Midterm 1 Review

ECON 30020: Intermediate Macroeconomics
Professor Sims
University of Notre Dame, Fall 2024

The first midterm will take place on Monday, September 30. The material covered includes Chapters 1 through 7 and Chapter 9. This means that the exam covers economic growth and the Solow model as well as the two-period consumption model. Material from Chapters 1-3 (on data definitions and why economists use models) is also fair game. The exam will consistent of 10 multiple choice questions (for which no justification is required), 10 true/false questions (for which no justification is required), and several "free response" questions which require either a written response in words, algebraic calculations, or graphical analysis. The free response questions ought to look something like the problem set questions. You may use a calculator (though you should not need one), but you may not access notes, phones, or anything else during the exam. This review lists some terms you should know, asks some basic questions, includes some new problems for practice, and references problems from GLS. It is meant to help you study; it is not a substitute for reviewing the text, lecture notes, and problem sets.

I start by listing some key variables / parameters that often come up:

- Y_t: output produced at time t
- C_t, I_t: consumption and investment at time t
- N_t: labor input (hours) at time t
- w_t: real wage (denominated in units of goods)
- R_t: real rental rate on capital (denominated in units of goods)
- α: exponent on capital in a Cobb-Douglas production function
- s: saving rate in the Solow model
- δ: depreciation rate on capital
- k_t: capital per worker at time t (i.e. $\frac{K_t}{N_t}$); similar (lowercase) for other variables
- A_t: neutral productivity (exogenous)
- Z_t: labor augmenting productivity (exogenous)
- $Z_t N_t$: efficiency units of labor
- $\widehat{k}_t$: capital per efficiency unit of labor (i.e. $\frac{K_t}{Z_t N_t}$); similar (lowercase with a hat) for other variables

- Variable with a $*$ superscript: steady state
- z: growth rate of labor augmenting productivity (exogenous)
- n: growth rate of labor input (used interchangeably with growth rate of population, though in principle these are distinct concepts)
- S_t: the stock of savings (with an s) a household takes from period t to period $t+1$ in the two period consumption model
- r_t: the interest rate in the two period consumption model
- β: discount factor
- Y_t, Y_{t+1}: income in period t and $t+1$ in the two period consumption model (exogenous)

Next, I list key expressions/formulae that frequently come up:

- $Y_t = C_t + I_t + G_t + NX_t$ (different expenditure categories; for much of what we do there is no government and no rest of world, so $G_t = NX_t = 0$)
- $K_{t+1} = I_t + (1-\delta)K_t$: capital accumulation equation. K_t is predetermined and exogenous within a period (it depends on past investment decisions)
- $Y_t = A_t F(K_t, N_t)$ or $Y_t = A_t F(K_t, Z_t N_t)$: Solow model production function
- $y_t = A_t f(k_t)$ or $\widehat{y}_t = A_t f(\widehat{k}_t)$ per worker or per efficiency units of workers production function
- $w_t = A_t F_N(K_t, N_t)$ and $R_t = A_t F_K(K_t, N_t)$: factors paid marginal products in Solow model, where $F_N(\cdot)$ and $F_K(\cdot)$ denote first partial derivatives with respect to arguments
- $k_{t+1} = sA_t f(k_t) + (1-\delta)k_t$: capital per worker accumulation equation
- $\widehat{k}_{t+1} = \frac{1}{(1+z)(1+n)}\left[sA_t f(\widehat{k}_t) + (1-\delta)\widehat{k}_t\right]$: capital per efficiency unit of labor accumulation equation
- $C_t + S_t = Y_t$ and $C_{t+1} = Y_{t+1} + (1+r_t)S_t$: the period t and $t+1$ flow budget constraints in the two period consumption model (imposing the terminal condition and assuming these hold with equality)
- $C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$: intertemporal budget constraint (IBC)
- $u'(C_t) = \beta(1+r_t)u'(C_{t+1})$: Euler equation
- $C_t = C^d(Y_t, Y_{t+1}, r_t)$: consumption function
- $C_t = \frac{1}{1+\beta}\left[Y_t + \frac{Y_{t+1}}{1+r_t}\right]$: consumption function in the two period model with log utility

Next are some questions for review. I will *not* provide answers to these questions, but am happy to discuss them in office hours.

