Consumption

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Ch. 9

Microeconomics of Macro

We now move from the long run (decades and longer) to the medium run (several years) and short run (months up to several years)

In long run, we did not explicitly model most economic decision-making – just assumed rules (e.g. consume a constant fraction of income)

Building blocks of the remainder of the course are decision rules of optimizing agents and a concept of equilibrium

This is micro but in an aggregate context

Basic Framework

We will start by studying optimal decision rules

Then we work up to aggregation and equilibrium

Framework is dynamic but only two periods (t, the present, and $t+1$, the future)

Will also work with representative agents: one household and one firm

Unrealistic but useful abstraction and can be motivated in world with heterogeneity through insurance markets

Consumption

Consumption the largest expenditure category in GDP (60-70 percent)

Representative household receives exogenous amount of income in periods t and $t+1$

Household may consume or save/borrow – must decide how to divide its income in t between consumption and saving/borrowing

Everything real – think about one good as “fruit”

Basics

Income of Y_t and Y_{t+1}. Future income known with certainty (allowing for uncertainty raises some interesting issues but does not fundamentally impact problem)

Consumes C_t and C_{t+1}

Begins life with no wealth, and can save $S_t = Y_t - C_t$ (can be negative, which is borrowing)

Earns/pays real interest rate r_t on saving/borrowing

Household a price-taker: takes r_t as given

Do not model a financial intermediary (e.g., a bank), but assume existence of option to borrow/save through this intermediary

Budget Constraints

Two flow budget constraints in each period:

$$C_t + S_t \leq Y_t$$
$$C_{t+1} + S_{t+1} - S_t \leq Y_{t+1} + r_t S_t$$

Saving vs. Savings: saving is a flow and savings is a stock. Saving is the change in the stock

As written, S_t and S_{t+1} are stocks

In period t, no distinction between stock and flow because no initial stock

$S_{t+1} - S_t$ is flow saving in period $t+1$; S_t is the stock of savings household takes from t to $t+1$, and S_{t+1} is the stock it takes from $t+1$ to $t+2$

$r_t S_t$: income earned on the stock of savings brought into $t+1$

Terminal Condition and the IBC

Household would not want $S_{t+1} > 0$. Why? There is no $t+2$. Don't want to die with positive assets

Household would like $S_{t+1} < 0$ – die in debt. Lender would not allow that

Hence, $S_{t+1} = 0$ is a terminal condition

Assume budget constraints hold with equality, and eliminate S_t, leaving:

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

This is called the intertemporal budget constraint (IBC). Says that present discounted value of the stream of consumption equals the present discounted value of stream of income.

Preferences

Household gets utility from how much it consumes

Utility function: $u(C_t)$. "Maps" consumption into utils

Assume: $u'(C_t) > 0$ (positive marginal utility) and $u''(C_t) < 0$ (diminishing marginal utility)

"More is better, but at a decreasing rate"

Example

Example utility function:

$$u(C_t) = \ln C_t$$
$$u'(C_t) = \frac{1}{C_t} > 0$$
$$u''(C_t) = -C_t^{-2} < 0$$

Utility is completely ordinal – no meaning to magnitude of utility (it can be negative). Only useful to compare alternatives

Lifetime Utility

Lifetime utility is a weighted sum of utility from period t and $t+1$ consumption:

$$U = u(C_t) + \beta u(C_{t+1})$$

$0 < \beta < 1$ is the discount factor – it is a measure of how impatient the household is.

Household Problem

Technically, household chooses C_t and S_t in first period subject to the first-period flow constraint. This effectively determines C_{t+1} from the second-period flow constraint

Think instead about choosing C_t and C_{t+1} in period t subject to the IBC

$$\max_{C_t, C_{t+1}} \quad U = u(C_t) + \beta u(C_{t+1})$$

s.t.

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

Euler Equation

First order optimality condition is famous in economics – the "Euler equation" (pronounced "oiler")

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$

This is just an MRS = price ratio condition!

Necessary but not sufficient for optimality

Doesn't determine level of consumption. To do that need to combine with IBC

Indifference Curve

Think of C_t and C_{t+1} as different goods (different in time dimension)

Indifference curve: combinations of C_t and C_{t+1} yielding fixed overall level of lifetime utility

Different indifference curve for each different level of lifetime utility. Direction of increasing preference is "northeast"

Slope of indifference curve at a point is the negative ratio of marginal utilities (or marginal rate of substitution, MRS):

$$\text{slope} = -\frac{u'(C_t)}{\beta u'(C_{t+1})}$$

Given assumption of $u''(\cdot) < 0$, steep near origin and flat away from it

Budget Line

Graphical representation of IBC

Shows combinations of C_t and C_{t+1} consistent with IBC holding, given Y_t, Y_{t+1}, and r_t

- Points inside budget line: do not exhaust resources
- Points outside budget line: infeasible

By construction, must pass through point $C_t = Y_t$ and $C_{t+1} = Y_{t+1}$ ("endowment point")

