d\widehat{X}_{1,t+1} + \epsilon\phi\Lambda\Pi^{\epsilon-1}\widehat{X}_1 d\Pi_{t+1} + \phi\Pi^{\epsilon}\widehat{X}_1 d\Lambda_{t,t+1}\right]$$

Given that $\Lambda = \beta$ and $\Pi = 1$, we can write this as:

$$\widehat{x}_{1,t} = \frac{1}{\widehat{X}_1}\left[mcY\widetilde{mc}_t + mcYy_t + \phi\beta\widehat{X}_1\widehat{x}_{1,t+1} + \epsilon\phi\beta\widehat{X}_1\pi_{t+1} + \phi\beta\widehat{X}_1\lambda_{t,t+1}\right]$$

Where $\lambda_{t,t+1} = \frac{d\Lambda_{t,t+1}}{\Lambda}$. We can write this further:

$$\widehat{x}_{1,t} = \frac{mcY}{\widehat{X}_1}\widetilde{mc}_t + \frac{mcY}{\widehat{X}_1}y_t + \phi\beta\mathbb{E}_t\widehat{x}_{1,t+1} + \epsilon\phi\beta\mathbb{E}_t\pi_{t+1} + \phi\beta\mathbb{E}_t\lambda_{t,t+1}$$

Note, that, in steady state, $\widehat{X}_1 = \frac{mcY}{1-\phi\beta}$ assuming $\Pi = 1$, so this reduces further to:

$$\widehat{x}_{1,t} = (1-\phi\beta)\widetilde{mc}_t + (1-\phi\beta)y_t + \phi\beta\mathbb{E}_t\widehat{x}_{1,t+1} + \epsilon\phi\beta\mathbb{E}_t\pi_{t+1} + \phi\beta\mathbb{E}_t\lambda_{t,t+1} \tag{72}$$

Now do log-linearization of $\widehat{X}_t$:

$$\ln \widehat{X}_{2,t} = \ln \left[Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} \widehat{X}_{2,t+1} \right]$$

Totally differentiate:

$$\frac{d\widehat{X}_{2,t}}{\widehat{X}_2} = \frac{1}{\widehat{X}_2} \left[dY_t + \phi \Lambda \Pi^{\epsilon-1} d\widehat{X}_{2,t} + (\epsilon - 1)\phi \Lambda \Pi^{\epsilon-2} \widehat{X}_2 d\Pi_{t+1} + \phi \Lambda \Pi^{\epsilon-1} d\lambda_{t,t+1} \right]$$

Which can be written:

$$\widehat{x}_{2,t+1} = \frac{1}{\widehat{X}_2} \left[Y y_t + \phi \beta \widehat{X}_2 \widehat{x}_{2,t+1} + (\epsilon - 1)\phi \beta \widehat{X}_2 \pi_{t+1} + \phi \beta \widehat{X}_t \lambda_{t,t+1} \right]$$

Where I have imposed steady state values where relevant: $\Lambda = \beta$ and $\Pi = 1$. Note that $\widehat{X}_2 = \frac{Y}{1-\phi\beta}$. Using this fact, we can write the above as:

$$\widehat{x}_{2,t+1} = (1 - \phi\beta) y_t + \phi\beta \mathbb{E}_t \widehat{x}_{2,t+1} + (\epsilon - 1)\phi\beta \mathbb{E}_t \pi_{t+1} + \phi\beta \mathbb{E}_t \lambda_{t,t+1} \tag{73}$$

Now subtract (73) from (72). We have:

$$\widehat{x}_{1,t} - \widehat{x}_{2,t} = (1 - \phi\beta)\widehat{mc}_t + \phi\beta \mathbb{E}_t \pi_{t+1} + \phi\beta \left(\mathbb{E}_t x_{1,t+1} - \mathbb{E}_t x_{2,t+1} \right)$$