1. Provide brief definitions (or, in some cases, a brief discussion) for the following terms:

    (i) Real vs. nominal variable
    (ii) Gross domestic product
    (iii) Income, expenditure, output
    (iv) Implicit price deflator
    (v) Representative agent
    (vi) Capital
    (vii) Constant returns to scale
    (viii) Diminishing returns
    (ix) Inada conditions
    (x) Steady State
    (xi) Golden rule
    (xii) Convergence
    (xiii) Dynamic inefficiency
    (xiv) Impulse response
    (xv) Terminal condition
    (xvi) Saving vs. savings
    (xvii) Utility function
    (xviii) Lifetime utility
    (xix) Discount factor
    (xx) Marginal rate of substitution
    (xxi) Euler equation
    (xxii) Indifference curve
    (xxiii) Budget line
    (xxiv) Endowment point
    (xxv) Income vs. substitution effect
    (xxvi) Consumption function
    (xxvii) Marginal propensity to consume (MPC)
    (xxviii) Random walk
    (xxix) Precautionary saving

2. Explain how the CPI and the GDP deflator price indexes are constructed. In the data, which of these yields the higher rate of inflation on average? Why does this make sense in light of the way in which they are constructed?

3. What have been the approximate annual growth rates of real GDP pre capita, nominal GDP, population, and inflation (as measured by the GDP deflator) over the last 50 years?

4. For what fraction of GDP do consumption, investment, and government spending typically account? Rank consumption and investment volatility relative to GDP volatility.

5. Which declines more during a recession: average hours worked (the intensive margin) or employment (the extensive margin)?

6. Write down the six stylized time series growth facts and the three cross-sectional stylized facts.

7. Consider the Solow model. A firm produces output according to $Y_t = AF(K_t, Z_tN_t)$, where $Z_t = (1+z)^t$, $N_t = (1+n)^t$, and $F(\cdot)$ is constant returns to scale. Assume that A is constant across time (hence no time subscript). The household consumes a fixed fraction of its income each period, equal to $(1-s)$. It invests the other fraction of its income, s, in new capital, with capital accumulating according to: $K_{t+1} = I_t + (1-\delta)K_t$.

   - Derive a capital accumulation equation relating K_{t+1} to K_t and exogenous variables and parameters.
   - Define $\widehat{k}_t = \frac{K_t}{Z_tN_t}$. Re-write the capital accumulation equation in terms of the redefined variable.
   - Graphically characterize the behavior of the economy, and argue that there exists a steady state in which $\widehat{k}_t = \widehat{k}_{t+1} = \widehat{k}^*$.
   - Graphically show what happens after a permanent surprise increase in s, assuming that the economy begins in a steady state. Trace out the dynamic responses (impulse responses) of capital per efficiency unit of labor, output per efficiency unit of labor, and consumption per efficiency unit of labor. If there is any ambiguity please state why. Also show impulse responses of capital per worker, output per worker, and consumption per worker.
   - Repeat this exercise, this time considering (separately) surprise, permanent increases in A, n, z, and δ.
   - Suppose that this economy sits in a steady state, but that a hurricane hits and destroys half of its capital stock. Show in the main diagram what happens, and trace out the dynamic responses of capital, consumption, and output (both in per efficiency unit terms as well as in per capita/worker terms).
   - Suppose that $F(K_t, Z_tN_t) = K_t^{\alpha}(Z_tN_t)^{1-\alpha}$. Derive an analytic expression for $\widehat{k}^*$.

- Continue to assume the Cobb-Douglas production structure. What will be the growth rate of K_t in the steady state in which $\widehat{k}_t$ is constant at $\widehat{k}^*$? Also derive the steady state growth rates of Y_t, C_t, R_t (equal to the marginal product of capital), and w_t (equal to the marginal product of labor). Comment on how well these results align with the stylized facts.
- Define the Golden Rule saving rate as the saving rate which maximizes steady state consumption per effective worker, $\widehat{c}_t = \frac{C_t}{Z_t N_t}$. For the Cobb-Douglas production function, find an expression for this Golden Rule saving rate.
- Is the Golden Rule saving rate affected by the level of A? Put differently, would the Golden rule saving rate be any different for a very rich economy (high A) relative to a very poor economy (low A)? Explain.