Slope of budget line is negative gross real interest rate:

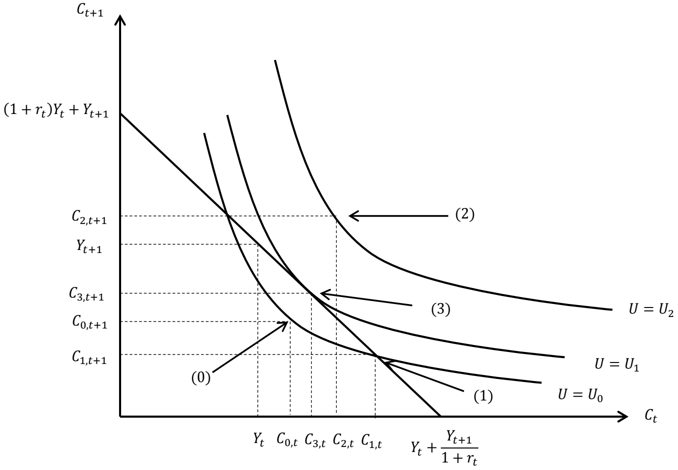
$$\text{slope} = -(1 + r_t)$$

Optimality

Objective is to choose a consumption bundle on highest possible indifference curve that does not violate IBC

At this point, indifference curve and budget line are tangent (which is same condition as Euler equation)

Optimality: Graphically



Consumption Function

What we want is a decision rule that determines C_t as a function of things which the household takes as given – Y_t, Y_{t+1}, and r_t

Consumption function:

$$C_t = C^d(Y_t, Y_{t+1}, r_t)$$

Can use indifference curve - budget line diagram to qualitatively figure out how changes in Y_t, Y_{t+1}, and r_t affect C_t (i.e., to sign partial derivatives)

Increases in Y_t and Y_{t+1}

An increase in Y_t or Y_{t+1} causes the budget line to shift out horizontally to the right

In new optimum, household will locate on a higher indifference curve with higher C_t and C_{t+1}

Consumption Smoothing

Household wants to increase consumption in both periods when income increases in either period

Wants its consumption to be "smooth" relative to its income

Achieves smoothing by adjusting saving behavior: increases S_t when Y_t goes up, reduces S_t (borrows, or saves less) when Y_{t+1} goes up

Can conclude that $\frac{\partial C^d}{\partial Y_t} > 0$ and $\frac{\partial C^d}{\partial Y_{t+1}} > 0$

Further, $\frac{\partial C^d}{\partial Y_t} < 1$

Call this the marginal propensity to consume, MPC

Increase in r_t

A little trickier

Causes budget line to become steeper, pivoting through endowment point

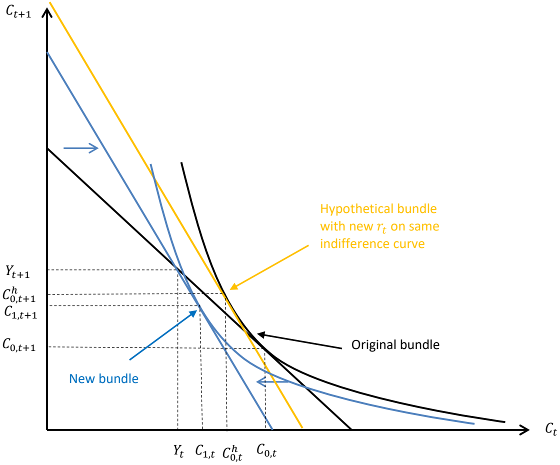
Competing income and substitution effects:

- Substitution effect: how would consumption bundle change when r_t increases and income is adjusted so that household would locate on unchanged indifference curve?
- Income effect: how does change in r_t allow household to locate on a higher/lower indifference curve?

Substitution effect always to reduce C_t, increase S_t

Income effect depends on whether initially a borrower ($C_t > Y_t$, income effect to reduce C_t) or saver ($C_t < Y_t$, income effect to increase C_t)

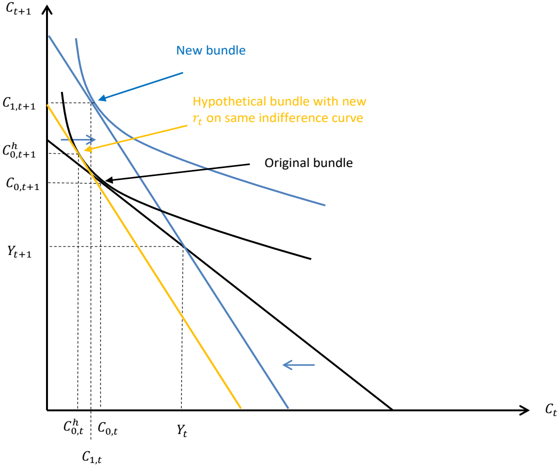
Borrower



Sub effect: $\downarrow C_t$. Income effect: $\downarrow C_t$

Total effect: $\downarrow C_t$

Saver



Sub effect: $\downarrow C_t$. Income effect: $\uparrow C_t$

Total effect: ambiguous

The Consumption Function

We will assume that the substitution effect always dominates for the interest rate

