Graduate Macro Theory II:
A Two-Agent New Keynesian (TANK) Model

Eric Sims
University of Notre Dame

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1 Introduction

Over the last several years, there has been significant interest in the implications of heterogeneity for macroeconomics. Both sides of the heterogeneity coin are of interest: how does heterogeneity impact transmission of aggregate shocks into the aggregate economy, and how does the presence of heterogeneity affect optimal policy design?

In this course, we have focused almost exclusively on representative agent frameworks. While unrealistic, the representative agent framework keeps models tractable, which is useful both for gaining analytic intuition as well and because it is much easier to solve models without heterogeneity. To a large extent, the surge of interest in macro implications of heterogeneity has been a response to increased computational power.

In this set of notes, I am going to lay out a very simple New Keynesian model with heterogeneity. In particular, I am going to study a Two-Agent New Keynesian Model (TANK). This is different than the standard setup, which is often times called RANK (Representative Agent New Keynesian Model). In the TANK model, there are two types of households: the standard representative household that can freely borrow and save, and another household that is shut out of financial markets. I am going to refer to the latter type of household as "constrained" or as "hand-to-mouth." These households are unable to borrow or save. As a consequence, they just consume their disposable income each period.

The TANK model is a bridge to a more complicated model with richer heterogeneity, the so-called HANK model (Heterogeneous Agent New Keynesian). In HANK models, households are subject to non-insurable idiosyncratic income shocks and are subject to some form of a borrowing constraint (typically that a household is not permitted to go into debt). The combination of uninsurable income shocks and a borrowing constraint makes the distribution of household wealth matter. Some households will always be at the borrowing constraint, behaving like the hand-to-mouth households in the TANK model. But there will be time variation in how many households are constrained, which could result in interesting non-linearities (e.g. certain types of shocks might affect output more when lots of agents are constrained compared to times when few are constrained).

It is also the case that there will be some precautionary behavior, in that agents not currently subject to the borrowing limit might behave preemptively in ways to avoid hitting the constraint. In a standard TANK model, there is no switching between types, and as a result there is no precautionary behavior. Still, the TANK model can help us gain some intuition for implications of a more complicated HANK framework. In particular, as we shall see below, the presence of constrained households makes the New Keynesian model behave a bit more like "old" Keynesian intuition – things like current disposable income and marginal propensity to consume suddenly matter. It is also the case that "supply" shocks will affect output less, whereas "demand" shocks are amplified. In that sense, the inclusion of hand-to-mouth agents is somewhat isomorphic to having nominal rigidities be stronger (i.e. the Phillips Curve be flatter).

2 Households

In the basic TANK model, there are two types of households: unconstrained households and constrained households. I'm going to assume that, in total, there exists a unit mass of households. A mass $\alpha \in [0,1]$ are unconstrained and the remaining mass $1-\alpha$ are unable to borrow or save. I will refer to unconstrained households as unconstrained, and to constrained households as either constrained or as hand-to-mouth.

2.1 Unconstrained Households

There is a representative unconstrained household. I will denote variables related to these households with a u subscript. The problem facing these households is entirely standard. Note that I am abstracting altogether from money, but could include it as an additively separable term in the flow utility function if I wanted.

The problem is:

$$\max_{C_{u,t},N_{u,t},B_{u,t}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{u,t}^{1-\sigma}}{1-\sigma} - \theta \frac{N_{u,t}^{1+\chi}}{1+\chi} \right]$$

s.t.

$$P_t C_{u,t} + B_{u,t} - B_{u,t-1} \leq W_t N_{u,t} + P_t D_{u,t} - P_t T_{u,t} + i_{t-1} B_{u,t-1}$$

Here P_t is the price of goods. $B_{u,t-1}$ is the stock of nominal bonds with which a household enters the period; these bonds pay net nominal interest i_t. W_t is the nominal wage. $D_{u,t}$ is distributed profit from ownership in firms and is taken as given by an unconstrained household. $T_{u,t}$ is a lump sum tax (or transfer) paid to the government, and is also taken as given by the household.

The first order conditions are standard. Let $w_t = W_t/P_t$ be the real wage and $\Pi_t = P_t/P_{t-1}$ be the gross inflation rate:

$$\theta N_{u,t}^{\chi} = C_{u,t}^{-\sigma} w_t \tag{1}$$

$$1 = \mathbb{E}_t \left[\Lambda_{u,t,t+1}(1+i_t)\Pi_{t+1}^{-1}\right] \quad (2)$$

$$\Lambda_{u,t-1,t} = \beta \left(\frac{C_t}{C_{t-1}}\right)^{-\sigma} \quad (3)$$

These are standard conditions. $\Lambda_{u,t,t+1}$ is the stochastic discount factor for the unconstrained household between t and $t+1$.

2.2 Constrained Households

Constrained households have identical preferences to unconstrained households. The important difference is that constrained households are not allowed to borrow or save (i.e. they cannot hold or issue bonds). They also do not hold ownership in firms. I shall refer to these households as hand-to-mouth, and denote them with an h subscript.

The problem of a constrained household is:

$$\max_{C_{h,t},N_{h,t}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{h,t}^{1-\sigma}}{1-\sigma} - \theta \frac{N_{h,t}^{1+\chi}}{1+\chi}\right]$$

s.t.

$$P_t C_{h,t} \leq W_t N_{h,t} - P_t T_{h,t}$$

The constrained household supplies labor and faces the same wage as the unconstrained household. The constrained household pays a lump sum tax (transfer) to the government, $T_{h,t}$, which is potentially different from what the unconstrained household pays. Because there are no state variables, the problem of a constrained household is effectively static. The first order condition is:

$$\theta N_{h,t}^{\chi} = C_{h,t}^{-\sigma} w_t \quad (4)$$

3 Production

The production side of the economy is *identical* to the standard New Keynesian model. The only thing to keep track of is that the relevant stochastic discount factor for the dynamic pricing problem of intermediate firms is the stochastic discount factor of the unconstrained households, $\Lambda_{u,t,t+1}$.