Now, since $\widehat{x}_{1,t} - \widehat{x}_{2,t} = \widetilde{p}_t^{\#}$, we can write this:

$$\widetilde{p}_t^{\#} = (1 - \phi\beta)\widehat{mc}_t + \phi\beta \mathbb{E}_t \pi_{t+1} + \phi\beta \mathbb{E}_t \widetilde{p}_{t+1}^{\#}$$

But from (70), we know that:

$$\widetilde{p}_t^{\#} = \frac{\phi}{1-\phi} \pi_t$$

Plug this in, and we have:

$$\frac{\phi}{1-\phi} \pi_t = (1 - \phi\beta)\widetilde{mc}_t + \phi\beta \mathbb{E}_t \pi_{t+1} + \frac{\phi^2 \beta}{1-\phi} \mathbb{E}_t \pi_{t+1}$$

But this is:

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi} \widetilde{mc}_t + (1-\phi)\beta \mathbb{E}_t \pi_{t+1} + \phi\beta \mathbb{E}_t \pi_{t+1}$$

But this is just:

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi} \widetilde{mc}_t + \beta \mathbb{E}_t \pi_{t+1} \tag{74}$$

(74) is called the *New Keynesian Phillips Curve* and is one of the most important equations in modern macro. With this equation in hand, we have eliminated (and no longer need to keep track of) “intermediate” variables like $\widetilde{p}_t^{\#}$, $\widehat{x}_{1,t}$, or $\widehat{x}_{2,t}$¿

To gain some intuition for this expression, solve it forward. We can express inflation as proportional to a present discounted value of real marginal cost:

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi}\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \widetilde{mc}_{t+s} \tag{75}$$

With monopolistic competition, intermediate good firms want their price to be a *markup* ($\epsilon/(\epsilon-1)$) over marginal cost. If current (or expected future) marginal cost is supposed to be high, firms who can need to raise prices today to hit their desired markup. If they don't adjust today, they will have lower than desired markups in the future. So marginal cost pressures put upward pressure on inflation. How much upward pressure depends on ϕ – if ϕ is small, the coefficient $\frac{(1-\phi)(1-\phi\beta)}{\phi}$ is large, and lots of firms will adjust, so inflation will react to marginal cost (current or future) a lot (and vice-versa if ϕ is low). In the limiting case where prices are flexible, $\frac{(1-\phi)(1-\phi\beta)}{\phi} \to \infty$, which necessitates $\widehat{mc}_{t+s} = 0$, which means all firms will have their desired markup each period.

7.1 Linearized Conditions in One Place

Below are the remaining conditions in one place, where I have substituted out $\widetilde{p}_t^{\#}$, $\widehat{x}_{1,t}$, $\widehat{x}_{2,t}$, $\widetilde{v}_t^p$, c_t, and $\lambda_{t,t+1}$. As noted early, price dispersion is constant to a first-order approximation about a zero inflation steady state, and $\widetilde{p}_t^{\#}$, $\widehat{x}_{1,t}$, $\widehat{x}_{2,t}$ are subsumed in the New Keynesian Phillips Curve. I no longer need to keep track of the stochastic discount factor either, and I have imposed the market-clearing condition that $c_t = y_t$. This leaves me with nine equations and nine variables (y_t, i_t, π_t, n_t, $\widetilde{w}_t$, $\widetilde{mc}_t$, $\widetilde{m}_t$, a_t, and $\widetilde{g}_t^M$) as follows:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \tag{76}$$

$$\chi n_t = \widetilde{w}_t - \sigma y_t \tag{77}$$

$$\widetilde{m}_t = \sigma y_t + \frac{i-1}{i} i_t \tag{78}$$

$$\widetilde{mc}_t = \widetilde{w}_t - a_t \tag{79}$$

$$a_t + n_t = y_t \tag{80}$$

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi}\widetilde{mc}_t + \beta \mathbb{E}_t \pi_{t+1} \tag{81}$$

$$\widetilde{g}_t^M = \rho_M \widetilde{g}_{t-1}^M + s_M \varepsilon_{M,t} \tag{82}$$

