Problem Set 4

Graduate Macro II, Spring 2024
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. When asked to solve a model in Dynare, unless otherwise instructed you (i) should solve via a first-order approximation, (ii) may give Dynare the equilibrium conditions and do not need to do the linearization by hand, (iii) should show impulse responses up to a horizon of 20 periods, and (iv) should plot impulse responses (or analyze moments) of logged variables. This problem set is due on Canvas by 5:00 pm on March 22.

1. Consider a real business cycle. There is only one shock in the model: a labor supply preference shock. In addition, there is variable capital utilization. Otherwise, the problem is standard. The household owns capital and leases capital services (the product of utilization and physical capital) to firms at rental rate R_t^k. The household also supplies labor. The household's problem may be written:

$$\max_{C_t, K_{t+1}, u_t, N_t, B_{t+1}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \theta_t \frac{N_t^{1+\chi}}{1+\chi} \right\}$$

s.t.

$$C_t + K_{t+1} - (1 - \delta(u_t))K_t + B_{t+1} - B_t \leq w_t N_t + R_t^k u_t K_t + \Pi_t + r_{t-1} B_t$$

Here, $I_t = K_{t+1} - (1 - \delta(u_t))K_t$ is investment. u_t is variable capital utilization. The cost of utilization is faster depreciation:

$$\delta(u_t) = \delta_0 + \phi_1(u_t - 1) + \frac{\phi_2}{2}(u_t - 1)^2$$

θ_t is a stochastic preference shock obeying:

$$\ln \theta_t = (1 - \rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + s_\theta \varepsilon_{\theta,t}$$

The firm produces output according to:

$$Y_t = \widehat{K}_t^{\alpha} N_t^{1-\alpha}$$

The firm can only choose $\widehat{K}_t$, which is equal to $u_t K_t$. Firm profit is:

$$\Pi_t = Y_t - w_t N_t - R_t^k \widehat{K}_t$$

There is no government, and the firm does not issue debt. Asset market clearing therefore requires $B_t = B_{t+1} = 0$.

(a) Derive the first order conditions for the household problem.

(b) Derive the first order conditions for the firm problem.

(c) Derive the aggregate resource constraint.

(d) Derive a parametric restriction on ϕ_1 such that steady state utilization, u, is 1.

(e) Using the above restriction to ensure $u = 1$, derive a parametric restriction on θ, the steady state value of the labor preference parameter, such that $N = 1/3$.

(f) Assume that $\alpha = 1/3$, $\beta = 0.99$, $\chi = 1$, $\delta_0 = 0.02$, $\rho_\theta = 0.95$, and $s_\theta = 0.03$ ($\varepsilon_{\theta,t}$ is drawn from a standard normal distribution). Solve the model in Dynare. Compute impulse responses of log output, log investment, log consumption, log hours, log wage, log utilization and the real interest rate to a preference shock for two values of ϕ_2: $\phi_2 = 0.01$ and $\phi_2 = 0.1$.

(g) Comment on how the value of ϕ_2 influences amplification in the model.

(h) Defined measured TFP as: $\ln TFP_t = \ln Y_t - \alpha \ln K_t - (1-\alpha)\ln N_t$. In the model, what is measured TFP equal to? Plot impulse responses of measured TFP to the preference shock for both of the above values of ϕ_2.

(i) Produce HP-filtered moments (smoothing parameter of 1600) from the model for output and measured TFP and the cyclicality of measured TFP for both values of ϕ_2. What is true about the volatility and cyclicality of measured TFP as a function of the value of ϕ_2? Discuss briefly.

2. Consider an economy in which production is exogenous (i.e. an endowment economy) in which a represent household holds money. The household problem is:

$$\max_{C_t, B_{t+1}, M_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \psi \frac{(M_t/P_t)^{1-\zeta}}{1-\zeta} \right]$$

s.t.

$$P_t C_t + B_{t+1} - B_t + M_t - M_{t-1} \leq P_t Y_t - P_t T_t + i_{t-1} B_t$$

Here, B_t is the stock of nominal bonds that the household brings from $t-1$ into t. i_{t-1} is the nominal net interest rate on bonds. M_{t-1} is the stock of money that the household chose in $t-1$, and M_t is the stock of money the household takes into $t+1$. P_t is the price of goods measured in terms of money. Y_t is the real endowment and is exogenous. Assume $\sigma > 0$ and $\zeta > 0$.

The government consumes an exogenous amount of output, G_t. It finances this by issuing debt, levying lump sum taxes, and creating money. Its budget constraint is:

$$P_t G_t + i_{t-1} D_t \leq P_t T_t + M_t - M_{t-1} + D_{t+1} - D_t$$

Gross inflation is defined as $\Pi_t = P_t / P_{t-1}$. Suppose that G_t is always proportional to output via:

$$G_t = g Y_t$$

Where $0 < g < 1$ is a fixed share of output consumed by the government. Suppose that the endowment, Y_t, follows some exogenous process.