The optimality conditions for the production and price-setting part of the problem are:

$$p_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{\widehat{X}_{1,t}}{\widehat{X}_{2,t}} \quad (5)$$

$$\widehat{X}_{1,t} = mc_t Y_t + \phi \mathbb{E}_t \Lambda_{u,t,t+1} \Pi_{t+1}^{\epsilon} \widehat{X}_{1,t+1} \quad (6)$$

$$\widehat{X}_{2,t} = Y_t + \phi \mathbb{E}_t \Lambda_{u,t,t+1} \Pi_{t+1}^{\epsilon-1} \widehat{X}_{2,t+1} \quad (7)$$

$$mc_t = \frac{w_t}{A_t} \quad (8)$$

4 Policy, Equilibrium, and Aggregation

Monetary policy is set according to a Taylor rule:

$$i_t = (1-\rho_i)i + \rho_i i_{t-1} + (1-\rho_i)\phi_\pi\left(\ln \Pi_t - \ln \Pi\right) + s_i \varepsilon_{i,t} \tag{9}$$

I am allowing for interest rate smoothing, but, for simplicity, I am ruling out any response to output (or the output gap). That is not hard to accommodate.[1]

In nominal terms, the government's budget constraint is:

$$P_t G_t + i_{t-1} B^G_{t-1} \leq \alpha P_t T_{u,t} + (1-\alpha) P_t T_{h,t} + B^G_t - B^G_{t-1}$$

Note that $T_{u,t}$ is the amount *each* unconstrained household pays (or receives, if it is negative). In aggregate, since there are α of these households, the government is receiving nominal revenue of $\alpha P_t T_{u,t}$ (and similarly for the constrained households). To write this in real terms, divide through by P_t, and define $b^G_t = B^G_t / P_t$ as real government debt. We have:

$$G_t + (1+i_{t-1})\Pi_t^{-1} b^G_{t-1} \leq \alpha T_{u,t} + (1-\alpha) T_{h,t} + b^G_t \tag{10}$$

Note that Ricardian Equivalence will *not* hold in this model; I shall return to this below.

The aggregate production function, price level, and price dispersion terms are the same as in the standard model. Written in terms of inflation rates, we have:

$$v^p_t Y_t = A_t N_t \tag{11}$$

$$v^p_t = (1-\phi)\left(p^\#_t\right)^{-\epsilon} + \phi \Pi_t^\epsilon v^p_{t-1} \tag{12}$$

$$1 = (1-\phi)\left(p^\#_t\right)^{1-\epsilon} + \phi \Pi_t^{\epsilon-1} \tag{13}$$

Define aggregate labor input as the sum of labor input across the two types of households. We therefore have:

$$N_t = \alpha N_{u,t} + (1-\alpha) N_{h,t} \tag{14}$$

We proceed in the same way for aggregate consumption:

$$C_t = \alpha C_{u,t} + (1-\alpha) C_{h,t} \tag{15}$$

Let us integrate over the budget constraints (at equality) for both types of households. Doing so, we have:

[1] With two households, this model is substantially more complicated than the standard model. Note that one can always solve for Y_t^f (the flexible price level of output) numerically in Dynare by including a hypothetical "flexible price block" in the model. Then one could write the Taylor rule with an addition $\phi_x(\ln Y_t - \ln Y_t^f)$ term.

$$\alpha P_t C_{u,t} + \alpha(B_{u,t} - B_{u,t-1}) = \alpha W_t N_{u,t} + \alpha P_t D_{u,t} - \alpha P_t T_{u,t} + \alpha i_{t-1} B_{u,t-1}$$

$$(1-\alpha) P_t C_{h,t} = (1-\alpha) W_t N_{h,t} - (1-\alpha) P_t T_{h,t}$$

Sum these together. We have:

$$P_t \left(\alpha C_{u,t} + (1-\alpha) C_{h,t}\right) + \alpha B_{u,t} = W_t \left(\alpha N_{u,t} + (1-\alpha) N_{h,t}\right) + \alpha P_t D_{u,t} - \alpha P_t T_{u,t} - (1-\alpha) P_t T_{h,t} + \alpha(1+i_{t-1}) B_{u,t-1}$$

But, using the definitions of aggregate consumption and labor input, these are:

$$P_t C_t + \alpha B_{u,t} = W_t N_t + \alpha P_t D_{u,t} - \alpha P_t T_{u,t} - (1-\alpha) P_t T_{h,t} + \alpha(1+i_{t-1}) B_{u,t-1}$$

Bond market clearing requires that unconstrained households (in aggregate) hold all government debt. This means:

$$\alpha B_{u,t} = B_t^G$$

The α term appears on the left hand side because the integral of bond holdings by unconstrained households is α times what they each hold. Solve the government's budget constraint at equality for $\alpha P_t T_{u,t}$:

$$\alpha P_t T_{u,t} = P_t G_t + (1+i_{t-1}) B_{t-1}^G - B_t^G - (1-\alpha) P_t T_{h,t}$$

Plug this into the combined household budget constrains at equality:

$$P_t C_t + \alpha B_{u,t} = W_t N_t + \alpha P_t D_{u,t} - \left(P_t G_t + (1+i_{t-1}) B_{t-1}^G - B_t^G - (1-\alpha) P_t T_{h,t}\right) - (1-\alpha) P_t T_{h,t} + (1+i_{t-1}) \alpha B_{u,t-1}$$

Given the bond market clearing condition, the terms involving debt cancel (i.e. $\alpha B_{u,t} = B_t^G$, and $(1+i_{t-1}) B_{t-1}^G = (1+i_{t-1}) \alpha B_{u,t-1}$. The term involving $T_{h,t}$ also cancels out. We have:

$$P_t C_t + P_t G_t = W_t N_t + \alpha P_t D_{u,t}$$

What are profits from firms? The final good firm earns nominal profit:

$$P_t D_t^F = P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj$$

Collectively, intermediate firms earn profits:

$$P_t D_t^I = \int_0^1 \left[P_t(j) Y_t(j) - W_t N_t(j)\right] dj = \int_0^1 P_t(j) Y_t(j) - W_t \int_0^1 N_t(j) dj$$

Total profits for distribution are:

$$P_tD_t = P_tD_t^F + P_tD_t^I = P_tY_t - \int_0^1 P_t(j)Y_t(j)dj + \int_0^1 P_t(j)Y_t(j) - W_t\int_0^1 N_t(j)dj$$

Using the labor market clearing condition, this is:

$$P_tD_t = P_tY_t - W_tN_t$$

These profits have to be distributed (in aggregate) to only unconstrained households. Therefore, we have:

$$P_tD_t = \alpha P_tD_{u,t}$$

Plugging this in above yields the aggregate resource constraint in real terms:

$$Y_t = C_t + G_t \tag{16}$$

Let's assume that G_t follows an exogenous AR(1) process in the log (and same, as usual, for A_t):

$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + s_G\varepsilon_{G,t} \tag{17}$$

$$\ln A_t = \rho_A \ln A_{t-1} + s_A\varepsilon_{A,t} \tag{18}$$

$\ln G$ is the log steady-state government spending level, and the steady state level of productivity is normalized to unity (so zero in the log).