$$a_t = \rho_A a_{t-1} + s_A \varepsilon_{A,t} \tag{83}$$

$$\widetilde{g}_t^M = \widetilde{m}_t - \widetilde{m}_{t-1} + \pi_t \tag{84}$$

In terms of the economics of what these equations are saying, (76) is the IS equation, which is essentially an aggregate demand condition (note that $r_t = i_t - \mathbb{E}_t \pi_{t+1}$). (77) is a labor supply condition. (78) is a money demand condition. The money supply equation is (82), and (84) is just a relationship between growth in the nominal money supply, real balances, and inflation. (79) defines real marginal cost; as discussed earlier, this is essentially a labor demand condition. (80) is the production function. (81) is the Phillips curve, which can be interpreted as an aggregate supply relationship (more below). Essentially, we have three markets – for goods, for labor, and for money – with demand and supply relationships going into each. In equilibrium, prices must adjust so that all three hold. (83) is just an exogenous process for productivity.

Note that, if instead of an exogenous money supply rule, we wanted to model monetary policy with an interest rate rule, we would just replace (82) with a Taylor rule. The money supply would then be exogenous, given a target for the interest rate. The linearized Taylor rule (which takes out the constants) would be:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + u_t \tag{85}$$

7.2 System Reduction, Flexible Price Output, the Output Gap, and the Natural Rate

We can actually reduce the system quite a bit further. Combining the production function with the labor supply condition to substitute out n_t, we have:

$$\chi(y_t - a_t) = \widetilde{w}_t - \sigma y_t$$

From the labor demand condition, we can eliminate $\widetilde{w}_t$ from this expression:

$$\chi(y_t - a_t) = \widetilde{mc}_t + a_t - \sigma y_t$$

But this can be written:

$$(\sigma + \chi) y_t = \widetilde{mc}_t + (1 + \chi)\widetilde{a}_t$$

Now, let us introduce an important concept, called the "flexible price level of output." Sometimes this is called "potential output."[2] The flexible price level of output is defined as the equilibrium level of output that would emerge in equilibrium in the hypothetical world in which prices are completely flexible (i.e. $\phi = 0$). We could get this using the conditions in levels, but it is straightforward to do so in the linearized model. When prices are flexible, $\widetilde{mc}_t = 0$ – real marginal cost will be constant (in levels, equal to $\frac{\epsilon - 1}{\epsilon}$). This would just mean that all firms charge a constant markup over marginal cost. Let's call $\widetilde{y}_t^f$ the flexible price level of output. From above, we have:

$$(\sigma + \chi) y_t^f = (1 + \chi) a_t$$

Or:

[2]Though one wants to be careful here. The flexible price level of output is not in general going to be *efficient* since there is a monopoly distortion. Even without price stickiness, labor would be paid less than its marginal product.

$$y_t^f = \frac{1+\chi}{\sigma+\chi} a_t$$

In other words, since productivity is the only (real) exogenous state variable, equilibrium output in the hypothetical world with flexible prices is just a linear function of it. But using this definition, we can actually sub out real marginal cost. We have:

$$\widetilde{mc}_t = (\sigma+\chi) y_t - (1+\chi) a_t$$

But since $a_t = \frac{\sigma+\chi}{1+\chi} y_t^f$, we have:

$$\widetilde{mc}_t = (\sigma+\chi)\left(y_t - y_t^f\right)$$

In other words, real marginal cost is just proportional to the *output gap*, $y_t - y_t^f$. Define $x_t \equiv y_t - y_t^f$ as the gap. If $x_t > 0$, then $mc_t > 0$, which means that the average markup is lower than desired. This puts upward pressure on inflation (and vice-versa). With this, we can write the Phillips Curve more compactly as:

