Growth

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Ch. 4 (facts)

GLS Ch. 5-6 (Solow Growth Model)

GLS Ch. 7 (cross-country income differences)

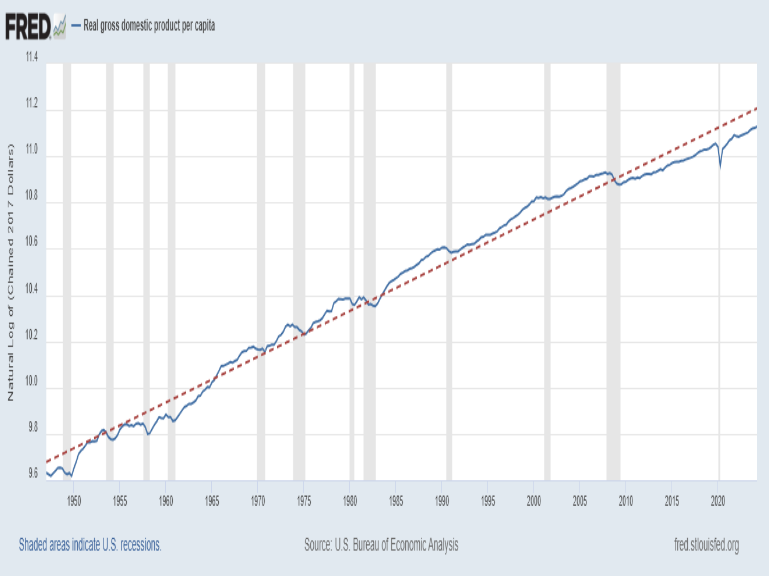
Economic Growth

When economists say "growth," typically mean average rate of growth in real GDP per capita over long horizons

- Long run: frequencies of time measured in decades
- Not period-to-period fluctuations in the growth rate

"Once one begins to think about growth, it is difficult to think about anything else" – Robert Lucas, 1995 Nobel Prize winner

US Real GDP per capita



Summary Stats

Average (annualized) growth rate of per capita real GDP: 1.8%

Implies that the level of GDP doubles roughly once every 40 years

- Growing just 0.2 percentage points faster (2% growth rate): level doubles every 35 years
- Rule of 70: number of years it takes a variable to double is approximately 70 divided by the growth rate
- Consider two countries that start with same GDP, but country A grows 2% per year and country B grows 1% per year. After 100 years, A will be 165% richer!

Small differences in growth rates really matter over long horizons

Key Question

What accounts for this growth?

In a mechanical sense, can only be two things:

- Growth in productivity: we produce more output given the same inputs
- Factor accumulation: more factors of production help us produce more stuff

Factors of Production

Two key factors of production on which we focus are capital and labor

- Capital: stuff we produce that we don't consume and instead use to produce other stuff
- Labor: measured in units of time

Hours worked per capita (i.e., labor) is roughly trendless – not a plausible source of growth in per-capita income

So what drives growth: capital accumulation or productivity improvements?

Relatedly, are rich countries rich because they have more capital than poorer countries, or because they are more productive?

Stylized Facts: Time Series

1. Output per worker grows at an approximately constant rate over long periods of time ▸ picture
2. Capital per worker grows at an approximately constant rate over long periods of time ▸ picture
3. The capital to output ratio is roughly constant over long periods of time ▸ picture
4. Labor's share of income is roughly constant over long periods of time ▸ picture
5. The return to capital is roughly constant over long periods of time ▸ picture
6. The real wage grows at approximately the same rate as output per worker over long periods of time ▸ picture

Stylized Facts: Cross-Section

1. There are large differences in income per capita across countries ▸ table
2. There are some examples where poor countries catch up (growth miracles), otherwise where they do not (growth disasters) ▸ table
3. Human capital (e.g. education) strongly correlated with income per capita ▸ table

Solow Model

Solow model (Solow, 1953): used to study long-run growth and cross-country income differences

Does nice job with stylized facts

Main implication of model: productivity is key

- Productivity key to sustained growth (not factor accumulation)
- Productivity key to understanding cross-country income differences (not level of capital)

Model takes productivity to be exogenous. What is it? How to increase it?