Because constrained households do not have access to credit markets, Ricardian Equivalence does not hold and we have to keep track of the constrained household's budget constraint, which is (at equality, and in real terms):

$$C_{h,t} = w_tN_{h,t} - T_{h,t}$$

Because $T_{h,t}$ is directly relevant for the constrained households' consumption, we cannot just ignore lump sum taxes altogether like in a standard model with no constraints. We also cannot ignore government debt or lump sum taxes levied on the unconstrained households. This means we have to assume something about all these fiscal variables (unlike in a standard representative agent with free access to credit markets). There are a number of ways to do this. The simplest way is to assume that the government never issues any debt, so $B_t^G = 0$ (which implies that b_t^G equals zero as well):

$$b_t^G = 0 \tag{19}$$

I am then going to assume that lump sum taxes to the constrained household follow an AR(1) in the *level* (I'm doing level, not log, because I want to allow these to go negative so that constrained households can receive transfers):

$$T_{h,t} = (1-\rho_h)T_h + \rho_h T_{h,t-1} + s_h \varepsilon_{h,t} \tag{20}$$

To see how this shock works, impose that $b_t^G = 0$. The government's budget constraint at eqaulity is:

$$G_t = \alpha T_{u,t} + (1-\alpha) T_{h,t}$$

Since G_t follows an exogenous process, this means that (i) shocks to $T_{h,t}$ are funded by offsetting taxes on unconstrained households – so a shock to $T_{h,t}$ is really a transfer shock; and, (ii) changes in government spending are funded by taxes on unconstrained households (not on constrained households). There are, of course, other fiscal rules that I could employ.

5 Equilibrium Conditions

I am collecting all equilibrium conditions below. We have 21 variables: $N_{u,t}$, $C_{u,t}$, w_t, $\Lambda_{u,t,t+1}$, i_t, Π_t, $N_{h,t}$, $C_{h,t}$, $T_{h,t}$, $T_{u,t}$, $p_t^{\#}$, $\widehat{X}_{1,t}$, $\widehat{X}_{2,t}$, mc_t, Y_t, A_t, N_t, C_t, G_t, v_t^p, and b_t^G. We also have 21 equations. These are shown below (grouped by type).

- Unconstrained households:

$$\theta N_{u,t}^{\chi} = C_{u,t}^{-\sigma} w_t \tag{21}$$

$$1 = \mathbb{E}_t \left[\Lambda_{u,t,t+1} (1+i_t) \Pi_{t+1}^{-1} \right] \tag{22}$$

$$\Lambda_{u,t-1,t} = \beta \left(\frac{C_{u,t}}{C_{u,t-1}} \right)^{-\sigma} \tag{23}$$

- Hand-to-mouth households

$$\theta N_{h,t}^{\chi} = C_{h,t}^{-\sigma} w_t \tag{24}$$

$$C_{h,t} = w_t N_{h,t} - T_{h,t} \tag{25}$$

- Intermediate firms:

$$p_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{\widehat{X}_{1,t}}{\widehat{X}_{2,t}} \tag{26}$$

$$\widehat{X}_{1,t} = mc_t Y_t + \phi \mathbb{E}_t \Lambda_{u,t,t+1} \Pi_{t+1}^{\epsilon} \widehat{X}_{1,t+1} \tag{27}$$

$$\widehat{X}_{2,t} = Y_t + \phi \mathbb{E}_t \Lambda_{u,t,t+1} \Pi_{t+1}^{\epsilon-1} \widehat{X}_{2,t+1} \tag{28}$$

$$mc_t = \frac{w_t}{A_t} \tag{29}$$

- Policy

$$i_t = (1-\rho_i)i + \rho_i i_{t-1} + (1-\rho_i)\phi_\pi (\ln \Pi_t - \ln \Pi) + s_i \varepsilon_{i,t} \tag{30}$$

$$G_t + (1+i_{t-1})\Pi_t^{-1} b_{t-1}^G = T_{u,t} + T_{h,t} + b_t^G + b_t^G \tag{31}$$

- Aggregate conditions

$$v_t^p Y_t = A_t N_t \tag{32}$$

$$v_t^p = (1-\phi)\left(p_t^{\#}\right)^{-\epsilon} + \phi \Pi_t^{\epsilon} v_{t-1}^p \tag{33}$$

$$1 = (1-\phi)\left(p_t^{\#}\right)^{1-\epsilon} + \phi \Pi_t^{\epsilon-1} \tag{34}$$

$$N_t = \alpha N_{u,t} + (1-\alpha) N_{h,t} \tag{35}$$

$$C_t = \alpha C_{u,t} + (1-\alpha) C_{h,t} \tag{36}$$

$$Y_t = C_t \tag{37}$$

- Exogenous

$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t} \tag{38}$$

$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + s_G \varepsilon_{G,t} \tag{39}$$

$$b_t^G = 0 \tag{40}$$

$$T_{h,t} = (1-\rho_h) T_h + \rho_h T_{h,t-1} + s_h \varepsilon_{h,t} \tag{41}$$

5.1 Steady State

We need to solve for the steady state. I'm going to approximate about a zero inflation steady state. This means $\Pi = 1$. We have $\Lambda_u = \beta$. This means that the steady state nominal interest rate satisfies:

$$1 + i = \beta^{-1}$$

As we have seen before, zero inflation steady state means:

$$p^{\#} = 1$$

$$v^p = 1$$

$$mc = w = \frac{\epsilon - 1}{\epsilon}$$

Solving for the remainder of the steady state is going to be a bit more challenging because there are more free steady state variables. Let's start by assuming that government spending is a fixed share, ψ_G, of steady state output:

$$G = \psi_G Y$$

Let's assume further that the steady state lump-sum taxes on both types of households are equal: $T_u = T_h = T$. From the government's budget constraint, since we are assuming no debt, we therefore have:

$$T = \psi_G Y$$

Then, from the unconstrained household's budget constraint, we have:

$$C_h = wN_h - \psi_G Y \tag{42}$$

Where $w = \frac{\epsilon-1}{\epsilon}$. From the FOC for labor for the same household, we can then write:

$$\theta N_h^{\chi} = (wN_h - \psi_G Y) \tag{43}$$

But we know that $Y = N$ in steady state, so this can be written:

$$\theta N_h^{\chi} = (wN_h - \psi_G N)^{-\sigma} w \tag{44}$$

Now go to the static labor supply FOC for the unconstrained household. We have:

$$\theta N_u^{\chi} = C_u^{-\sigma} w$$

But we know that: $N_u = \frac{1}{\alpha}N - \frac{1-\alpha}{\alpha}N_h$ and $C_u = \frac{1}{\alpha}C - \frac{1-\alpha}{\alpha}C_h$. But we also know that $C = (1-\psi_G)Y$. So $C_u = \frac{1-\psi_G}{\alpha}Y - \frac{1-\alpha}{\alpha}C_h$. But we know C_h, so we have: $C_u = \frac{1-\psi_G}{\alpha}N - \frac{1-\alpha}{\alpha}(wN_h - \psi_G N)$. We therefore have:

$$\theta \left(\frac{1}{\alpha}N - \frac{1-\alpha}{\alpha}N_h\right)^{\chi} = \left(\frac{1-\psi_G}{\alpha}N - \frac{1-\alpha}{\alpha}(wN_h - \psi_G N)\right)^{-\sigma} w \tag{45}$$