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} \tag{86}$$

Where $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}(\sigma+\chi)$. This is in-line with empirical Phillips Curves, which show some relationship between economic activity and inflation (typically in terms of an unemployment rate, which would naturally be negatively related to an output gap). With this, we can reduce the system further:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \tag{87}$$

$$\widetilde{m}_t = \sigma y_t + \frac{i-1}{i} i_t \tag{88}$$

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} \tag{89}$$

$$\widetilde{g}_t^M = \rho_M \widetilde{g}_{t-1}^M + s_M \varepsilon_{M,t} \tag{90}$$

$$a_t = \rho_A a_{t-1} + s_A \varepsilon_{A,t} \tag{91}$$

$$\widetilde{g}_t^M = \widetilde{m}_t - \widetilde{m}_{t-1} + \pi_t \tag{92}$$

$$y_t^f = \frac{1+\chi}{\sigma+\chi} a_t \tag{93}$$

$$x_t = y_t - y_t^f \tag{94}$$

In this formulation, (89) summarizes the supply-side of the economy: production, labor demand, labor supply, and price-setting. (87) summarizes demand for goods, and (88) the demand for money.

We can actually go even further. Add and subtract y_t^f and $\mathbb{E}_t y_{t+1}^f$ from the (87):

$$y_t - y_t^f + y_t^f = \mathbb{E}_t y_{t+1} - \mathbb{E}_t y_{t+1}^f + \mathbb{E}_t y_{t+1}^f - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right)$$

Which can be written more compactly:

$$x_t = \mathbb{E}_t x_{t+1} + \mathbb{E}_t y_{t+1}^f - y_t^f - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right)$$

Now, one might wonder where this is going. Define the *natural rate of interest*, r_t^f (or sometimes r_t^*, or "r-star") as the hypothetical real interest rate that would clear the goods market with no price rigidity. This would satisfy:

$$\frac{1}{\sigma} r_t^f = \mathbb{E}_t y_{t+1}^f - y_t^f$$

But then we can write the IS equation:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right)$$

The term $i_t - \mathbb{E}_t \pi_{t+1} - r_t^f = r_t - r_t^f$ is a (real) interest rate gap – the gap between the equilibrium real interest rate and the natural rate. We can actually write the entire system as a function of r_t^f and drop y_t^f and a_t. How? Note that:

$$r_t^f = \sigma\left(\mathbb{E}_t y_{t+1}^f - y_t^f\right)$$

But, from above, this may be written:

$$r_t^f = \sigma\left(\frac{1+\chi}{\sigma+\chi}\mathbb{E}_t a_{t+1} - \frac{1+\chi}{\sigma+\chi} a_t\right) = \frac{\sigma(1+\chi)}{\sigma+\chi}\left(\mathbb{E}_t a_{t+1} - a_t\right)$$

But since $\mathbb{E}_t a_{t+1} = \rho_A a_t$, this is:

$$r_t^f = \frac{\sigma(1+\chi)(\rho_A - 1)}{\sigma+\chi} a_t$$

But solving backwards, we can write this as:

$$r_t^f = \frac{\sigma(1+\chi)(\rho_A - 1)}{\sigma+\chi}\rho_A a_{t-1} + \frac{\sigma(1+\chi)(\rho_A - 1)}{\sigma+\chi} s_A \varepsilon_{A,t}$$

But since $a_{t-1} = \frac{\sigma+\chi}{\sigma(1+\chi)(\rho_A-1)} r_{t-1}^f$, we have:

$$r_t^f = \rho_A r_{t-1}^f + s_r \varepsilon_{A,t} \tag{95}$$

Where:

$$s_r = \frac{\sigma(1+\chi)(\rho_A - 1)}{\sigma+\chi} s_A$$

Then the full system is:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right) \tag{96}$$

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} \tag{97}$$

$$\widetilde{m}_t = \sigma y_t + \frac{i-1}{i} i_t \tag{98}$$

$$\widetilde{g}_t^M = \rho_M \widetilde{g}_{t-1}^M + s_M \varepsilon_{M,t} \tag{99}$$

$$\widetilde{g}_t^M = \widetilde{m}_t - \widetilde{m}_{t-1} + \pi_t \tag{100}$$

$$r_t^f = \rho_A r_{t-1}^f + s_r \varepsilon_{A,t} \tag{101}$$

(96) is an aggregate demand relationship (IS equation) and (97) is an aggregate supply relationship. (98)-(100) describe the money market, and (101) is an exogenous process for the natural rate of interest.