Model Basics

Time runs from t (the present) onwards into infinite future

Representative household and representative firm

Everything real, one kind of good (fruit)

Production Function

Production function:

$$Y_t = A_t F(K_t, N_t)$$

- K_t: capital. Must be itself produced, used to produce other stuff, does not get completely used up in production process
- N_t: labor
- Y_t: output
- A_t: productivity (exogenous)

Think about output as units of fruit. Capital is stock of fruit trees. Labor is time spent picking from the trees

Properties of Production Function

Both inputs necessary: $F(0, N_t) = F(K_t, 0) = 0$

Increasing in both inputs: $F_K(K_t, N_t) > 0$ and $F_N(K_t, N_t) > 0$

Concave in both inputs: $F_{KK}(K_t, N_t) < 0$ and $F_{NN}(K_t, N_t) < 0$

Constant returns to scale: $F(qK_t, qN_t) = qF(K_t, N_t)$

Capital and labor are paid marginal products:

$$w_t = A_t F_N(K_t, N_t)$$
$$R_t = A_t F_K(K_t, N_t)$$

Example Production Function: Cobb-Douglas

$$F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1$$

Consumption and Investment

Fruit can either be eaten (consumption) or re-planted in the ground (investment)

Investment yields another tree (capital) with a one-period delay

A constant fraction of output is invested, $0 \leq s \leq 1$. "Saving rate" or "investment rate"

Means $1 - s$ of output is consumed

Resource constraint:

$$Y_t = C_t + I_t$$

Labor & Capital

Abstract from endogenous labor supply – labor supplied inelastically

Current capital stock is exogenous – depends on past decisions

- We often refer to variables like this as state variables

Capital accumulation, $0 < \delta < 1$ depreciation rate:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

Equations of Model

$$Y_t = A_t F(K_t, N_t)$$

$$Y_t = C_t + I_t$$

$$K_{t+1} = I_t + (1 - \delta) K_t$$

$$I_t = s Y_t$$

$$w_t = A_t F_N(K_t, N_t)$$

$$R_t = A_t F_K(K_t, N_t)$$

Six endogenous variables (Y_t, C_t, K_{t+1}, I_t, w_t, R_t), six equations, and three exogenous variables (A_t, K_t, N_t)

Central Equation

First four equations can be combined into one:

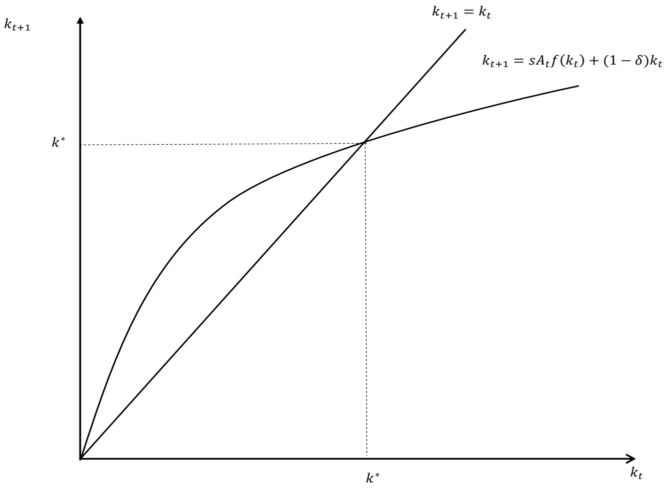
$$K_{t+1} = sA_tF(K_t, N_t) + (1-\delta)K_t$$

Define lowercase variables as "per worker." $k_t = \frac{K_t}{N_t}$. In per-worker terms:

$$k_{t+1} = sA_tf(k_t) + (1-\delta)k_t$$

One equation describing dynamics of k_t. Once you know dynamic path of capital, you can recover everything else

Plot of the Central Equation



The Steady State

The steady-state capital stock is the value of capital at which $k_{t+1} = k_t$. Label this k^*

Graphically, this is where the curve (the plot of k_{t+1} against k_t) crosses the 45-degree line (a plot of $k_{t+1} = k_t$)

Via assumptions of the production function along with auxiliary assumptions (the Inada conditions), there exists one non-zero steady-state capital stock

The steady state is "stable" in the sense that for any initial $k_t \neq 0$, the capital stock will converge to this point

"Once you get there, you sit there"

Since capital governs everything else, all other variables go to a steady state determined by k^*

Algebraic Example

Suppose $f(k_t) = k_t^{\alpha}$. Suppose A_t is constant at A^*. Then:

$$
\begin{aligned}
k^* &= \left(\frac{sA^*}{\delta}\right)^{\frac{1}{1-\alpha}} \\
y^* &= A^* k^{*\alpha} \\
c^* &= (1-s)A^* k^{*\alpha} \\
i^* &= sA^* k^{*\alpha} \\
R^* &= \alpha A^* k^{*\alpha-1} \\
w^* &= (1-\alpha)A^* k^{*\alpha}
\end{aligned}
$$