(44) and (45) constitute two equations in two unknowns – N and N_h. There is not a straightforward analytical solution, but we can solve for N and N_h numerically. Once we have N and N_h, it is straightforward to recover the rest of the steady staet. In particular:

$$N_u = \frac{1}{\alpha}N - \frac{1-\alpha}{\alpha}N_h \tag{46}$$

$$Y = N \tag{47}$$

$$G = \psi_G N \tag{48}$$

$$C = (1-\psi_G)N \tag{49}$$

$$C_h = wN_h - \psi_G N \tag{50}$$

$$C_u = \frac{1}{\alpha}C - \frac{1-\alpha}{\alpha}C_h \tag{51}$$

$$T = T_u = T_h = \psi_G N \tag{52}$$

6 Quantitative Analysis

To solve the model, I need to specify values for parameters. I am going to choose "standard" values for many parameters, and will then play around with α (the share of unconstrained households).

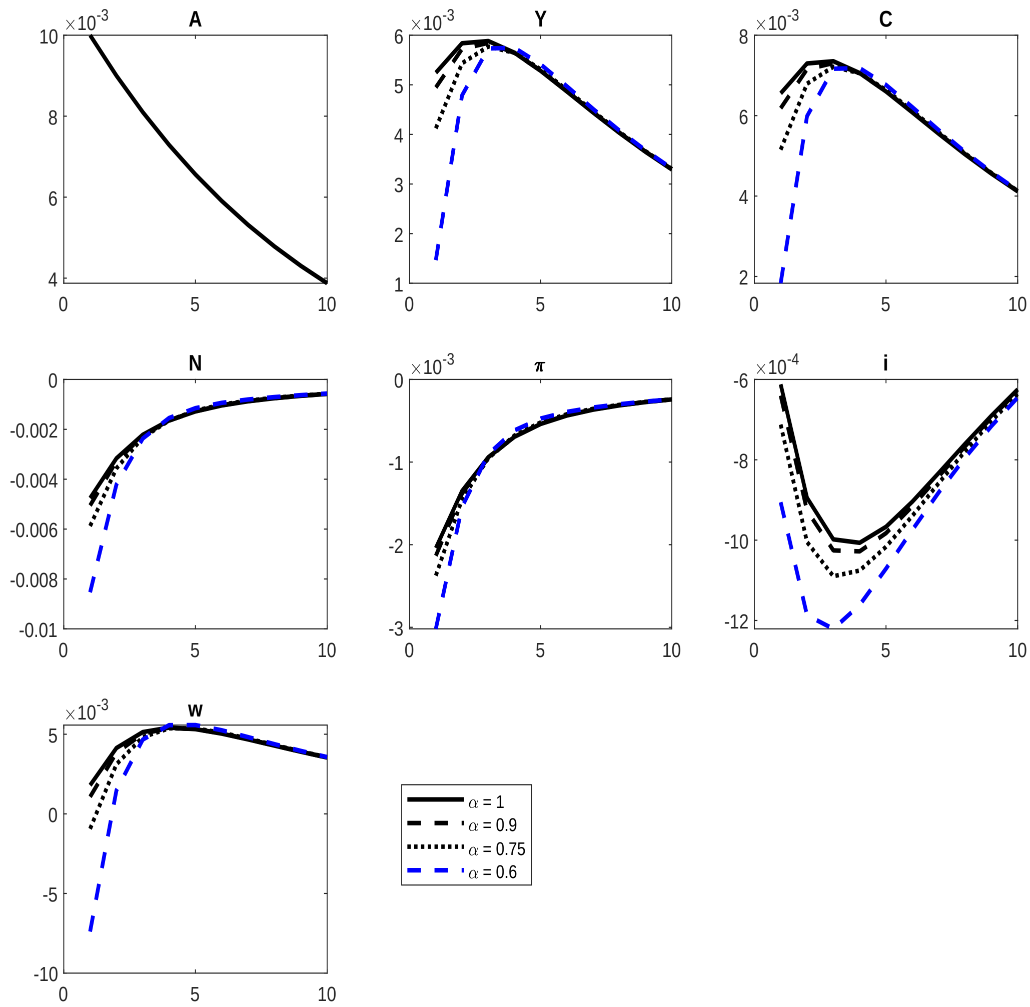
I set $\beta = 0.99$, $\epsilon = 11$, $\sigma = \chi = 1$, $\theta = 1$, and $\phi = 3/4$. I pick $\psi_G = 0.2$. I assume that the AR(1) parameters are $\rho_A = 0.9$ and $\rho_G = \rho_h = \rho_i = 0.8$. I set $\rho_i = 0.8$ and $\phi_\pi = 1.5$. The shock standard deviations are $s_A = s_G = 0.01$, $s_i = 0.0025$, and $s_t = 0.02$.

I consider different values of α: 1, 0.9, 0.75, and 0.6. $1-\alpha$ is the share of constrained households. Hence, when $\alpha = 1$, we are in the standard NK model. As α gets smaller, a larger share of agents are constrained.[2] I compute impulse responses to the four different shocks for these four different values of α below. Solid black lines are the standard NK model ($\alpha = 1$), the dashed black line corresponds to $\alpha = 0.9$, the dotted black line to $\alpha = 3/4$, and the dashed blue line to $\alpha = 3/5$.

Consider first impulse responses to a productivity shock. I am plotting responses of aggregate variables. What we observe is that output reacts *less* to the productivity shock the smaller the share of unconstrained agents (i.e. the smaller is α; equivalently, the bigger is the share of constrained agents).