7.3 The Three-Equation NK Model

Although I have done it with money, it is uncommon to even bother modeling money in most applications. Rather, most of the time researchers assume some kind of interest rate rule. A modified version of the Taylor rule (that reacts to the output gap, rather than the log deviation of output from steady state) is:

$$i_t = \phi_\pi \pi_t + \phi_x x_t + u_t \tag{102}$$

Where u_t obeys a potentially persistent process. One also often sees people build in interest rate persistence via a lagged interest rate term:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \pi_t + (1-\rho_i)\phi_x x_t + u_t \tag{103}$$

Here, $0 \leq \rho_i < 1$, and I have just scaled the coefficients on inflation and the output gap by $1-\rho_i$ (more on why later). In (103), we would typically assume that u_t is an iid shock, but one could entertain persistence. Either way, if one includes (102) or (103), one gets a three-equation model:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right) \tag{104}$$

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} \tag{105}$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t + u_t \tag{106}$$

This is three equations in three endogenous variables – inflation, the output gap, and the interest rate. We could include the money demand equation, (99), and the relationship between nominal money growth, real balances, and inflation, (100), if we wanted to (note we would drop the exogenous money supply rule in favor of the interest rate rule), but we actually don't need to in order to determine x_t and i_t. We also need the exogenous process for r_t^f (and potentially u_t), but (104)-(106) summarize the endogenous variables of the model concisely in three equations.

This three equation model is nice. It's nice for several reasons. First, it is easy to work with and build intuition for. Second, it has an AD-AS flavor from an undergraduate textbook. Third, it focuses in on exactly the variables central banks care about. In the US, the Federal Reserve has a "dual mandate" for "price stability" and "full employment." We can interpret π_t (inflation) as measuring price stability and x_t (the output gap) as a measure of how close we are to full employment. The Fed's principal policy tool is the short-term interest rate, i_t. So these three equations describe the two targets modern central banks focus on (inflation and the output gap / unemployment gap) and its principal instrument (a nominal interest rate). Kind of cool!

7.4 Cost-Push Shocks

It is common to include a "cost-push" shock into the NKPC:

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t^p \tag{107}$$

Where u_t^p follows a stationary AR(1) process:

$$u_t^P = \rho_p u_{t-1}^p + s_p \varepsilon_{p,t} \tag{108}$$

A shock like this can be motivated as time-variation in desired markups (i.e. a shock to ϵ_t in the full, non-linear model). People also sometimes think about a shock like this as representing an oil price shock. In any event, as we will see, this shock is "nice" in that it introduce a non-trivial tradeoff for a central bank that wants to stabilize both inflation and the output gap.

8 Method of Undetermined Coefficients

The method of undetermined coefficients is a way to solve for linearized policy functions by hand. In principle, it could be used in a more complicated model, but it is especially useful in a small-scale model with no endogenous state variables. It allows us to build better intuition for how the model economy works than simply plugging equations into a computer.

The basic gist of the methodology is to guess that forward-looking jump variables are linear functions of state variable(s). In principle, we could do this with many exogenous states, but it's most straightforward with just one. Consider the basic NK model from above, (104)-(106). Suppose that inflation and output are linear function of the exogenous state variable, r_t^f:

$$\pi_t = \theta_\pi r_t^f$$
$$x_t = \theta_x r_t^f$$

Given that we have assumed r_t^f (which can be thought of as exogenous) is stationary, we do not need to worry about imposing that inflation and the gap do not explode. These linear functions will be time-invariant (i.e. they hold at all times). Plug them into the NKPC:

$$\theta_\pi r_t^f = \gamma \theta_x r_t^f + \beta \theta_\pi \mathbb{E}_t r_{t+1}^f$$