Dynamic Effects of Changes in Exogenous Variables

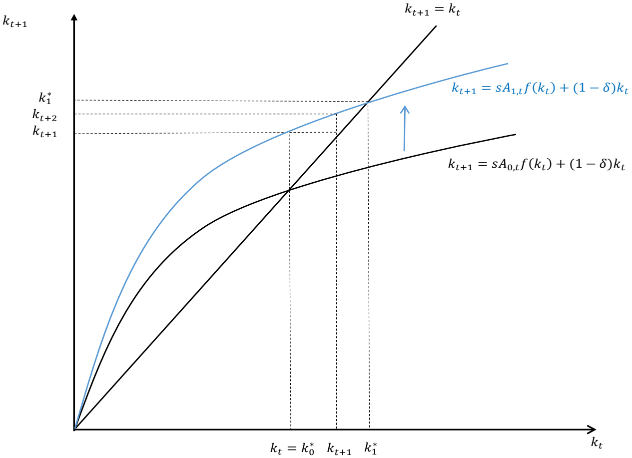
Want to consider the following exercises:

- What happens to endogenous variables in a dynamic sense after a permanent change in A^*
- What happens to endogenous variables in a dynamic sense after a permanent change in s (the saving rate)?

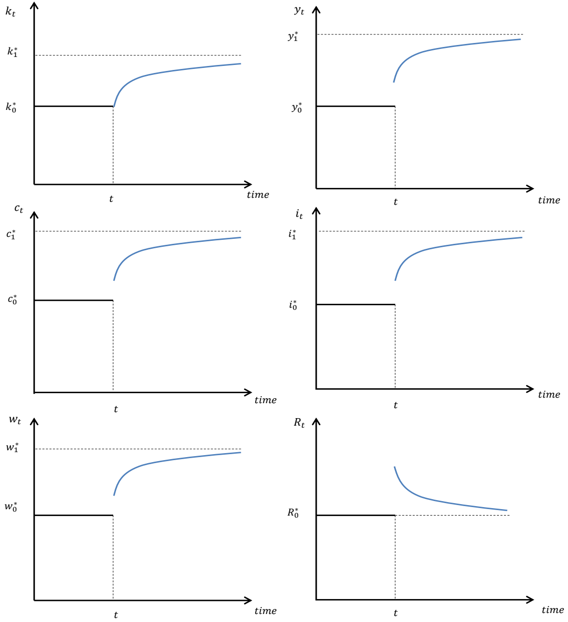
For these exercises:

1. Assume we start in a steady state
2. Graphically see how the steady state changes after the change in productivity or the saving rate
3. Current capital stock cannot change (it is predetermined/exogenous). But $k_t \neq k^*$. Use dynamic analysis of the graph to figure out how k_t reacts dynamically
4. Once you have that, you can figure out what everything else is doing