8. Discuss several different things that could influence total factor productivity, A in the notation of the problem above.

9. (adapted from Mankiw, *Macroeconomics*, 5th edition, problem 7.8) Consider how unemployment would affect the Solow model. Suppose that output is produced according to $Y_t = AK_t^{\alpha}((1-u)N_t)^{1-\alpha}$. $0 \leq u \leq 1$ is the "unemployment rate," or fraction of workers who are not engaged in productive activity each period. It is taken to be exogenous. Capital accumulates according to $K_{t+1} = I_t + (1-\delta)K_t$, and $I_t = sY_t$. There is no population or productivity growth.
   - Write the variables in per worker terms (i.e. $k_t = \frac{K_t}{N_t}$). Combine these equations to reduce the model to the standard Solow equation, expressing k_{t+1} as a function of k_t and exogenous variables and parameters.
   - Solve for the steady state levels of k^* and y^*. How does the u affect the steady states?
   - Suppose that a change in government policy permanent reduces u. In a diagram, show how this will affect the economy, both immediately and in a new steady state. Will the effect on y_t be largest immediately (in the period in which u declines) or in the "long run" (once we transition to a new steady state)?

10. Suppose that a household has the following economic problem:

$$\max_{C_t, S_t} \quad U = u(C_t) + \beta u(C_{t+1})$$

s.t.

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1 + r_t)S_t$$

   (a) Explain why there can be no positive or negative saving in the second period.

(b) Combine the two-within period budget constraints into one, and re-write the household problem as one of choosing C_t and C_{t+1} at time t (as opposed to choosing C_t and S_t)

(c) Use calculus to find the first order condition (or Euler equation) characterizing an optimal consumption plan. Provide some intuition for this condition

(d) Characterize the optimal consumption plan using an indifference-curve budget line diagram. Carefully label this diagram, being sure to note what the slopes of the indifference curve and budget lines are, as well as noting the horizontal and vertical axis intercepts of the budget line

(e) Graphically show what will happen to current and future consumption when Y_t increases

(f) Do the same for (separate) increases in Y_{t+1} and r_t. Discuss why there is some ambiguity with regards to the effect of the real interest rate on current consumption.

(g) Suppose that $u(C_t) = \ln C_t$. Use this, in conjunction with the Euler equation and the budget constraint to derive the consumption function. Calculate the partial derivatives of consumption with respect to Y_t, Y_{t+1}, and r_t.

11. Suppose that you have two different countries, $i = 1$ or $i = 2$, which are characterized by the Solow model. These countries are identical except for their saving rates. The central equation of the Solow model expresses capital per worker in period $t + 1$ as a function of capital per worker in period t. We assume a Cobb-Douglas production function and a constant value of A:

$$k_{i,t+1} = s_i A k_{i,t}^{\alpha} + (1 - \delta) k_{i,t}, \quad i = 1, 2$$

$k_{i,t}$ is capital per worker in country i. The countries have the same A, α, and δ. They only differ in their saving rates. Suppose that $A = 1$, $\alpha = 0.33$, and $\delta = 0.1$. Suppose that the saving rate in country 1 is $s_i = 0.2$. Suppose that both economies have converged to their steady states. Suppose that steady state output in country 1 is twice as big as in country 2, i.e. $y_1^*/y_2^* = 2$.

(a) What would the saving rate in country 2 have to be for this to be true?

(b) Calculate the steady state values of R in each economy with these saving rates (the saving rate given to you for country 1, and the saving rate for country 2 you solved for in (a)).

(c) If capital were mobile (i.e. people in country 1 could send their capital to country 2, and vice-versa), given your answer on (b), what do you think would happen? Would capital flow from country 1 to 2, from 2 to 1, or not at all? Why?