The last line follows because $\mathbb{E}_t \pi_{t+1} = \theta_\pi \mathbb{E}_t r_{t+1}^f$. But $\mathbb{E}_t r_{t+1}^f = \rho_A r_t^f$. So we have:

$$\theta_\pi r_t^f = \gamma \theta_x r_t^f + \beta \theta_\pi \rho_A r_t^f$$

This is one equation in two unknowns. r_t^f drops out. Solving for θ_π as a function of θ_x, we have:

$$\theta_\pi = \frac{\gamma}{1 - \beta \rho_A} \theta_x$$

Now go to the IS equation and do the same. Plug in the Taylor rule to write i_t in terms of π_t and x_t (and assume $u_t = 0$, which is not without loss of generality given the linearity of the model – we will get the same policy functions in response to r_t^f whether we allow for the monetary policy shock or not):

$$\theta_x r_t^f = \theta_x \mathbb{E}_t r_{t+1}^f - \frac{1}{\sigma} \left(\phi_\pi \pi_t + \phi_x x_t - \theta_\pi \mathbb{E}_t r_{t+1}^f \right) + \frac{1}{\sigma} r_t^f$$

Note again that $\mathbb{E}_t r_{t+1}^f = \rho_A r_t^f$, and plug in inside the Taylor rule to eliminate π_t and x_t. We have:

$$\sigma \theta_x r_t^f = \sigma \theta_x \rho_A r_t^f - \phi_\pi \theta_\pi r_t^f - \phi_x \theta_x r_t^f + \theta_\pi \rho_A r_t^f + r_t^f$$

We can now drop the r_t^f. We have:

$$\theta_x \left(\sigma (1 - \rho_A) + \phi_x \right) + \theta_\pi (\phi_\pi - \rho_A) = 1$$

We now have two equations in two unknowns. Plug in for θ_π using the expression we found from the NKPC:

$$\theta_x \left(\sigma (1 - \rho_A) + \phi_x \right) + \theta_x \frac{\gamma (\phi_\pi - \rho_A)}{1 - \beta \rho_A} = 1$$

Which is:

$$\theta_x \left[(1 - \beta \rho_A) \left(\sigma (1 - \rho_A) + \phi_x \right) + \gamma (\phi_\pi - \rho_A) \right] = 1 - \beta \rho_A$$

Or:

$$\theta_x = \frac{1 - \beta \rho_A}{(1 - \beta \rho_A) \left(\sigma (1 - \rho_A) + \phi_x \right) + \gamma (\phi_\pi - \rho_A)} \tag{109}$$

But then:

$$\theta_\pi = \frac{\gamma}{(1-\beta\rho_A)\left(\sigma(1-\rho_A)+\phi_x\right)+\gamma(\phi_\pi-\rho_A)} \tag{110}$$

These are pretty intuitive. First, suppose that $\gamma \to \infty$ (prices are completed flexible). Then $\theta_x = 0$ (but θ_π will not be, since γ is in both the numerator and the denominator). In other words, if prices are flexible, then the output gap is always zero (i.e. $y_t = y_t^f$). Second, suppose that $\phi_\pi \to \infty$. Then *both* θ_x and θ_π go to 0. Third, suppose that $\phi_x \to \infty$. Again, *both* θ_π and θ_x go to zero. This is a manifestation of something called the "Divine Coincidence" – in the basic NK model, even though there is on instrument (the policy rate) and two targets (inflation and the output gap), it is possible to stabilize both targets at the same time. To see this better, note that if $\phi_\pi \to \infty$, then $\pi_t = 0$ for the Taylor rule to feature a non-explosive policy rate. But, from the Phillips Curve, if inflation is always zero, then $x_t = 0$ as well.