Define money growth, $g_t^M = \ln M_t - \ln M_{t-1}$. Suppose that money growth follows an exogenous process:

$$g_t^M = (1 - \rho_M) g^M + \rho_M g_{t-1}^M + s_M \varepsilon_{M,t}$$

g^M is the steady state value of money growth. We assume nothing about lump sum taxes and government debt, other than that they are such that the government's budget constraint holds. Real money balances are defined as:

$$m_t = \frac{M_t}{P_t}$$

(a) Derive the first order optimality conditions for the household. Write the FOC for bonds as an asset pricing condition in terms of the stochastic discount factor and gross inflation. What is the Fisher relationship? Write the FOC for money holdings as an asset pricing condition in terms of the stochastic discount factor.

(b) What is the asset market clearing condition? Use this to derive the aggregate resource constraint.

(c) Solve for exact expressions for the steady state values of C, m, i, r, and Π as a function of parameters and steady state values of exogenous variables (e.g. steady state Y).

(d) What value of g^M would maximize household utility in the steady state? Provide some intuition for why.

(e) The quantity equation *defines* velocity via the relationship $M_t V_t = P_t Y_t$ – money times velocity equals nominal GDP. Use the optimality conditions above to derive an exact expression for V_t in the model.

3. Suppose you have a RBC model with two stochastic shocks: a shock to productivity and a shock to government spending. We will consider two different preference specifications: standard separable preferences and GHH preferences.

For generic preferences, the household problem can be written:

$$\max_{C_t, K_{t+1}, N_t, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t)$$

s.t.

$$C_t + K_{t+1} - (1 - \delta) K_t + B_{t+1} \leq w_t N_t + R_t^k K_t + \Pi_t - T_t + (1 + r_{t-1}) B_t$$

The firm problem is the same in both setups:

$$\max_{N_t, K_t} \quad \Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t^k K_t$$

Suppose that G_t and A_t follows stationary AR(1) processes in the log:

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t}$$
$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + s_G \varepsilon_{G,t}$$

The steady state value of productivity is normalized to 1. There is an arbitrary steady state value of government spending, G.

(a) For an arbitrary specification of household preferences, $u(C_t, N_t)$, find the first order conditions for a solution to the problem.

(b) Find the first order conditions for the firm problem.

(c) The government chooses its spending exogenously and balances its budget each period, $G_t = T_t$. What then must be true about bond-holding by the household in equilibrium? Use this to write down the aggregate resource constraint.

(d) For now, suppose that preferences are given by the standard separable form:

$$U(C_t, N_t) = \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi}$$

Suppose that the non-stochastic steady state value of A_t is $A = 1$, while the steady state value of government spending is $G = \omega Y$, where Y is the steady state value of output and $0 < \omega < 1$. Using these preferences, derive an expression for the value of θ consistent with steady state hours of $N = 1/3$, and provide expressions for the steady state values of Y, C, I, K, w, and R as a function of parameters.

(e) Solve the model using a first order log-linear approximation in Dynare using the following parameter values: $\beta = 0.99$, $\delta = 0.02$, $\alpha = 1/3$, $\chi = 1$, $\rho_A = 0.97$, $\rho_G = 0.95$, $s_A = 0.01$ (standard deviation of productivity shock), $s_G = 0.01$ (standard deviation of government spending shock), $\omega = 0.20$, and the value of θ consistent with $N = 1/3$. Produce impulse response graphs of the log deviations of Y_t, C_t, I_t, N_t, w_t, and r_t to each shock over a 20 period horizon. Calculate the "government spending multiplier," defined as the ratio of the impact response of the level of output to the impact response of the level of government spending following a government spending shock. Also calculate a version of the fiscal multiplier defined as the ratio of the sum of the output level response (over 20 periods) to the sum of the government spending response over the same horizon.

(f) Now instead suppose that preferences are given by the GHH variety:

$$U(C_t, N_t) = \ln\left(C_t - \theta \frac{N_t^{1+\chi}}{1+\chi}\right)$$

Using these preferences, derive an expression for the value of θ consistent with steady state hours of $N = 1/3$, and provide expressions for the steady state values of Y, C, I, K, w, and R as a function of parameters.

(g) Solve the model using a first order log-linear approximation using the same parameter values as above. Produce impulse response graphs of the log deviations of Y_t, C_t, I_t, N_t, w_t, and r_t to each shock over a 20 period horizon. Again calculate two versions of the fiscal multiplier: one defined as the ratio of the impact response of the level of output to the level of government spending, the other defined as the sum of the output level response over a 20-period horizon to the sum of the government spending response over the same horizon.

(h) Compare and contrast your impulse responses to both the productivity and government spending shocks with the two different preference specifications. Provide some intuition for your findings.