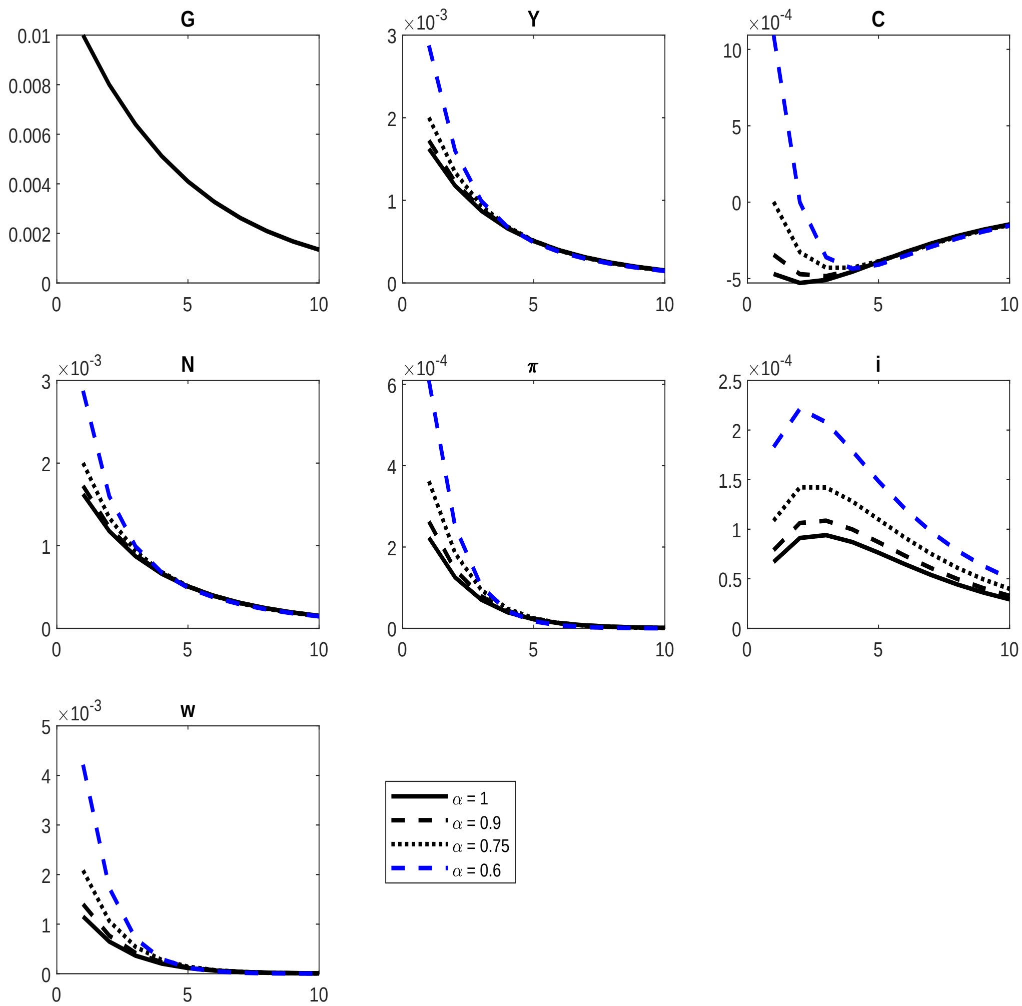
[2]Given the forward-looking nature of inflation, if α is too low there are determinacy issues in the model.

Figure 1: IRFs to a Productivity Shock



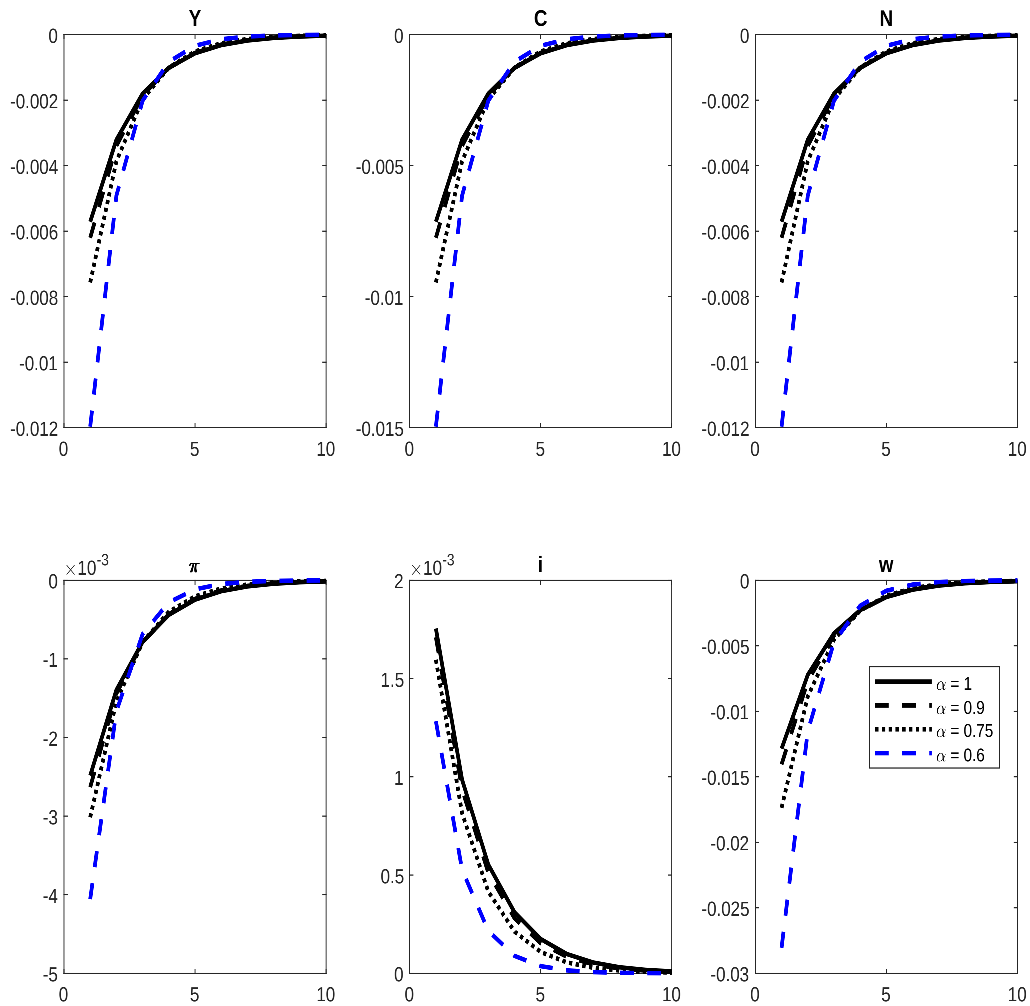
Next, consider responses to a government spending shock. This is kind of the inverse of the productivity shock – output reacts *more* to government spending the higher the share of constrained agents (i.e. the lower is α).