We can use these coefficients to recover an expression for the nominal interest rate (which is effectively a static variable, given that it only appears in the equilibrium conditions at date t. We have:

$$i_t = \left(\phi_\pi\theta_\pi + \phi_x\theta_x\right) r_t^f$$

Which is:

$$i_t = \frac{\phi_\pi\gamma + \phi_x(1-\beta\rho_A)}{(1-\beta\rho_A)\left(\sigma(1-\rho_A)+\phi_x\right)+\gamma(\phi_\pi-\rho_A)} r_t^f \tag{111}$$

9 Alternative Price Stickiness Model: Rotemberg (1982) Pricing

We have thus far modeled price stickiness via the Calvo (1983) assumption, wherein each period firms face a constant hazard of being able to adjust their price. This means that in equilibrium firms are heterogeneous, but the model is rigged in such a way that aggregation works out nicely. An alternative pricing assumption is based on Rotemberg (1982). In the Rotemberg model, firms face a quadratic cost of price adjustment. In equilibrium they all end up behaving identically. To a first order approximation about a zero inflation steady state, the Rotemberg and Calvo models can be parameterized to be identical. They are not identical to order higher than one, and they have different implications for micro data (e.g. in the Calvo model only a fraction of firms will adjust their price in a given period, but in the Rotemberg model all firms will be changing their prices each period).

In the Rotemberg model, there are many intermediate goods firms. Firms face the same demand curve from the final good firm as earlier. They produce output according to $Y_t(j) = A_t N_t(j)$, just as in the Calvo model. Nominal flow profit for producer j is given by:

$$\Pi_t(j) = P_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t - W_t N_t(j) - \frac{\psi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t \tag{112}$$

The parameter $\psi_p \geq 0$ measures the cost of price adjustment, and it is measured in units of the final good. We can write the profit function in real dollars as:

$$\Pi_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - w_t N_t(j) - \frac{\psi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \tag{113}$$

Since $N_t(j) = Y_t(j)/A_t$, we can write this as:

$$\Pi_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - \frac{w_t}{A_t} Y_t(j) - \frac{\psi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \tag{114}$$

But since $mc_t = w_t/A_t$, this is just:

$$\Pi_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - mc_t Y_t(j) - \frac{\psi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \tag{115}$$

Plugging in the demand function for variety j, we get:

$$\Pi_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - mc_t P_t(j)^{-\epsilon} P_t^{\epsilon} Y_t - \frac{\psi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \tag{116}$$

Each period, firms choose price to maximize the expected present discounted value of flow profit, where discounting is by the household's stochastic discount factor. The problem is dynamic because the adjustment cost function means that current prices are relevant for future profits. The optimality condition for price-setting can be written:

$$\begin{aligned}(\epsilon - 1) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} = & \\ & \epsilon mc_t \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon-1} \frac{Y_t}{P_t} - \psi_p \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right) \frac{Y_t}{P_{t-1}(j)} + \\ & \psi_p \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{P_{t+1}(j)}{P_t(j)} - 1 \right) \left(\frac{P_{t+1}(j)}{P_t(j)} \right) \left(\frac{Y_{t+1}}{P_t(j)} \right) \right]\end{aligned} \tag{117}$$

This first order condition holds for all firms. We will therefore converge to a symmetric equilibrium where all firms behave identically, so $P_t(j) = P_t$ for all j. We can therefore write the FOC as:

$$(\epsilon - 1) \frac{Y_t}{P_t} = \epsilon mc_t \frac{Y_t}{P_t} - \psi_p \left(\Pi_t - 1 \right) \frac{Y_t}{P_t} + \psi_p \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\Pi_{t+1} - 1 \right) \Pi_{t+1} \frac{Y_{t+1}}{P_t} \right]$$

Which can be written:

$$\epsilon - 1 = \epsilon mc_t - \psi_p \pi_t \Pi_{t+1} + \psi_p \mathbb{E}_t \Lambda_{t,t+1} \pi_{t+1} \Pi_{t+1} \frac{Y_{t+1}}{Y_t}$$