(d) Given that capital is fairly mobile in the 21st century, what would you conclude about the plausibility that different saving rates account for different standards of living across the globe?

(e) If it's not differences in s that account for different standards of living, what is the most plausible candidate?

12. Suppose a household has the following lifetime utility function:

$$U = C_t^{1/2} + \beta C_{t+1}^{1/2}$$

(a) Find expressions for the partial derivatives of lifetime utility, U, with respect to period t and period $t+1$ consumption. Is marginal utility of consumption in both periods always positive?

(b) Find expressions for the second derivatives of lifetime utility with respect to period t and $t+1$ consumption, i.e. $\frac{\partial^2 U}{\partial C_t^2}$ and $\frac{\partial^2 U}{\partial C_{t+1}^2}$. Are these second derivatives always negative for any positive values of period t and $t+1$ consumption?

(c) Derive an expression for the indifference curve associated with lifetime utility level U_0 (i.e. derive an expression for C_{t+1} as a function of U_0 and C_t). What is the slope of the indifference curve? How does the magnitude of the slope vary with the value of C_t?

(d) Suppose that the household faces two within period budget constraints of the form:

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1 + r_t)S_t$$

Combine the two period budget constraints into one intertemporal budget constraint.

(e) Use the intertemporal budget constraint and this utility function to derive the Euler equation characterizing an optimal consumption plan.

(f) Use this Euler equation and the intertemporal budget constraint to derive a consumption function expressing C_t as a function of Y_t, Y_{t+1}, and r_t

(g) Use your consumption function to find the partial derivatives of C_t with respect to Y_t, Y_{t+1}, and r_t.

(h) Is the MPC positive and less than 1? Does it depend on the value of r_t here?

(i) Is consumption decreasing in the real interest rate?

(j) Is consumption increasing in Y_{t+1}?

13. Suppose that you have a household which lives for *three* periods. Its lifetime utility is:

$$U = \ln C_t + \beta \ln C_{t+1} + \beta^2 \ln C_{t+2}, \quad 0 < \beta < 1$$

The utility flows between any two adjacent periods are discounted by β – i.e. the weight on utility flows from $t+2$ relative to $t+1$ is β, which means that the weight on utility flows in $t+2$ relative to t is β^2. The household faces a sequence of three flow budget constraints.

The household can save across periods. For simplicity, assume that $r_t = r_{t+1} = r$ (i.e. the real interest rate between t and $t+1$ is the same as the real interest rate between $t+1$ and $t+2$):

$$C_t + S_t = Y_t$$

$$C_{t+1} + S_{t+1} - S_t = Y_{t+1} + rS_t$$

$$C_{t+2} + S_{t+2} - S_{t+1} = Y_{t+2} + rS_{t+1}$$

(a) What will have to be true about S_{t+2} given preferences of both the household and the (unmodeled) financial intermediary? Explain briefly.

(b) Impose the terminal condition from (a) and combine the three flow budget constraints into one intertemporal budget constraint. Comment on how this is a natural extension of the intertemporal budget constraint in the two period case.

(c) Derive *two* Euler equations which are necessary for a solution to the household's problem. Again comment on how your answer is a natural extension of the two period case.

(d) Combine the Euler equations with the intertemporal budget constraint to derive the consumption function – i.e. an expression for C_t as a function of variables the household takes as given (Y_t, Y_{t+1}, Y_{t+2}, and r).

(e) What is the marginal propensity to consume (MPC) in this problem? How does the MPC compare to what the MPC would be for the same preference specification in a two period problem? Provide some intuition for your answer.

14. GLS, Chapter 5, Questions 6-7

15. GLS, Chapter 5, Exercise 1

16. GLS, Chapter 5, Exercise 3

17. GLS, Chapter 6, Exercise 1

18. GLS, Chapter 6, Exercise 2

19. GLS, Chapter 9, Questions 5-8

20. GLS, Chapter 9, Exercise 1

21. GLS, Chapter 9, Exercise 2

22. GLS, Chapter 9, Exercise 4