Figure 2: IRFs to a Government Spending Shock



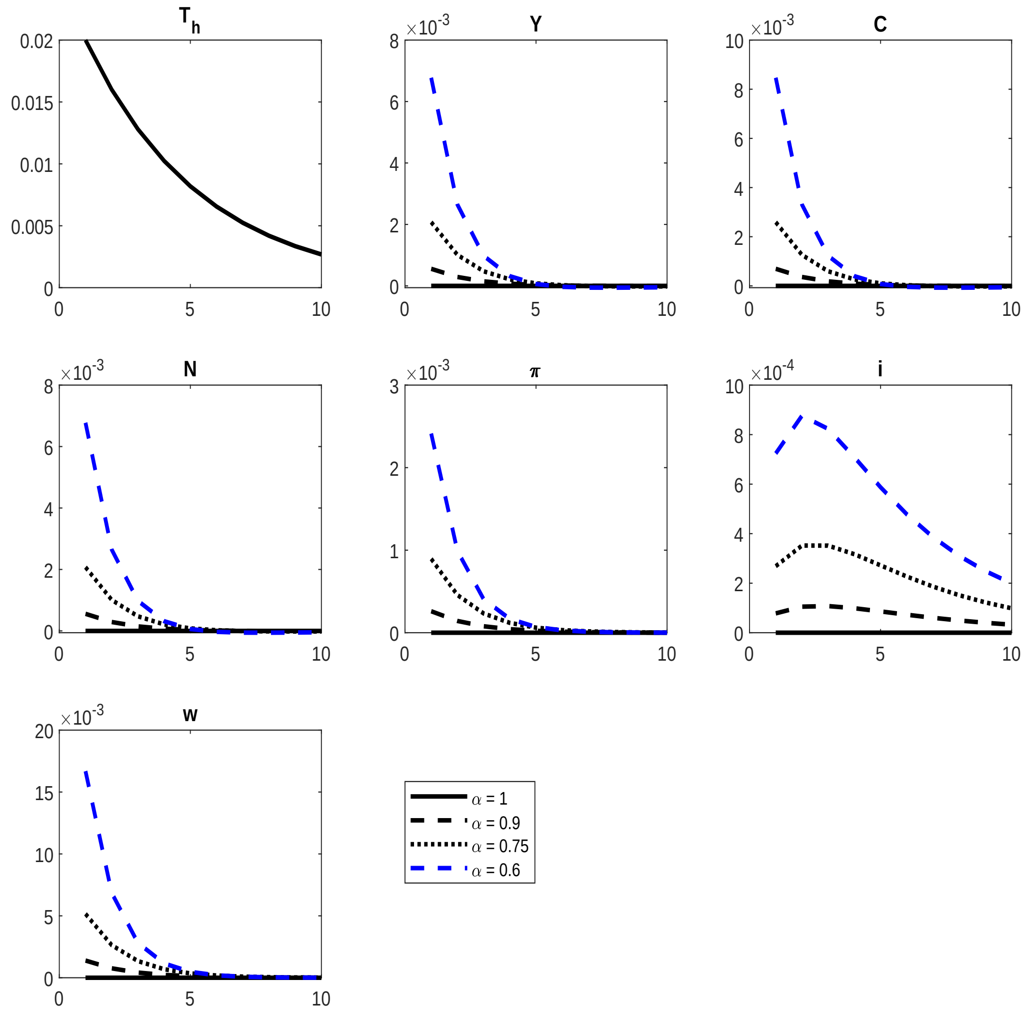
Next up, a monetary shock. We see a similar pattern – output reacts more to the shock the bigger the share of constrained agents.

Figure 3: IRFs to a Monetary Policy Shock



Finally, consider a transfer shock. Given the way I have specified the fiscal block of the model, with a negative sign on the shock I am taxing unconstrained agents to give a transfer to constrained agents. This has the effect of causing output to rise. With $\alpha = 1$, there would be no impact of the transfer shock on the aggregate equilibrium – in other words, Ricardian Equivalence would hold.

Figure 4: IRFs to a Transfer Shock



A basic message emerges here. The presence of hand-to-mouth agents *dampens* the output response to a "supply shock" (a shock that would move output and inflation in opposite directions) and *amplifies* responses to a "demand shock" (a shock that moves output and inflation in the same direction). In this way, the presence of hand-to-mouth agents is kind of like having more nominal rigidity.

6.1 Quick and Dirty Intuition

Let's try to open the black box above to understand some intuition for these results. Note again that the supply side of the model is unaffected with the presence of constrained agents. The linearized

Phillips Curve, written in terms of marginal cost, is:

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi}\widetilde{mc}_t + \beta \mathbb{E}_t \pi_{t+1} \tag{53}$$

But the demand side is affected by the presence of hand-to-mouth agents. Let's log-linearize the demand side of the model to get some intuition. The linearized bond Euler equation for unconstrained households is standard:

$$c_{u,t} = \mathbb{E}_t c_{u,t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \tag{54}$$

We have $C_t = \alpha C_{u,t} + (1-\alpha)C_{h,t}$. Take logs of this:

$$\ln C_t = \ln\left[\alpha C_{u,t} + (1-\alpha)C_{h,t}\right]$$

Totally differentiate:

$$c_t = \frac{1}{C}\left[\alpha dC_{u,t} + (1-\alpha)dC_{h,t}\right]$$

Or:

$$c_t = \frac{\alpha C_u}{C}c_{u,t} + \frac{(1-\alpha)C_h}{C}c_{h,t} \tag{55}$$

From the aggregate resource constraint, in linearized terms we have:

$$y_t = (1-\psi)c_t + \psi g_t$$

Or:

$$c_t = \frac{1}{1-\psi}y_t - \frac{\psi}{1-\psi}g_t \tag{56}$$

Combining (56) with (55), we have:

$$\frac{1}{1-\psi}y_t - \frac{\psi}{1-\psi}g_t = \frac{\alpha C_u}{C}c_{u,t} + \frac{(1-\alpha)C_h}{C}c_{h,t}$$

Or:

$$c_{u,t} = \frac{C}{C_u}\frac{1}{\alpha(1-\psi)}y_t - \frac{C}{C_u}\frac{\psi}{\alpha(1-\psi)}g_t - \frac{1-\alpha}{\alpha}\frac{C_h}{C_u}c_{h,t} \tag{57}$$

From the equilibrium conditions, we know that consumption of the hand-to-mouth equals their *disposal income*. In particular, let $DISP_{h,t}$ be the disposal income of the hand-to-mouth agents:

$$C_{h,t} = w_t N_{h,t} - T_{h,t} = DISP_{h,t} \tag{58}$$

In linearized terms, using this change of variables, we have:

$$c_{h,t} = disp_{h,t} \tag{59}$$

Combine (61) with (57) to have:

$$c_{u,t} = \frac{C}{C_u}\frac{1}{\alpha(1-\psi)}y_t - \frac{C}{C_u}\frac{\psi}{\alpha(1-\psi)}g_t - \frac{1-\alpha}{\alpha}\frac{C_h}{C_u}disp_{h,t} \tag{60}$$

