Problem Set 2

ECON 30020, Intermediate Macroeconomics, Fall 2024
The University of Notre Dame
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Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, you may turn in one problem set, but make sure that the names of all group members are clearly legible at the top of your assignment. Problem sets should be handed in during class and stapled in the upper left corner. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers / figures from Excel – you need not turn in Excel code. This problem set is due at the beginning of class on September 25.

1. **The Plague:** Consider a standard Solow model with a Cobb-Douglas production function and a constant productivity variable, $A_t = A = 1$. The production function is therefore:

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

In terms of capital per work, $k_t \equiv K_t/N_t$, the model is summarized by the central equation:

$$k_{t+1} = s k_t^{\alpha} + (1-\delta) k_t$$

(a) Solve for an analytic expression for the steady-state capital stock per worker. Also solve for an analytic expression for steady-state output per work ($y_t \equiv Y_t/N_t$).

(b) Assume that factors of production (N_t and K_t) are paid their marginal products (w_t and R_t, respectively). Derive expressions for these factor prices in terms of the capital stock per worker. Then derive expressions for the steady-state values of the factor prices using your answer from (a).

(c) Suppose that $N_t = 2$ and is (initially) fixed at that level. Derive analytic expressions for the steady state *levels* of capital and output.

(d) Suppose that the economy initially sits in a steady state as described above. Draw the main Solow diagram initially. Suppose that a plague comes in and permanently wipes out half the population, so $N_t = 1$ and is expected to forever remain at that level. Use the main diagram to show how the capital stock per worker will react (both immediately and dynamically going forward). Draw qualitative impulse response diagrams for both capital per worker and output per worker.

(e) In response to the immediate and sudden halving of the population, what happens (both on impact and dynamically) to the *levels* of capital and output? What about factor prices (w_t and R_t)? Draw out qualitative impulse response diagrams for these variables in response to the plague shock.

(f) Now create an Excel file (or Google Sheet file). Suppose that $\alpha = 1/3$, $\delta = 0.1$, and $s = 0.2$. Solve for *numeric* steady-state values of k^*, y^*, w^*, and R^* prior to the plague, when $N_t = 2$.

(g) Now, use your Excel file to trace out *numeric* dynamics of variables. In particular, suppose that the economy initially sits in the steady state associated with $N_t = 2$. Then, N_t suddenly drops to 1 and is expected to remain forever at that level. Produce *numeric* impulse response graphs of capital per worker, output per worker, the real wage, the rental rate on capital, and the levels of both output and the capital stock. Please turn in those plots, and confirm that they look (more or less) like your qualitative impulse response diagrams you drew above. To be very precise. Create rows corresponding to periods, with periods ranging from -5, -4, ... 0, 1, 2, ... 100. Prior to period 0, assume that all variables sit in the steady state. Then, in period 0, population gets halved due to the plague. Plot all variables against time, with time running from -5 to 100.

2. **Optimal Saving:** Suppose we have a Cobb-Douglas production function, and we normalize the level of productivity to $A_t = A^* = 1$. The central equation of the model is:

$$k_{t+1} = sk_t^{\alpha} + (1-\delta)k_t$$

Assume that $s = 0.2$, $\alpha = 1/3$, and $\delta = 0.1$.

(a) Create an Excel file. Solve for a *numeric* value of the steady-state value of consumption per work, c^*.

(b) Suppose that the economy initially sits in a steady state associated with these parameters. Then, the saving rate suddenly and permanently increases to $s = 0.3$. In your Excel file, trace out the dynamic response of consumption per worker. As in the earlier problem, assume that the economy sits in steady state for periods -5, -4, ..., -1. The change in the saving rate occurs in period 0. Plot the dynamic path of consumption per work from period -5 to 100. Show your plot in your answer.

(c) Is the household better off after this increase in the saving rate in the short run? What about the long run? What about overall? Explain briefly.

(d) Create a grid of values of s ranging from 0.05 to 0.5, with a space of 0.01 in between. For each possible value of s, numerically solve for the steady state value of c^*. Create a plot of c^* against s. Turn your plot in. What is (approximately) the value of s that maximizes c^*?

3. **Solow Model with Trend Growth:** Consider a Solow model with a Cobb-Douglas production function. Normalize the level of $A_t = A^* = 1$. The production function is:

$$Y_t = K_t^{\alpha} (Z_t N_t)^{1-\alpha}$$

The central equation of the model (again, in levels) is:

$$K_{t+1} = sK_t^{\alpha} (Z_t N_t)^{1-\alpha} + (1-\delta)K_t$$

Factor prices equal marginal products (i.e. $w_t = \frac{\partial Y_t}{\partial N_t}$ and $R_t = \frac{\partial Y_t}{\partial K_t}$). N_t grows at a constant rate, with $n \geq 0$:

$$N_{t+1} = (1+n)N_t$$

Z_t also grows at a constant rate, with $z \geq 0$:

$$Z_{t+1} = (1+z)Z_t$$

(a) Define "per efficiency unit" variables as $\widehat{k}_t \equiv K_t/(Z_t N_t)$ (and similarly for output). Re-write the central equation of the model in terms of per efficiency unit variables.

(b) Derive expressions for factor prices (w_t and R_t). Re-write the expressions for factor prices in terms of $\widehat{k}_t$.

(c) Solve for an analytic expression for the steady-state capital stock per efficiency units of labor.

(d) Suppose that the economy arrives at the steady state. Use your work from above to analytically show that the model as laid out above is consistent with all six of the time-series stylized facts discussed in class.

4. **Income and Substitution Effects in a Two-Period Consumption-Saving Model:** Suppose there is a household that lives for two periods (t, the present, and $t+1$, the future). The household begins life with no wealth. It receives an exogenous endowment of income in each period, Y_t and Y_{t+1}. The future endowment is known with certainty in the present. The household can borrow or save at real interest rate r_t, which it takes as given. The household's lifetime utility is:

$$U = \ln C_t + \beta \ln C_{t+1}$$

It faces the sequence of budget constraints:

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1+r_t)S_t$$

(a) Explain, in words, what the parameter β is meant to capture.

(b) Combine the two budget constraints into one to derive the intertemporal budget constraint (IBC).

(c) Use calculus to derive the consumption Euler equation.

(d) Use the Euler equation and the budget constraint to derive a consumption function.

(e) Derive an analytic expression for the marginal propensity to consume (MPC, i.e., $\frac{\partial C_t}{\partial Y_t}$).

(f) Derive an analytic expression for the partial derivative of consumption with respect to the real interest rate (i.e., $\frac{\partial C_t}{\partial r_t}$).

(g) Suppose that the household has no future endowment, i.e., $Y_t > 0$ and $Y_{t+1} = 0$. What is the partial derivative of the consumption function with respect to the real interest rate?

(h) Suppose, instead, that the household has a future endowment but no current endowment, i.e., $Y_t = 0$ and $Y_{t+1} > 0$. What is the partial derivative of the consumption function with respect to the real interest rate?

(i) Briefly discuss your answers to (g) and (h) and relate them to *income* and *substitution effects.*

5. GLS, Chapter 9, Exercise 4.