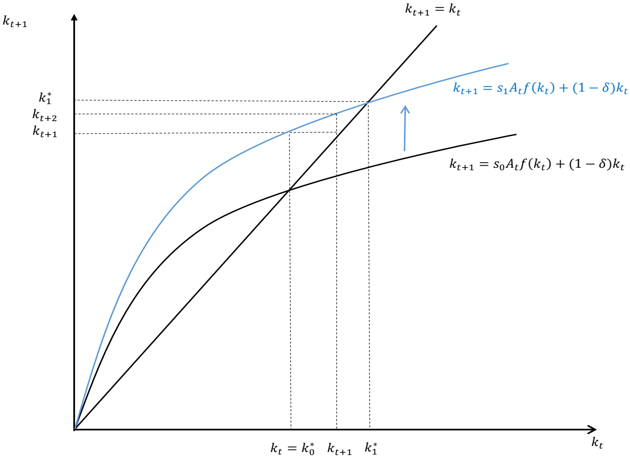
Permanent Increase in A^*



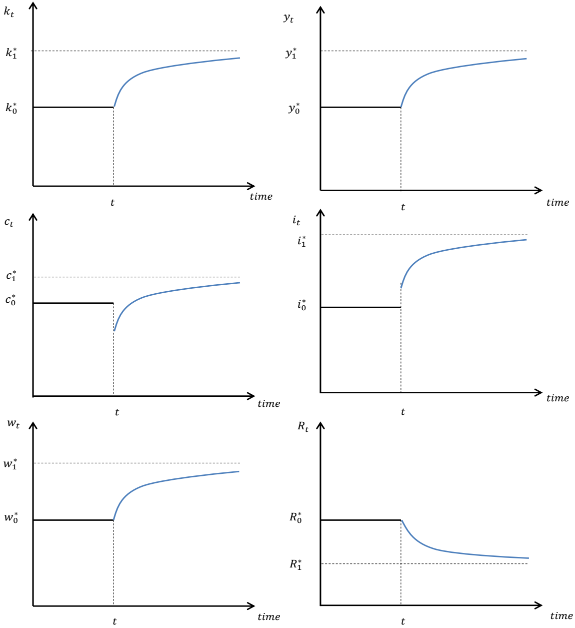
Impulse Response Functions: Permanent Increase in A^*



Permanent Increase in s



Impulse Response Functions: Permanent Increase in s



Discussion

Neither changes in A^* nor s trigger sustained increases in growth

Each triggers faster growth for a while while the economy accumulates more capital and transitions to a new steady state

In the long run, there is no growth in this model – it goes to a steady state!

We'll fix that. You can kind of see, however, that sustained growth must come from increases in productivity. Why?

- No limit on how high A can get – it can just keep increasing. Upper bound on s
- Repeated increases in s would trigger continual decline in R_t, inconsistent with stylized facts

The Bottom Line

Sustained growth must be due to productivity growth, not factor accumulation

You can't save your way to more growth

Key model assumption: diminishing returns to capital

Optimal Saving Rate

What is the "optimal" saving rate, s?

Utility comes from consumption, not output

Higher s has two effects – the "size of the pie" and the "fraction of the pie":

- More capital $\rightarrow$ more output $\rightarrow$ more consumption (bigger size of the pie)
- Consume a smaller fraction of output $\rightarrow$ less consumption (eat a smaller fraction of the pie)

The Golden Rule

The Golden Rule saving rate: value of s that maximizes steady-state consumption, c^*

- $s = 0$: $c^* = 0$
- $s = 1$: $c^* = 0$

Implicity characterized by $A^* f'(k^*) = \delta$. Graphical intuition.

Dynamic Inefficiency

Being "below" the Golden Rule does not necessarily mean that an economy is not saving enough

- There is a dynamic tradeoff: $\uparrow s$ today means less consumption today, but more in the future
- Whether that is good or not depends on how the future is valued relative to the present (i.e., discounting)

But being "above" the Golden Rule cannot be optimal. We say that it is dynamically inefficient

- By reducing the saving rate, could get more consumption both in present and in the future

Little or no evidence to suggest any modern economy is dynamically inefficient

Growth

Wrote down a model to study growth

But model converges to a steady state with no growth

Isn't that a silly model?

It turns out, no

Augmented Solow Model

Production function is:

$$Y_t = A_t F(K_t, Z_t N_t)$$

- Z_t: labor-augmenting productivity
- $Z_t N_t$: efficiency units of labor
- Assume Z_t and N_t both grow over time (initial values in period 0 normalized to 1):

$$Z_t = (1+z)^t$$
$$N_t = (1+n)^t$$

  - $z = n = 0$: case we just did
  - Z_t not fundamentally different from A_t. Convenient to use Z_t to control growth while A_t controls level of productivity

Per Efficiency Unit Variables

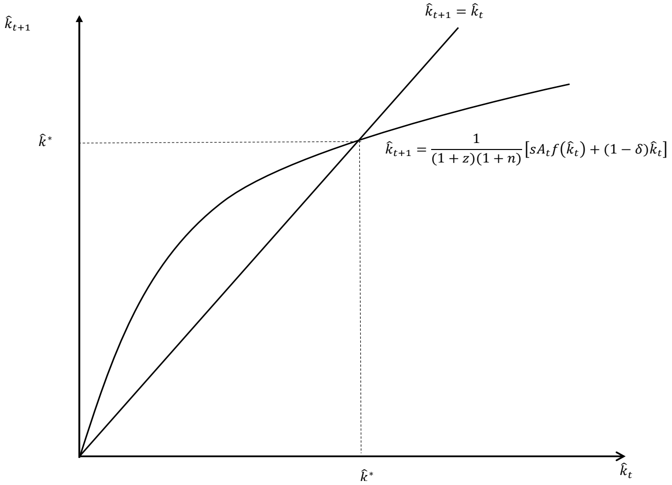
Define $\widehat{k}_t = \frac{K_t}{Z_t N_t}$ and similarly for other variables. Lower case variables: per-capita. Lower case variables with "hats": per efficiency unit variables

