Problem Set 6

Graduate Macro II, Spring 2024
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. When asked to solve a model in Dynare, unless otherwise instructed you (i) should solve via a first-order approximation, (ii) may give Dynare the equilibrium conditions and do not need to do the linearization by hand, (iii) should show impulse responses up to a horizon of 20 periods, and (iv) should plot impulse responses (or analyze moments) of logged variables. This problem set is due on Canvas by 5:00 pm on May 1.

1. Consider a basic New Keynesian model. The Phillips Curves and IS equation may be written as:

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$
$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right)$$

Here, u_t is a cost-push shock and r_t^f is the natural rate of interest. Both are exogenous and obey stationary AR(1) processes:

$$u_t = \rho_u u_{t-1} + s_u \varepsilon_{u,t}$$
$$r_t^f = \rho_r r_{t-1}^f + s_r \varepsilon_{r,t}$$

Suppose that the central bank wants to minimize the following loss function:

$$\mathbb{L} = \frac{1}{2}\pi_t^2 + \frac{\alpha_x}{2} x_t^2 + \frac{\alpha_i}{2}\left(i_t - i_{t-1}\right)^2$$

Here, $\alpha_x \geq 0$ and $\alpha_i \geq 0$ are the (relative) weights on fluctuations in the output gap and the change in the interest rate. The central bank acts under discretion; it takes the past and future as given, and it does not incorporate how changes in policy today impact the future.

(a) Derive a first order condition for the optimal targeting rule under discretion.

(b) Show that the first order condition is equivalent to a Taylor-type rule under which the *first difference* of the interest rate reacts positively to fluctuations in inflation and the output gap.

(c) In Dynare, compute impulse responses of the gap, inflation, and nominal interest rate to both the natural rate shock and the cost-push shock for two different sets of weights: ($\alpha_x = 1$, $\alpha_i = 0$) and ($\alpha_x = 1$, $\alpha_i = 1$). For other parameter values, assume $\gamma = 0.1$, $\beta = 0.99$, $\sigma = 1$, $\rho_r = 0.9$, and $\rho_u = 0.9$. Compute impulse responses to one-unit shocks to both the natural rate and the cost-push term. Comment on how the responses look different depending on the welfare weights.

2. Consider a New Keynesian model with government spending. Linearized about a zero (net) inflation steady state. The linearized equilibrium conditions that are unrelated to policy are:

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi}\widetilde{mc}_t + \beta \mathbb{E}_t \pi_{t+1}$$
$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right)$$
$$\widetilde{mc}_t = \widetilde{w}_t$$
$$y_t = n_t$$
$$y_t = (1-\psi)c_t + \psi g_t$$
$$\chi n_t = -\sigma c_t + \widetilde{w}_t$$

Here, ϕ is the Calvo parameter and β is a subjective discount factor. σ is the inverse elasticity of substitution, and χ is the inverse Frisch labor supply elasticity. ω is the steady state share of government spending in output. There is no productivity shock.

(a) Take the above linearized conditions and re-write them as the following system:

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1}$$
$$x_t = \mathbb{E}_t x_{t+1} - \eta\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right)$$

Here, $x_t = y_t - y_t^f$, where y_t^f is the hypothetical equilibrium level of output where prices are flexible. Similarly, r_t^f is the hypothetical real interest rate if prices were flexible. Derive expressions for γ and η.

(b) Assume that government spending (in log deviations) obeys an AR(1) process:

$$g_t = \rho_G g_{t-1} + \varepsilon_{G,t}$$

Show that you can write r_t^f as an exogenous process of the form:

$$r_t^f = \rho_r r_{t-1}^f + s_r \varepsilon_{G,t}$$

Derive analytic expressions for ρ_r and s_r.

(c) Suppose that, in the present (i.e. period t), the nominal interest rate is pegged at $i_t = 0$. Each period thereafter, there is a probability α of the interest rate remaining pegged and a probability $1-\alpha$ of the peg lifting. Whenever the peg lifts, the central bank implements a strict inflation target (i.e. $\pi_t = 0$) and continues to do so forever. Use the method of undetermined coefficients to find linear policy functions for the output gap and the inflation rate as a function of r_t^f during the interest rate peg, i.e.:

$$x_t = \theta_x r_t^f$$
$$\pi_t = \theta_\pi r_t^f$$

Derive analytic expressions for θ_x and θ_π as a function of underlying parameters and the probability of the peg remaining in place (α).

(d) Use your expressions above to derive an analytic expression for the (impact) fiscal multiplier, $\frac{dY_t}{dG_t}$. Compute the multiplier as a function of α. Use values $\rho_G = 0.9$, $\gamma = 0.05$, $\psi = 0.2$, $\beta = 0.99$, $\sigma = 1$, $\chi = 1$. Assume values of α ranging from 0 to 0.9, with a space of 0.001 in between Plot the multiplier as a function of the *expected duration of the ZLB* (which is a function of α).

(e) Now, instead of a stochastic interest rate peg as above, solve the model using a deterministic peg length (as discussed in class). Consider peg lengths ranging from $H = 1$ to $H = 10$. For each, calculate the value of the fiscal multiplier. Plot the fiscal multiplier against H under a deterministic peg *on the same graph* as the multiplier under the deterministic peg (against the *expected* duration of the peg). Briefly comment on the similarities and differences between the two plots.