Now, combine (60) with (54), the Euler equation for the unconstrained agents. We have:

$$\frac{C}{C_u}\frac{1}{\alpha(1-\psi)}y_t - \frac{C}{C_u}\frac{\psi}{\alpha(1-\psi)}g_t - \frac{1-\alpha}{\alpha}\frac{C_h}{C_u}disp_{h,t} = \frac{C}{C_u}\frac{1}{\alpha(1-\psi)}\mathbb{E}_t y_{t+1} - \frac{C}{C_u}\frac{\psi}{\alpha(1-\psi)}\mathbb{E}_t g_{t+1} - \frac{1-\alpha}{\alpha}\frac{C_h}{C_u}\mathbb{E}_t disp_{h,t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t r_{t+1}\right)$$

This can be written:

$$y_t - \psi g_t - (1-\alpha)(1-\psi)\frac{C_h}{C}disp_{h,t} = \mathbb{E}_t y_{t+1} - \psi \mathbb{E}_t g_{t+1} - (1-\alpha)(1-\psi)\frac{C_h}{C}\mathbb{E}_t disp_{h,t+1} - \frac{\alpha(1-\psi)}{\sigma}\frac{C_u}{C}\left(i_t - \mathbb{E}_t \pi_{t+1}\right)$$

This can be simplified further to yield:

$$y_t = \mathbb{E}_t y_{t+1} + \psi(1-\rho_G)g_t - (1-\alpha)(1-\psi)\frac{C_h}{C}\left(\mathbb{E}_t disp_{h,t+1} - disp_{h,t}\right) - \frac{\alpha(1-\psi)}{\sigma}\frac{C_u}{C}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \tag{61}$$

In the above, I note that $\mathbb{E}_t g_{t+1} = \rho_G g_t$. If there were no unconstrained agents, we would have $\alpha = 1$, which would also imply that $C_u = C$. In this case, the IS equation could be written in a standard way (noting, of course, that there are many ways to write the IS equation, e.g. in terms of the gap rather than output as we have done already):

$$y_t = \mathbb{E}_t y_{t+1} + \psi(1-\rho_G)g_t - \frac{1-\psi}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \tag{62}$$

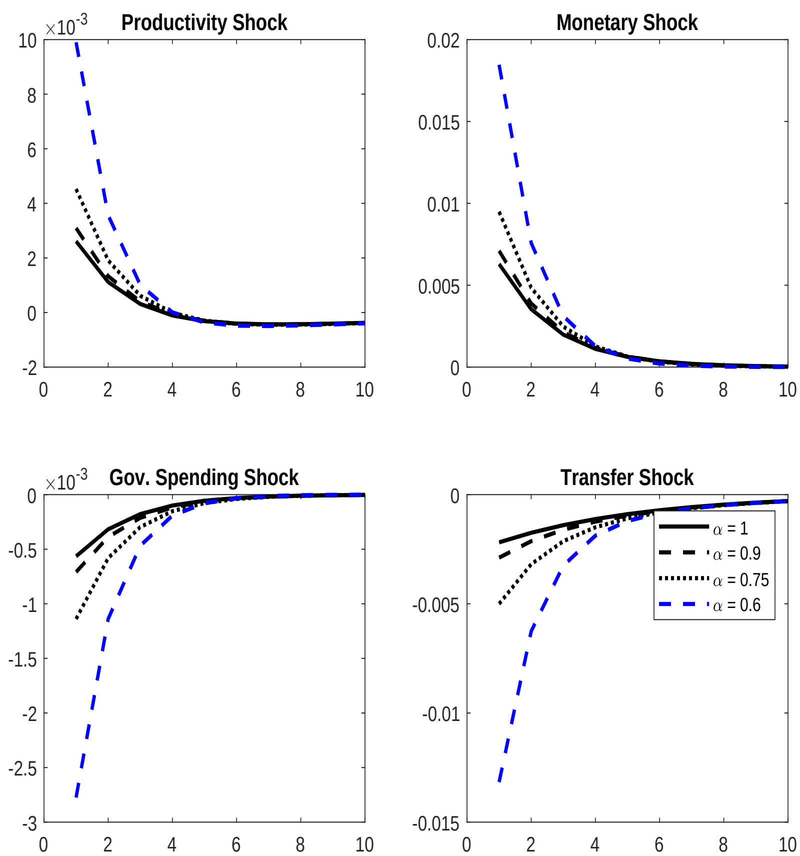
Let's think about what $\alpha < 1$ does by comparing (62) to (61). $\alpha < 1$ *lowers* the coefficient on the real interest rate (directly through $\alpha < 1$, and indirectly because $\alpha < 1$ means $C_u < C$). And it *increases* the coefficient on $\mathbb{E}_t disp_{h,t+1} - disp_{h,t}$, which is the expected growth rate of disposable income for hand-to-mouth agents. If $\alpha = 1$, the expected growth rate of disposable income does not appear at all in the linearized IS equation.

Effectively what is going on here is that we have two types of agents. Constrained agents simply eat their income each period (i.e. their marginal propensity to consume is unity). Unconstrained agents obey the standard Euler equation. The aggregate IS equation is, roughly speaking, a convex combination of these two forces – a weight $(1-\alpha)$ times the expected growth of disposable income and a weight α times the real interest rate (with some other constants floating around, of course). Another way to think about things: the IS equation is combining both elements of an "old"

Keynesian model (based on the marginal propensity to consume, or MPC) and a "new" Keynesian model (forward-looking, and based on intertemporal substitution, i.e. a reaction to the real interest rate). The relative weighting of the two channels depends on the population shares of the two kinds of agents.

To gain some more intuition for how α impacts aggregate dynamics, I computed impulse responses of expected disposable income growth for hand-to-mouth agents. These are in response to each of the same four shocks considered above: a productivity shock, a monetary/Taylor rule shock, a government spending shock, and a transfer shock. I do so for the same different values of α as above. Note that I am plotting the responses of expected disposable income *growth*, not the level of disposable income. It is the expected growth rate that appears in the IS equation.

Figure 5: IRFs of Expected Disposable Income Growth of Hand-to-Mouth Agents



In response to the productivity shock, expected disposable income growth (i.e. $\mathbb{E}_t disp_{h,t+1} - disp_{h,t}$) *increases* after a positive productivity shock. This increase is because of the response of output itself, which is hump-shaped. The important point here is that expected disposable income growth goes *up*. From the IS equation, (61), this exerts *negative* pressure on y_t from the demand side. This explains why output reacts *less* to a positive productivity shock the more unconstrained agents there are – since expected disposable income growth exerts a negative effect, the more hand-to-mouth agents there are (i.e. the smaller is α), the less output reacts to the shock.