Modified central equation of model is:

$$\widehat{k}_{t+1} = \frac{1}{(1+z)(1+n)} \left[sA_t f(\widehat{k}_t) + (1-\delta)\widehat{k}_t \right].$$

Practically the same as before

Plot of Modified Central Equation



Steady-State Growth I

Via similar arguments to earlier, there exists a steady state $\widehat{k}^*$ at which $\widehat{k}_{t+1} = \widehat{k}_t$. Economy converges to this point from any non-zero initial value of $\widehat{k}_t$

Economy converges to a steady state in which per efficiency unit variables do not grow. What about actual and per capita variables? If $\widehat{k}_{t+1} = \widehat{k}_t$, then:

$$\frac{K_{t+1}}{Z_{t+1}N_{t+1}} = \frac{K_t}{Z_t N_t}$$

$$\frac{K_{t+1}}{K_t} = \frac{Z_{t+1}N_{t+1}}{Z_t N_t} = (1+z)(1+n)$$

$$\frac{k_{t+1}}{k_t} = \frac{Z_{t+1}}{Z_t} = 1+z$$

Steady-State Growth II

Level of capital stock grows at approximately sum of growth rates of Z_t and N_t

Per-capita capital stock grows at rate of growth in Z_t

This growth is manifested in output and the real wage, but not the return on capital

Steady State Growth and Stylized Facts

Once in steady state, we have:

$$\frac{y_{t+1}}{y_t} = 1 + z$$

$$\frac{k_{t+1}}{k_t} = 1 + z$$

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{K_t}{Y_t}$$

$$\frac{w_{t+1}N_{t+1}}{Y_{t+1}} = \frac{w_t N_t}{Y_t}$$

$$R_{t+1} = R_t$$

$$\frac{w_{t+1}}{w_t} = 1 + z$$

These are the six time series stylized facts!

Understanding Cross-Country Income Differences

Solow model can reproduce time series stylized facts if it is assumed that productivity grows over time

Let's now use the model to think about cross-country income differences

What explains these differences?

Hypotheses

Three hypotheses for why cross-country income differences exist:

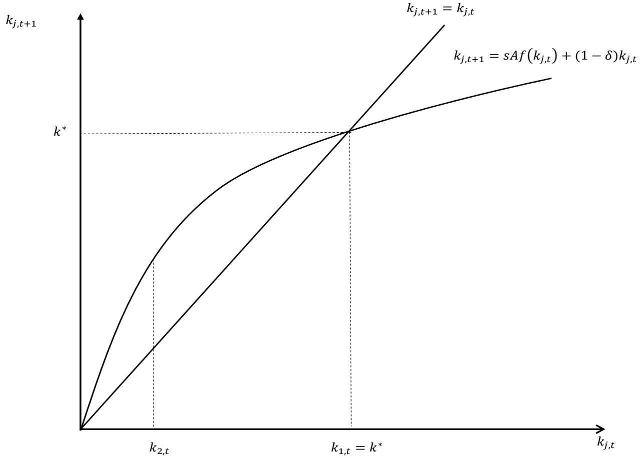
1. Countries initially endowed with different levels of capital
2. Countries have different saving rates
3. Countries have different productivity levels

Like sustained growth, most plausible explanation for cross-country income differences is productivity

Convergence

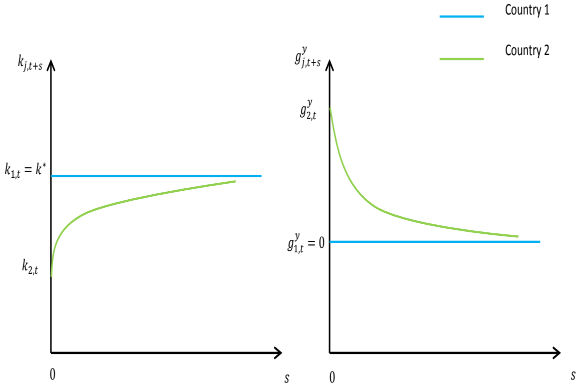
Suppose two countries are otherwise identical, and hence have the same steady state

But suppose that country 2 is initially endowed with less capital – $k_{2,t} < k_{1,t} = k^*$

