# Problem Set 5

Graduate Macro II, Spring 2024
The University of Notre Dame
Professor Sims

**Instructions:** You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. When asked to solve a model in Dynare, unless otherwise instructed you (i) should solve via a first-order approximation, (ii) may give Dynare the equilibrium conditions and do not need to do the linearization by hand, (iii) should show impulse responses up to a horizon of 20 periods, and (iv) should plot impulse responses (or analyze moments) of logged variables. This problem set is due on Canvas by 5:00 pm on April 16.

1. Consider a New Keynesian model built up from first principles. A representative household solves the following problem:

$$\max_{C_t, N_t, B_{t+1}, M_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} + \psi \ln \left( \frac{M_t}{P_t} \right) \right]$$

s.t.

$$C_t + \frac{B_{t+1} - B_t}{P_t} + \frac{M_t - M_{t-1}}{P_t} \leq w_t N_t + i_{t-1} \frac{B_t}{P_t} + D_t - T_t$$

Here, $M_{t-1}$ is the stock of money with which the household enters period $t$, and $B_t$ is the stock of bonds with which the household enters the period. $D_t$ is (real) dividends from ownership in firms, and $T_t$ is the (real) lump sum tax obligation due to the government. $w_t$ is the real wage.

There are a continuum of intermediate goods producers, indexed by $j \in [0, 1]$. They each produce $Y_t(j)$ and sell at $P_t(j)$. The final output good is produced via a CES aggregate of intermediates:

$$Y_t = \left[ \int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

Intermediate goods producers produce output using a linear technology in labor (there is no productivity shock):

$$Y_t(j) = N_t(j)$$

Intermediate goods producers cannot freely adjust their prices each period. In particular, each period there is a $1-\phi$, $\phi \in [0, 1)$, probability that a producer can adjust its price. This also coincides with the fraction of producers who can update. With probability $\phi$, an intermediate producer cannot adjust its price.

The government faces a consolidated budget constraint:

$$G_t + i_{t-1}\frac{B_t^G}{P_t} \leq T_t + \frac{B_{t+1}^G - B_t^G}{P_t} + \frac{M_t - M_{t-1}}{P_t}$$

Government spending obeys an exogenous stochastic process, with $G$ the non-stochastic steady state:

$$\ln G_t = (1-\rho_G)\ln G + \rho_G \ln G_{t-1} + s_G \varepsilon_{G,t}$$

The nominal money supply is set according to an exogenous AR(1) process in the growth rate, where $g_t^M = \ln M_t - \ln M_{t-1}$, with $g^M$ the non-stochastic steady state growth rate of the nominal money supply:

$$g_t^M = (1-\rho_M)g^M + \rho_M g_{t-1}^M + s_M \varepsilon_{M,t}$$

(a) Derive the first order conditions for the representative household.

(b) Setup the profit maximization problem for the final good producer to derive an expression for the aggregate price index and the demand for each intermediate good.

(c) Suppose, for the time being, that $\phi = 0$ (so that prices are flexible). Derive the optimality condition for an intermediate goods firm. Argue that all intermediate producers will choose the same price that will be a fixed markup over marginal cost.

(d) Now, assume that $\phi > 0$, so prices are sticky. Write down the problem of an intermediate goods producer who can update its price. Show that all updating firms update to a common price.

(e) Write down the market clearing conditions for bonds, money, and labor. Write down the definition of an equilibrium. Derive an expression for the aggregate resource constraint and the aggregate production function.

(f) Use properties of Calvo pricing to write the entire set of equilibrium conditions without reference to $j$ indexes. Write the conditions in terms of gross aggregate inflation, $\Pi_t = P_t/P_{t-1}$, and real money balances, $m_t = M_t/P_t$.

(g) In steady state, assume that $G = gY$, where $g \in [0,1)$. Solve for analytic expressions for the non-stochastic steady state in terms of parameters (take $\theta$ as given, rather than targeting some value of $N$).

(h) Write a Dynare code to solve the model and produce impulse responses to both the government spending shock and a monetary shock. Assume $\epsilon = 10$, $\theta = 1$, $\psi = 1$, $\beta = 0.99$, $\phi = 0.75$, $\chi = 1$, $g = 0.2$, assuming $g^M = 0$, $\rho_M = 0$, $s_M = 0.01$, $\rho_G = 0.8$, $s_G = 0.01$. Show impulse responses of log output, log hours, the inflation rate, the nominal interest rate, the real interest rate, real money balances, the log nominal money supply, and the log price level to each shock. Calculate both the impact government spending multiplier ($dY_t/dG_t$) and the sum multiplier over a 20-quarter horizon.