Consider next the government spending shock. In the aggregate IRFs, we see that output reacts more (i.e. the multiplier is bigger) the higher the fraction of unconstrained agents (i.e. the lower is α). The IRF of expected disposable income growth helps us understand why. As we see above, expected disposable income growth goes down. Note that disposable income itself goes up, but follows a mean-reverting pattern. This means that expected disposable income growth falls. But this exerts a *positive* impact on output from the IS equation. Hence, output reacts *more* to a government spending shock when there are more hand-to-mouth agents.

Next, let's focus on the monetary policy shock. As we see in the aggregate IRFs, output reacts *more* (i.e. falls more) the bigger is the share of constrained agents (i.e. the smaller is α). Why is this? Because expected disposable income growth goes up – disposable income for the constrained agents falls on impact, but mean reverts, so the expected growth rate goes up. This exerts a negative impact on output. One might be tempted to think that, because a smaller α makes the coefficient on the real rate smaller in (61), the monetary shock might have smaller aggregate effects. But this channel is overwhelmed by the "MPC channel" coming through expected disposable income growth.

Finally, consider the transfer shock. As defined, the transfer shock shifts resources from unconstrained to constrained agents. We see that expected disposable income growth for hand-to-mouth households declines; as with the government spending and monetary shocks, the level of disposable income goes up, but follows a mean-reverting pattern, so expected growth declines. This exerts a positive effect on output, and hence aggregate output reacts more to the shock.

The basic conclusion here is that the presence of constrained agents *dampens* the responses of aggregate variables to "supply" shocks (e.g. productivity), and *amplifies* aggregate responses to "demand" shocks (e.g. government spending, monetary, and transfer). Qualitatively, that's similar to having more price rigidity. Hand-to-mouth agents make the model more "old" Keynesian where there is a marginal propensity to consume channel.

7 From TANK to HANK

We have spent a bunch of bandwidth studying a RANK model, where RANK stands for "representative agent New Keynesian" model. The model is purely forward-looking and has lots of powerful insights. There has been increasing interest of late in the implications of micro-level heterogeneity for aggregate fluctuations. Much of this literature is within a HANK context, where HANK stands for "Heterogeneous Agent New Keynesian" model. The TANK framework laid out in these notes (Two-Agent New Keynesian Model) is sort of an in-between – it captures some (though not all) of the mechanisms and insights of a HANK model in a reasonably tractable way that is easy to compare to the RANK benchmark.

In a typical HANK model, there is a continuum of households who are subject to idiosyncratic income shocks. As an example, the budget constraint of household k ($k \in [0, 1]$) would look like:

$$P_t C_t(k) + B_t(k) - B_{t-1}(k) \leq \exp[\varepsilon_{k,t}] W_t N_t(k) + P_t D_t(k) - P_t T_t(k) + i_{t-1} B_{t-1}(k)$$

This is exactly the same budget constraint for unconstrained agents as above, except there is an idiosyncratic component to wages, given by $\exp[\varepsilon_{k,t}]$, where $\mathbb{E}_t \varepsilon_{k,t} = 0$. As long as there is some variance, some households will get good income shocks (positive draws of $\varepsilon_{k,t}$) and some will get bad shocks (negative draws of $\varepsilon_{k,t}$). As long as households can freely borrow and save via bonds, this won't end up mattering.

But what is typically done is that a borrowing constraint is imposed on households. Though one could consider alternative forms, the simplest one would say that households cannot go into debt:

$$B_t(k) \geq 0$$

When the borrowing constraint binds, $B_t(k) = 0$, the Euler equation won't hold (there would be a Lagrange multiplier on the constraint floating around). If a household gets a sequence of negative $\varepsilon_{k,t}$ shocks, it will eventually bump up against the borrowing constraint and will behave like a hand-to-mouth agent. And, in a global solution at least, a household would behave in a precautionary way away from the borrowing constraint – it would try to save up to avoid hitting the constraint in the first place.

Solving a model like this is actually substantially more complicated than a standard model, because we have to keep track of the distribution of household wealth. The idiosyncratic productivity shock will essentially generate some steady state (or average) share of households who are borrowing constrained. On top of this, macro shocks will generate time variation in the fraction who are constrained – when the economy is poor, households would like to borrow and will draw down their savings, putting more of them close to the borrowing constraint.

The TANK model captures a lot of the essential intuition that comes out of a HANK framework, but not all. In the HANK model, aggregate dynamics are going to be different than RANK because of the presence of borrowing constrained (i.e. hand-to-mouth) agents. These folks are going to have a high marginal propensity to consume and the "old" Keynesian intuition discussed above in the TANK model will be at play. But the HANK model does more – there will be time-variation in how many agents are constrained or not, for example giving rise to interesting non-linearities. And, away from the borrowing constraints, agents will behave in precautionary ways. This is a long-winded way of saying that a TANK model captures some of the essential intuition that comes out of a HANK framework, but the HANK model is far richer and has more going on. But studying a full-fledged HANK model is beyond the scope of this course.

In addition to quantitative complexities (which are easy enough to deal with, if tedious), there are a number of economic modeling choices that matter in a full-fledged HANK model that are also issues in the TANK model. These choices result from the fact that some agents are prohibited from borrowing. Below are three related issues that arise that are unimportant in a RANK model, but quite important in a TANK and especially in a HANK framework.

- Because of the failure of Ricardian Equivalence, the method of government finance matters. When government spending goes up, for example, it matters whether it is financed with debt or taxes. If financed via debt, it matters when and how tax revenues are raised to pay for that debt in the future. If financed via taxes – even when taxes are just lump sum – it matters whether taxes are levied on unconstrained or constrained households. More generally, the fiscal block of the model starts to matter a lot more.

- I swept this under the rug in the TANK model, but in general it also matters how profits from firms are distributed. In the TANK model, I swept it under the rug in the sense that I assumed that only unconstrained agents own firms, which means the timing of profit distribution is irrelevant. But if one is operating in a full-on HANK framework, in principle both constrained and unconstrained agents will own firms. This then makes dividend distribution policies matter – the constrained agents would prefer to be paid dividends immediately, for example, and they will consume any additional dividends received.

- Relatedly, there is another issue that arises when both constrained and unconstrained agents are owners in firms. Whose stochastic discount factor does one use to discount cash flows? I again swept this under the rug above – by assuming that only unconstrained households own firms, this wasn't an issue. But if both types of agents have ownership shares in firms, this becomes a non-trivial issue.

I don't wish to characterize these issues above as "bad" or anything like that. They are real issues, and a model with some degree of heterogeneity and market incompleteness forces one to grapple with them. But the choices that one makes regarding fiscal and distributional rules can matter a *a lot* for how TANK and/or HANK models perform, which can make results not particularly robust (or transparent) in a way that is not true in a RANK framework.