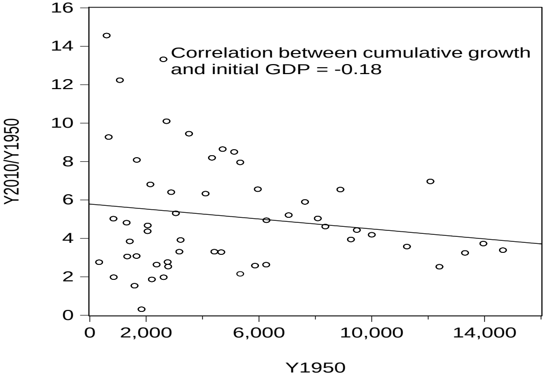


Catch Up

If country 2 is initially endowed with less capital, it should grow faster than country 1, eventually catching up with country 1 – they have the same steady state

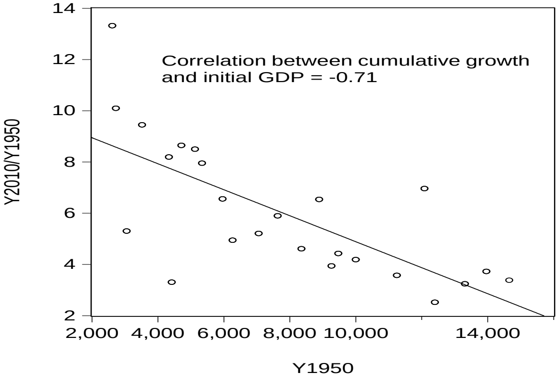


Is There Convergence in the Data?



Correlation between growth and initial GDP is weakly negative when focusing on all countries

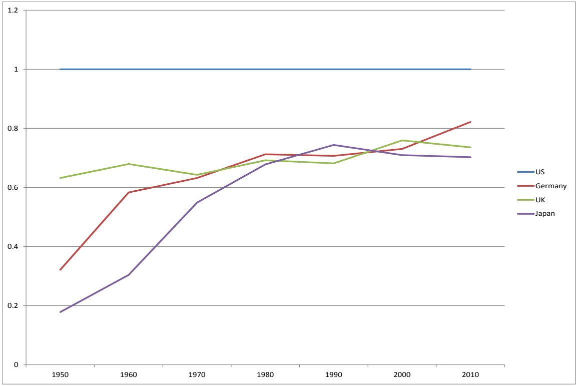
Focusing on a More Select Group of Countries



Focusing only on OECD countries (more similar) story looks more promising for convergence

Still, catch up seems too slow for initial low levels of capital to be the main story

Pseudo Natural Experiment: WWII



WWII losers (Germany and Japan) grew faster for 20-30 years than the winners (US and UK)

But don't seem to be catching up all the way to the US: conditional convergence. Countries have different steady states

Differences in s and A^*

Most countries seem to have different steady states

For simple model with Cobb-Douglas production function, relative outputs:

$$\frac{y_1^*}{y_2^*} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{1-\alpha}} \left(\frac{s_1}{s_2}\right)^{\frac{\alpha}{1-\alpha}}$$

Can differences in s plausibly account for large income differences?

No (for plausible values of α)

Differences in s

Suppose A^* the same in both countries. Suppose country 1 is US, and country 2 is Mexico: $\frac{y_1^*}{y_2^*} = 4$. We have:

$$s_2 = 4^{\frac{\alpha-1}{\alpha}} s_1$$

A plausible value of $\alpha = 1/3$. Means $\frac{\alpha-1}{\alpha} = -2$

Mexican saving rate would have to be 0.0625 times US saving rate

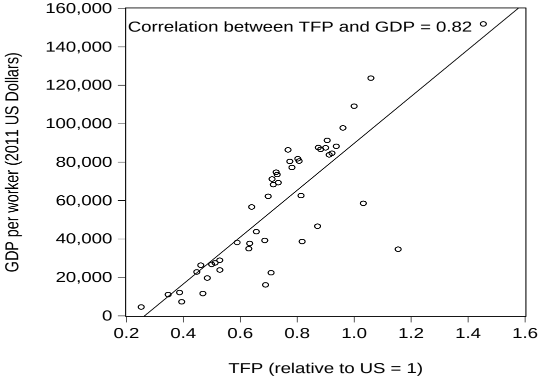
This would be something like a saving rate of one percent (or less)! Not plausible

Becomes more plausible if α is much bigger

What Could It Be?