(i) Please provide some analytically-based intuition for the impulse response of the nominal interest rate, $i_t$, to both shocks.

(j) Now produce impulse responses to the same two shocks, but assume two different values of $\phi$: 0.6 and 0.9. How does the value of $\phi$ impact the transmission of monetary and government spending shocks? How does it impact the value of the government spending multiplier?

(k) Now solve the model (again assuming $\phi = 0.75$), but with two alternative values of $\rho_G$: 0.65 and 0.95. Comment on how the multiplier changes with the persistence of the shock. Is this similar or different than in a real business cycle model? Explain briefly.

2. Consider a basic New Keynesian model. There is no productivity shock, so the flexible price level of output is fixed (as is the natural rate of interest). This mean that output equals the output gap. The linearized equations of the model may be written:

$$\pi_t = \gamma y_t + \beta \mathbb{E}_t \pi_{t+1}$$
$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right)$$

The first is the linearized Phillips Curve, while the second is the linearized IS equation. Assume that monetary policy is set according to the rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + u_t$$

Assume $\phi_\pi > 1$. $u_t$ is a shock term that follows an AR(1) process, with $0 \leq \rho < 1$:

$$u_t = \rho u_{t-1} + s_i \varepsilon_{i,t}$$

(a) Use the method of undetermined coefficients to solve for linearize policy functions for $\pi_t$ and $x_t$:

$$\pi_t = \theta_\pi u_t$$
$$y_t = \theta_y u_t$$

Solve for exact analytical expressions for $\theta_\pi$ and $\theta_x$.

(b) Given the analytical expressions, explain, in words and with reference to math, how the slope of the Phillips Curve impacts how inflation and output react to a monetary shock. In particular, do they respond more (or less) to a shock as a function of how big (or small) $\gamma$ is? What is the intuition for your answer?

(c) Use your analytical expressions for $\theta_\pi$ and $\theta_y$ to derive linear mappings between $u_t$ and the nominal, $i_t$, and real, $r_t$, interest rates.

(d) The Neo-fisherian worldview suggests that to get inflation to come down, the nominal interest rate must decrease (rather than increase). Is this model consistent with a Neo-Fisherian view? If so, under what parameter configuration might this be the case? Explain the logic of your answer.

3. Consider a linearized New Keynesian model. The flexible price level of output is constant (as is the natural rate of interest). The linearized Phillips Curve and IS equations are:

$$\pi_t = \gamma y_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$
$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right)$$

$u_t$ is a cost-push shock. It obeys a stationary AR(1) process ($0 \leq \rho < 1$):

$$u_t = \rho u_{t-1} + s_i \varepsilon_{i,t}$$

(a) Suppose that the interest rate is set according to a Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t$$

Assume $\phi_\pi > 1$ and $\phi_y \geq 0$. Guess:

$$\pi_t = \theta_\pi u_t$$
$$y_t = \theta_y u_t$$

Use the method of undetermined coefficients to solve for analytical expressions for $\theta_\pi$ and $\theta_y$. Use your analytical expressions to calculate impulse responses of output, inflation, the nominal interest rate, and the real interest rate ($r_t = i_t - \mathbb{E}_t \pi_{t+1}$) to a one-unit cost-push shock (i.e. $s_u \varepsilon_t = 1$) assuming the following parameter values: $\rho = 0.8$, $\gamma = 0.1$, $\beta = 0.99$, $\sigma = 1$, $\phi_\pi = 1.5$, $\phi_y = 0.5$.

(b) Suppose, instead of a Taylor rule, that the central bank follows a targeting rule that keeps inflation fixed:

$$\pi_t = 0$$

Given this targeting rule, solve for a new expression for $\theta_y$ and for the required response of the nominal interest rate. With the same parameter values (minus the Taylor rule parameters, which are not relevant), plot the IRFs to the same-sized cost-push shock. Compare and contrast the IRFs under this targeting rule relative to the Taylor rule.

(c) Repeat the above, but instead of targeting constant inflation, suppose that the central bank targets constant output:

$$y_t = 0$$

(d) Under what parameterizations of the Taylor rule will the equilibrium dynamics in response to the cost push shock be the same as both targeting rules? Explain briefly.