Qualitative consumption function (with signs of partial derivatives)

$$C_t = C(\underset{+}{Y_t}, \underset{+}{Y_{t+1}}, \underset{-}{r_t}).$$

Technically, partial derivative itself is a function

However, we will mostly treat the partial with respect to first argument as a constant parameter we call the MPC. This works with log utility

Algebraic Example with Log Utility

Suppose $u(C_t) = \ln C_t$

Euler equation is:

$$C_{t+1} = \beta(1 + r_t)C_t$$

Combining with IBC, consumption function is:

$$C_t = \frac{1}{1+\beta}\left[Y_t + \frac{Y_{t+1}}{1+r_t}\right]$$

MPC: $\frac{1}{1+\beta}$. Go through other partials

Permanent Income Hypothesis (PIH)

Our analysis consistent with Friedman (1957) and the PIH

Consumption ought to be a function of "permanent income"

Permanent income: annuity value of the present value of lifetime income

Special case: $r_t = 0$ and $\beta = 1$: consumption equal to *average* lifetime income

PIH Implications

1. Consumption forward-looking. Consumption should not react to changes in income that were predictable in the past
2. MPC less than 1
3. Longer you live, the lower is the MPC

Important empirical implications for econometric practice of the day. Regression of C_t on Y_t will <u>not</u> identify MPC (which is relevant for things like fiscal multiplier) if in historical data changes in Y_t are persistent

Applications and Extensions

Book considers several applications / extensions:

You are responsible for this material though we will only briefly discuss these in class

1. Wealth (GLS Ch. 9.4.1):
   - Can assume household begins life with some assets other than strict savings (e.g. housing, stocks) and potentially allow household to accumulate more wealth
   - Unsurprising implication: increases in value of wealth (e.g. increase in house prices) can result in more consumption/less saving
2. Permanent vs. transitory changes in income (GLS Ch. 9.4.2)
   - Household will adjust consumption more (and saving less) to shocks to income the more *persistent* these are (persistent in sense of change in Y_t being correlated with change in Y_{t+1} of same sign)

Consumption Under Uncertainty

GLS Ch. 9.4.4-9.4.5

Suppose that future income is <u>uncertain</u>

Suppose it can take on two values: $Y^h_{t+1} \geq Y^l_{t+1}$

Let $p \in [0, 1]$ be the probability of the high state and $1 - p$ the probability of the low state

Expected value of income is: $E(Y_{t+1}) = pY^h_{t+1} + (1 - p)Y^l_{t+1}$

Uncertainty of future income translates into uncertainty over future consumption

Budget Constraints with Uncertainty

Everything dated t (the present) is known

Period $t+1$ budget constraint must hold in both states of the world:

$$C_{t+1}^h \leq Y_{t+1}^h + (1+r_t)S_t$$
$$C_{t+1}^l \leq Y_{t+1}^l + (1+r_t)S_t$$

Expected Utility

Expected lifetime utility:

$$\mathbb{E}(U) = u(C_t) + \beta \times \left[pu(C_{t+1}^h) + (1-p)u(C_{t+1}^l) \right]$$

This is equivalent to:

$$\mathbb{E}(U) = u(C_t) + \beta \mathbb{E}\left[u(C_{t+1})\right]$$

Key insight: expected value of a function is not equal to the function of expected value (unless the function is linear)

Euler Equation

Euler equation looks almost same under uncertainty but has expectation operator:

$$u'(C_t) = \beta(1 + r_t)\mathbb{E}\left[u'(C_{t+1})\right]$$

With log utility:

$$\frac{1}{C_t} = \beta(1 + r_t)\left[p\frac{1}{C^h_{t+1}} + (1 - p)\frac{1}{C^l_{t+1}}\right]$$

Precautionary saving: if $u'''(\cdot) > 0$, then $\uparrow$ uncertainty over future income results in $\downarrow C_t$. Go through example.

Random Walk Hypothesis

Continue to allow future income to be uncertain

But instead assume that $u'''(\cdot) = 0$ (no precautionary saving; quadratic utility)

Further assume that $\beta(1 + r_t) = 1$. Then Euler equation implies:

$$\mathbb{E}\left[C_{t+1}\right] = C_t$$

Consumption expected to be constant – simple implication of desire to smooth consumption applied to model with uncertainty

Random Walk Implications

Consumption ought not react to changes in Y_{t+1} that were predictable from perspective of period t:

- e.g. retirement, Social Security withholding throughout year, monthly paychecks
- After Hall (1978), this is one of the most tested implications in macroeconomics
- Generally fails – potential evidence of liquidity constraints (GLS Ch. 9.4.6)