If countries have different steady states and differences in s cannot plausibly account for this, must be differences in productivity

Seems to be backed up in data: rich countries are highly productive



Productivity is King

Productivity is what drives everything in the Solow model

Sustained growth must come from productivity

Large income differences must come from productivity

But what is productivity? Solow model doesn't say

Factors Influencing Productivity

Including but not limited to:

1. Knowledge and education
2. Climate
3. Geography
4. Institutions
5. Finance
6. Degree of openness
7. Infrastructure

Policy Implications

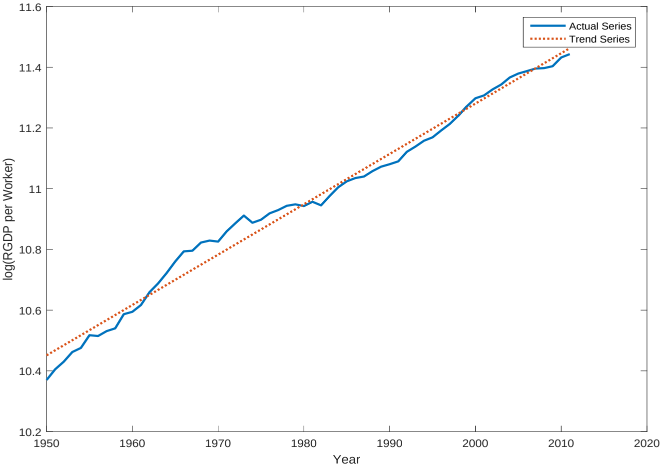
If a country wants to become richer, need to focus on policies that promote productivity

Example: would giving computers (capital) to people in sub-Saharan Africa help them get rich?

- Not without the infrastructure to connect to the internet, the knowledge of how to use the computer, and the institutions to protect property rights

Also has implications when thinking about poverty within a country (e.g., UBI)

Output Per Worker over Time



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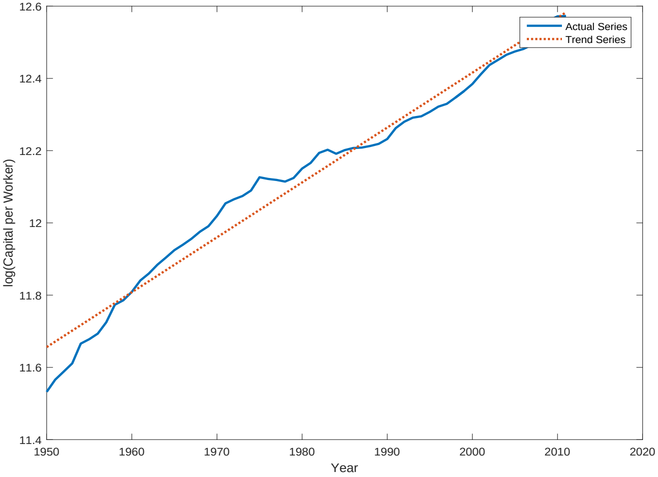
Capital Per Worker over Time



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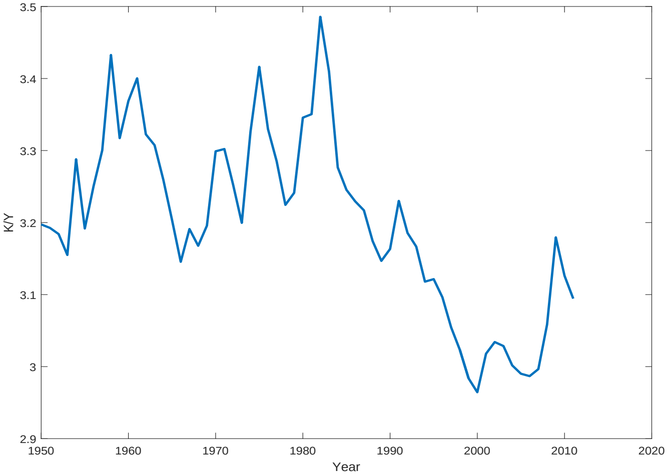
Capital to Output Ratio over Time



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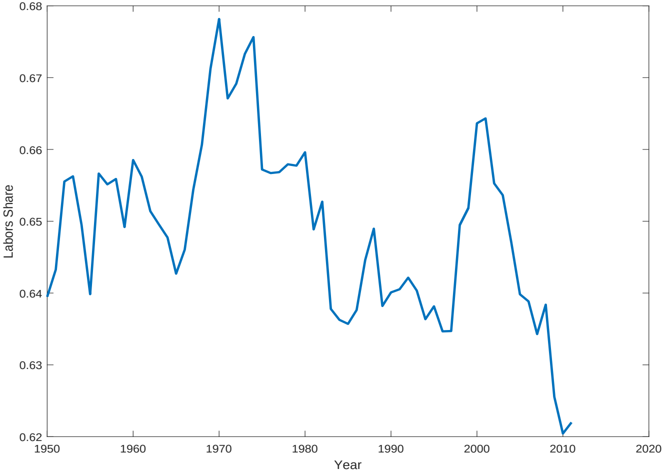
Labor Share over Time