Here I am using $\Pi_t = \frac{P_t}{P_{t-1}}$ and $\pi_t = \Pi_t - 1$. To make life especially straightforward, suppose that we have log utility over consumption, so $\Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}}$. Since $C_t = Y_t$ will be the aggregate resource constraint, all terms involving Y_t and Y_{t+1} drop out, leaving:

$$\epsilon - 1 = \epsilon mc_t - \psi_p \pi_t \Pi_{t+1} + \psi_p \beta \mathbb{E}_t \pi_{t+1} \Pi_{t+1} \tag{118}$$

If there is no price rigidity ($\psi_p = 0$), then this reduces to $mc_t = \frac{\epsilon-1}{\epsilon}$, which again just says that price is a constant markup over marginal cost (equivalently, labor is paid a markdown relative to its marginal product).

Let's now log-linearize (118) about a zero inflation steady state. Take logs of both sides:

$$\ln(\epsilon - 1) = \ln\left[\epsilon mc_t - \psi_p \pi_t \Pi_{t+1} + \psi_p \beta \mathbb{E}_t \pi_{t+1} \Pi_{t+1}\right]$$

Totally differentiate:

$$0 = \frac{1}{\epsilon - 1}\left[\epsilon dmc_t - \psi_p \pi d\Pi_{t+1} - \psi_p \Pi d\pi_t + \psi_p \pi d\Pi_{t+1} + \psi_p \beta \Pi d\pi_{t+1}\right]$$

We are linearizing about a zero (net) inflation steady state: this means $\pi = 0$, $\Pi = 1$, $d\pi_t = \pi_t$, and $d\Pi_t = \pi_t$. So we have:

$$0 = \frac{1}{\epsilon - 1}\left[mc\epsilon \frac{dmc_t}{mc} - \psi_p \pi_t + \psi_p \beta \mathbb{E}_t \pi_{t+1}\right]$$

Which can be written:

$$0 = mc\epsilon \widetilde{mc}_t - \psi_p \pi_t + \psi_p \beta \mathbb{E}_t \pi_{t+1}$$

In steady state, $mc = \frac{\epsilon-1}{\epsilon}$, so we can write this further as:

$$\psi_p \pi_t = (\epsilon - 1)\widetilde{mc}_t + \psi_p \beta \mathbb{E}_t \pi_{t+1}$$

Or:

$$\pi_t = \frac{\epsilon - 1}{\psi_p}\widetilde{mc}_t + \beta \mathbb{E}_t \pi_{t+1} \tag{119}$$

Although the coefficient on real marginal cost is different, this is exactly the same expression as (74)! We can pick ψ_p to generate *exactly* the same slope of the NKPC if we want. We again have that, if prices are quite flexible, then ψ_p is low, and the coefficient on marginal cost is big (and vice-versa).

The rest of the model works out similarly without adjustment. The one area where there is some potential adjustment is in the aggregate resource constraint. Because the adjustment cost for changing prices comes out of profit, this will show up in the aggregate resource constraint. In particular, the aggregate resource constraint will be:

$$Y_t = C_t + \frac{\psi_p}{2}\pi_t^2 Y_t \tag{120}$$

In other words, there is some resource loss due to inflation moving around. But if you linearize this about a zero inflation steady state, it drops out:

$$\ln Y_t = \ln\left[C_t + \frac{\psi_p}{2}\pi_t^2 Y_t\right]$$

$$\widetilde{Y}_t = \frac{dC_t}{Y} + \frac{\psi_p}{2}\pi^2\widetilde{Y}_t + 2\psi_p\pi\widetilde{\pi}_t$$

If we are linearizing about the point $\pi = 0$, then the latter terms drop out, and $C = Y$, leaving:

$$\widetilde{Y}_t = \widetilde{C}_t \tag{121}$$

This is analogous to how, in the Calvo model, price dispersion potentially lowers productivity, but this term disappears to first order about a zero inflation steady state.