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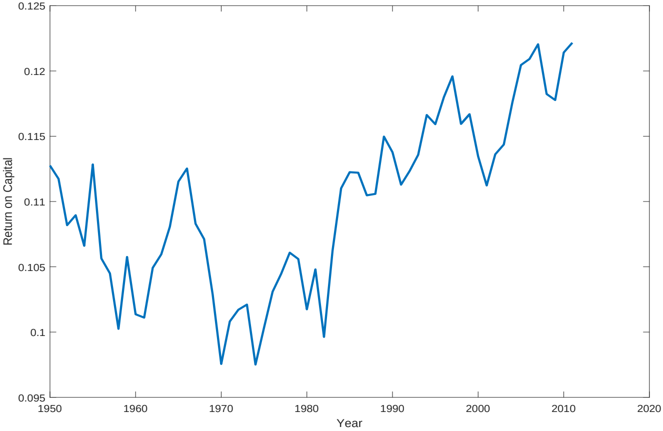
Return on Capital over Time



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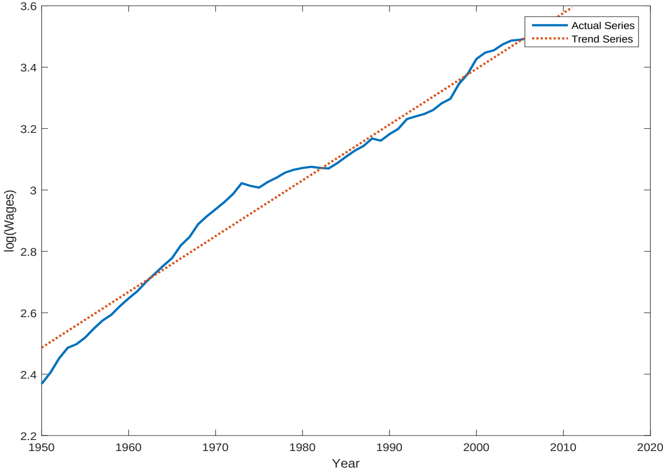
Real Wage over Time



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Income Differences

		GDP per Person
High income countries		
	Canada	$35,180
	Germany	$34,383
	Japan	$30,232
	Singapore	$59,149
	United Kingdom	$32,116
	United States	$42,426
Middle income countries		
	China	$8,640
	Dominican Republic	$8,694
	Mexico	$12,648
	South Africa	$10,831
	Thailand	$9,567
	Uruguay	$13,388
Low income countries		
	Cambodia	$2,607
	Chad	$2,350
	India	$3,719
	Kenya	$1636
	Mali	$1,157
	Nepal	$1,281

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Growth Miracles and Disasters

	Growth Miracles		
	1970 Income	2011 Income	% change
South Korea	$1918	$27,870	1353
Taiwan	$4,484	$33,187	640
China	$1,107	$8,851	700
Botswana	$721	$14,787	1951
	Growth Disasters		
Madagascar	$1,321	$937	-29
Niger	$1,304	$651	-50
Burundi	$712	$612	-14
Central African Republic	$1,148	$762	-34

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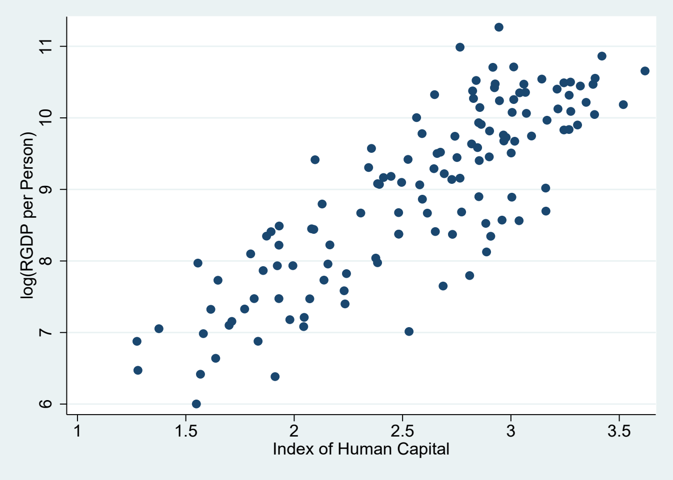
Education and Income Per Capita



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